MODELING AND OPTIMIZATION OF MANUFACTURING LOT SIZE IN AGGREGATE PRODUCTION PLANNING UNDER DEMAND UNCERTAINTY: A CASE STUDY OF MOVIT PRODUCTS (U) LIMITED

BY

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DECLARATION

Declaration by the Candidate

This thesis is my original work and has not been presented for a degree in any other University or Institution of higher learning. Any similarities of the content herein and other sources are duly acknowledged. Not a part of this thesis should be replicated without the preceding written consent of the author and/or Moi University.

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DEDICATION

Dedicated to my family:

My husband: Deo Ssempijja

My children: Daniella Kwagala, Michelle Namata, Esther Mirembe and Daniel Muggaga, for their invariable love, patience, and support.

All glory and honor to the Lord God Almighty

ACKNOWLEDGEMENTS

Let me take this opportunity to show my thoughtful sense of gratitude to those that have helped me throughout my research work and therefore the writing of this thesis.

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May the Almighty God richly bless you and meet your heart’s desires.

ABSTRACT

Demand uncertainty is eminent in many manufacturing industries giving a task of establishing the optimum manufacturing lot sizes in the production planning, leading to overstocking or understocking of the finished products. Different methods can be used in the reduction of these complexities and among these is modelling demand uncertainty of the production planning problem. The main objective was to develop an optimization model that predicts optimal manufacturing lot size in production planning under demand uncertainty. Specific objectives were: to characterize the existing production planning system with respect to manufacturing lot sizes, to define & formulate the manufacturing lot size problem in production planning under demand uncertainty at Movit products (U) Ltd, and to develop the manufacturing lot-size model under demand uncertainty that predicts optimal manufacturing lot sizes and then validating it. Two approaches were applied: Markov chains to formulate the possible states of demand under the condition of uncertainty; and stochastic goal programming to determine the number of units to be produced considering the over-achievement or under-achievement of the manufacturing lot size priorities desired. Using a framework on quarterly basis, the study undertook a case study on a manufacturing facility that manufactures, distributes and sells skin care, hair & nail care products to apply the mathematical model developed and demonstrate the proposed decision-making framework. The priorities for the model were established, the objective function was defined and the goal constraints were formulated for each of the five products. The 'stochastic goal programming' model for 'manufacturing lot size' was then established for all the products. The developed model was solved using MATLABTM software where an optimal solution was obtained. Results from the study indicated optimal levels of manufacturing lot size $(X_{FF}, X_{FU}, X_{UF}, X_{UU})$ as demand changes from one state to another as 0, 2.3729, 0, 104.0840 for product A, 6.7720, 0, 0, 109.6800 for B, 0, 1.7602, 0, 181.8117 for C, 0, 369.4800, 0, 4975.1000 for D and 0, 13.3956, 0.6835, 6286.3000 for E. The under achievement was established as 8137.7000, 4555.6000, 12103.0000, 5478.7000, 56.2688 for products A, B, C, D, and E respectively and there was no over-achievement for all the products in a quarter of the year. The model was validated giving optimal results for aggregate production planning of the products (manufacturing lot size as 0, 182.02, 0, and 2.8341 and under achievement 4546.38). All the objectives in this study were achieved and an optimization model that predicts optimal manufacturing lot size in production planning (PP) under demand uncertainty was developed. In conclusion, the production planning system was characterized as a batch and make-to-stock strategy with standardization of product and process sequence. The manufacturing lot size was defined and formulated as determining the optimal manufacturing lot size minimizing the total production cost. Varying demand was then modeled as a two-state Markov chain where the optimality was state-dependent and then validated. The study recommends the adoption of the stochastic goal programming model to assist manufacturing companies that operate under demand uncertainty to accurately project production levels in order to sustain demand.

TABLE OF CONTENTS

DECLARATION ii
DEDICATION iii
ACKNOWLEDGEMENTS iv
ABSTRACT v
TABLE OF CONTENTS vi
LIST OF TABLES x
LIST OF FIGURES xii
ACRONYMS AND ABBREVIATIONS xiii
CHAPTER ONE: INTRODUCTION 1
1.1 Background of the Study 1
1.2 Statement of the Problem 4
1.3 Objectives of the Study 5
  1.3.1 General objective 5
  1.3.2 Specific objectives 5
1.4 Research Questions 6
1.5 Justification of the study 6
1.6 Significance of the Study 7
1.7 Scope 7
CHAPTER TWO: LITERATURE REVIEW 8
2.1 Introduction 8
2.2 Uncertainties in Production Environments 8
2.3 Production Planning 10
2.4 Optimization of Manufacturing Lot-Sizes 15
  2.4.1 Manufacturing lot-sizing concept 16
  2.4.2 Manufacturing lot sizing variants 17
  2.4.3 Solution to manufacturing lot-sizing problems 20
2.5 Markov chains 21
  2.5.1 Markov Chain Model States 23
  2.5.2 Stochastic Process 24
  2.5.3 Transition Probability Matrix 25
2.6 Stochastic Goal Programming (SGP) 26
  2.6.1 Stochastic Programming (SP) 27

2.6.2 Goal Programming (GP) 29
2.7 Mathematical Modelling & Optimization 32
2.7.1 Optimization 33
2.8 Conceptual Framework 34
2.9 Theory 36
2.10 Literature Review Gap 36
2.11 Model Validation 37
2.11.1 In-sample and Out-of-sample validation 40
CHAPTER THREE: METHODOLOGY 43
3.1 Introduction 43
3.2 Research Design 43
3.3 Research Instruments 43
3.3.1 Data gathering sheets 44
3.3.2 Interview guide 44
3.4 Study Population 44
3.5 Data Coding 44
3.6 Sampling Procedures 45
3.7 Data Collection 47
3.8 Data Analysis 48
3.8.1 Quantitative Data Analysis 48
3.8.2 Qualitative Data Analysis 48
3.9 Ethical Considerations 49
3.10 Model development flow chart 49
3.11 Mathematical Model Formulation 50
3.12 Goal Programming 54
3.13 Model Validation 55
3.14 Methodology Limitations 56
CHAPTER FOUR: RESULTS AND DISCUSSIONS 57
4.1 Introduction 57
4.2 Characterization of the existing PP system at Movit Products (U) Ltd 57
4.3 Data classification 60
4.4 “State” transitions and “on-hand” inventory 71
4.5 Demand Transition Probabilities 74
4.6 Demand, Inventory & Production-Inventory Cost Matrix 81

4.6.1 Demand matrix..81
4.6.2 Inventory matrix...82
4.6.3 Production-inventory cost matrix 83
4.7 Expected Demand..87
4.8 Expected Inventory..89
4.9 Expected Production-Inventory Costs 91
4.10 Expected Manufacturing Lot Size 94
4.11 Stochastic Goal Programming Model...96
4.12 Stochastic Goal Programming Model for Manufacturing Lot Size..................101
4.13 Results of the stochastic goal programming model 105
4.14 Discussion of Results..107
4.15 Model validation..123
4.16 Discussion..137
CHAPTER FIVE: CONCLUSION AND RECOMMENDATIONS 139
5.1 Conclusion 139
5.2 Scientific Contribution..140
5.3 Recommendations..140
5.3.1 Further Research 141
REFERENCES 142
APPENDICES 151
Appendix 1, Paper 1: "Application of Markov chains in manufacturing systems: A review" 151
Appendix 2, Paper 2: "Manufacturing Lot Size Optimization under Demand Uncertainty: A Stochastic Goal Programming Approach" 166
Appendix 3, Paper 3: "Multi-Objective Optimization of Manufacturing Lot Size under Stochastic Demand" 183
Appendix 4, Paper 4: "A Review on Stochastic Goal Programming Approach in Production Planning in Manufacturing"..199
Appendix 5: Plagiarism Digital Receipt 209
Appendix 6: Similarity Index Report...210
Appendix 7: Data Collected Tools Used 211
Appendix 8: Data collected for fast moving products at MOVIT products (U) Ltd 217
Appendix 9: Photos at MOVIT products (U) Ltd during data collection.............219

Appendix 10: MATLAB Solutions..222

LIST OF TABLES

Table 3.1: Research Data Coding 45
Table 3.2: Sample size at the factory 47
Table 3.3: "Key notations" used in the Markov model 51
Table 4.1: "Data classification" by state of demand for product A 61
Table 4.2: 'Overstocking' & "understocking" with holding & shortage costs for product A 61
Table 4.3: "Data classification" by state of demand for product B 63
Table 4.4: 'Overstocking' & "understocking" with holding & shortage costs for product B 64
Table 4.5: "Data classification" by state of demand for product C 65
Table 4.6: 'Overstocking' & "understocking" with holding & shortage costs for product C 66
Table 4.7: "Data classification" by state of demand for product D 67
Table 4.8: 'Overstocking' & "understocking" with holding & shortage costs for product D 68
Table 4.9: "Data classification" by state of demand for product E 69
Table 4.10: 'Overstocking' & "understocking" with holding & shortage costs for product E 70
Table 4.11: Average 'on-hand' inventory for product A 71
Table 4.12: Average "on-hand" inventory for product B 72
Table 4.13: Average "on-hand" inventory for product C 72
Table 4.14: Average "on-hand" inventory for product D 73
Table 4.15: Average "on-hand" inventory for product E 73
Table 4.16: Summary of Average "on-hand" inventory and "state" transitions of *products* 74
Table 4.17: Data "classification" by 'state' transition for product A 75
Table 4.18: Data "classification" by 'state' transition for product B 76
Table 4.19: Data "classification" by "state" transition for product C 77
Table 4.20: Data "classification" by "state" transition for product D 78
Table 4.21: Data "classification" by "state" transition for product E 79
Table 4.22: Demand transition matrix 80
Table 4.23: Demand, Inventory and Production-inventory cost matrices 87

Table 4.24: Expected “demand, Inventory, production-inventory” cost & manufacturing lot size 96

Table 4.25: Optimum result from MATLAB 106

Table 4.26: Expected goal values for product A, “stochastic” solution with “over & under” achievement 110

Table 4.27: Expected goal values for product B, “stochastic” solution with “over & under” achievement 113

Table 4.28: Expected goal values for product C, “stochastic solution with over & under” achievement 117

Table 4.29: Expected goal values for product D, “stochastic solution with over & under” achievement 120

Table 4.30: Expected goal values for product E, “stochastic” solution with “over & under” achievement 123

Table 4.31: Data “classification” by state of demand 124

Table 4.32: “Overstocking & understocking” with holding and “shortage” costs for product Y 124

Table 4.33: Average “on-hand” inventory 126

Table 4.34: Data classification by state-transition 127

Table 4.35: Optimal solution from MATLAB 133

Table 4.36: Expected goal values for product Y, “stochastic solution with over & under achievement” 136

LIST OF FIGURES

Figure 1.1: Some manufacturing companies in Uganda including the case study 4
Figure 1.2: Fluctuations and demand uncertainties at MOVIT products (U) Ltd 5
Figure 2.1: Types of demand classification(Systems & Ziukov, 2015) 9
Figure 2.2: Concept design of Enterprise Resource Planning (Elbahri et al., 2019) ...11
Figure 2.3: APP relationship with other types of PPC activities(Jamalnia et al., 2019) 14
Figure 2.4: Markov Chain Diagram (Leigh et al., 2017) 23
Figure 2.5: Conceptual framework of modeling and optimization of manufacturing lot-size...35
Figure 2.6: Diagram of modelling 11 data points showing an overfit (Approaches to model validation by Sally Hunton) 42
Figure 3.1: Model development flow chart...49
Figure 4.1: Batch production manufacturing system at Movit products (U) limited...58
Figure 4.2: Make to stock 59
Figure 4.3: Overstocking & understocking of product A 62
Figure 4.4: Holding & Shortage costs of product A 62
Figure 4.5: Overstocking & understocking of product B 64
Figure 4.6: Holding & Shortage costs of product B 65
Figure 4.7: Overstocking & understocking of product C 66
Figure 4.8: Holding & Shortage costs of product C 67
Figure 4.9: Overstocking & understocking of product D 68
Figure 4.10: Holding & Shortage costs of product D..69
Figure 4.11: Overstocking & understocking of product E...70
Figure 4.12: Holding & Shortage costs of product E...71
Figure 4.13: Average 'on-hand' inventory and 'state' transitions...............................74
Figure 4.14: Manufacturing lot size with over or under achievement in each state transition 106
Figure 4.15: Overstocking & understocking of product Y 125
Figure 4.16: Holding & Shortage costs of product Y 125
Figure 4.17: Average "on-hand" inventory and state "transitions" 126
Figure 4.18: Manufacturing lot size with over or under achievement in each state transition 133

ACRONYMS AND ABBREVIATIONS

APP	Aggregate Production Planning
APP	Aggregate Production Planning
BOM	Bill of Materials
EOQ	Economic Order Quantity
EPQ	Economic Production Quantity
ERP	Enterprise Resource Planning
GP	Goal Programming
JIT	Just-In-Time
MLSP	Manufacturing Lot-Sizing Problem
MPS	Master Production Schedule
MRO	Maintenance Repair and Operations
MRP II	Manufacturing Resource Planning
MRP	Material Requirements Planning
PP	Production Planning
SGP	Goal Programming
VMI	Vendor Managed Inventory

CHAPTER ONE: INTRODUCTION

1.1 Background of the Study

Engineering companies are ceaselessly attempting to seek out productivity to defeat the challenges associated with the market dynamics. Production planning is a necessary component in improving the general manufacturing system's performance, especially if the system is working under uncertainty. One in every of the corporate varieties of instability that characterizes production situations is instability in product demand. Instability may well be a state of inadequate data, and this may well be seen in three forms: inexactness, unreliability, and border with ignorance (Mula et al., 2006). It is in this manner imperative that these uncertain parameters be considered when generating a sturdy production plan inside the production planning process, since once ignored, production effectiveness and the performance of the organization are going to be affected (Kazemi Zanjani et al., 2010).

Manufacturing organizations set up their production plans upheld outside requests with the center point of deciding the amount to be created given each period whereas fulfilling the stress and reducing the overall costs (Masmoudi et al., 2017).

In manufacturing, when creating production plans, making the correct choices concerning the lot-size is greatly critical since it straightforwardly impacts the framework execution and efficiency (Mohammadi & Tap, 2012) and this will be key for any engineering firm that wishes to compete inside the market. As usually frequently complex however as imperative, it's been exceedingly examined in spite of the fact that, there's still a spot around appearing the commitments to clarify the appropriateness of these strategies utilized concerning each sensibly fundamental manufacturing setting (with respect to varieties in demand and peaks of regularity) (Florim et al., 2019).

Many studies have been conducted by researchers over the years to solve the production lot sizing and scheduling problem using a range of techniques having the classical Economic Order Quantity model marking the beginning of research on lot-sizing problems (Erlenkotter, 2014). The Economic Order Quantity (EOQ), one of the foremost recognizable inventory models up to now, was first introduced by Ford W. Harris in 1913 and has inspired a range of fixed- quantity extensions. Since then, the sector has grown exponentially to incorporate fixed-interval models, zero-inventory models, Just-In-Time (JIT) models, and Vendor Managed Inventory (VMI), among others (Mosca et al., 2019).

However, most manufacturing companies have not implemented production planning approaches to mitigate the negative effects of uncertainties in demand.

In Uganda for instance, traditional approaches to production planning under demand uncertainty are largely based on rules of the thumb and are somehow unrealistic as it leads to overstocking or understocking (Bollapragada & Rao, 2006) which is costly on either side. In the light of this uncertainties characterized by the present market environment call for a dynamic new approach; for instance in case a manufacturing company produces a higher quantity of a product than it can sell, then there is an added holding cost per unit meaning it is less profitable to produce too many units, but too few also means lost sales (Mubiru, 2013). In order to accommodate the reality of constantly changing demand, production planners frequently need to put together sets of unconnected static models to ensure proper levels of production lot sizes.

Since demand varies over time, there has to be adjustment in production planning as it is difficult to precisely forecast demand at the most detailed level of the end products. The marketing plan must be revised at regular intervals and, as a consequence, the production plan as well (Graves, 2011).

In addition, an organization's overall operations over a planning horizon can be scheduled to satisfy demand while minimizing costs using an Aggregate Production Plan (APP). Optimizing the APP problem implies minimizing the cost over a finite planning horizon which can be done by adjusting production load as well as inventory and employment levels over a certain period of time to achieve the lowest cost while satisfying demand (Fahimnia et al., 2005). This is a very important tool in production decision making because the concept gives a basis for experimental investigation which provides information in less time and less cost.

Nevertheless, the use of Markov chains and Stochastic goal programming have not been exhaustively applied to model PP due to demand uncertainty, especially in the case of Ugandan manufacturing sector. In this regard, developing a suitable mathematical model putting into consideration the possible uncertainties relating to the manufacturing lot sizing problem in production planning into a reasonable framework.

This research therefore focuses on a multi-period, multi-product, manufacturing lot size problem under demand uncertainty. This was achieved by developing realistic mathematical models using Markov chains to formulate the possible states of demand under the condition of demand uncertainty. This study further, uses stochastic goal programming techniques to determine the number of units to be produced as demand changes from state to state so that the total production cost and inventory is minimized.

This study undertook a case study on a manufacturing facility that makes, disperses and offers skin care, hair & nail care products (Movit Products (U) Limited) to test the mathematical model and thereafter, validation was done to support the proposed model framework.

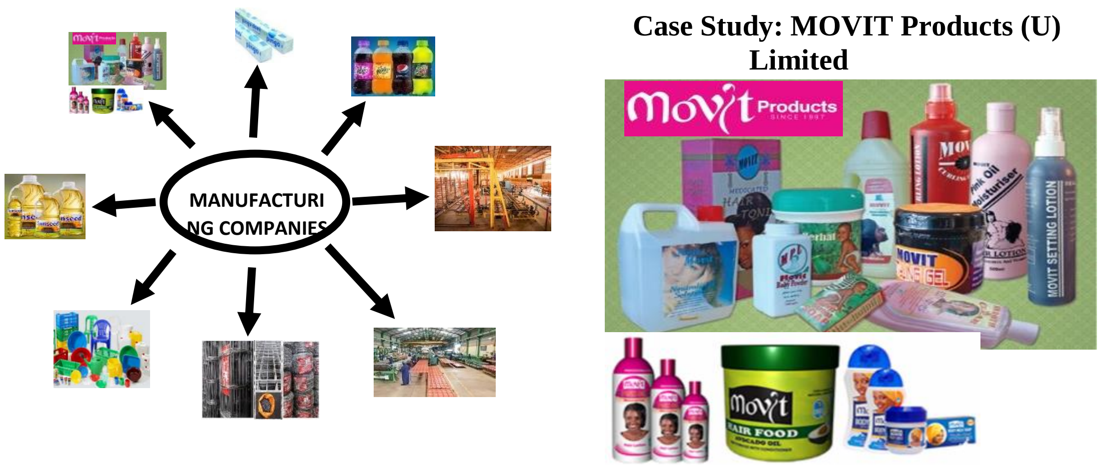

Figure 1.1: Some manufacturing companies in Uganda including the case study

Figure 1.1 shows some of the manufacturing companies or industries in Uganda ranging from plastics, steel, soft drinks, etc all these dealing with products with demand uncertainty; singling out the Movit products (U) limited which was used as the case study for this research.

1.2 Statement of the Problem

Characterized by the fluctuations and demand uncertainties, many manufacturing companies in Uganda continuously are faced with the challenge of establishing the optimal 'manufacturing lot sizes' in production planning systems. This is costly as it leads to overstocking (holding costs) or understocking (lost sales) of the finished manufactured products.

A realistic optimization model is highly sought to assist manufacturing industries in production planning of items with demand uncertainty.

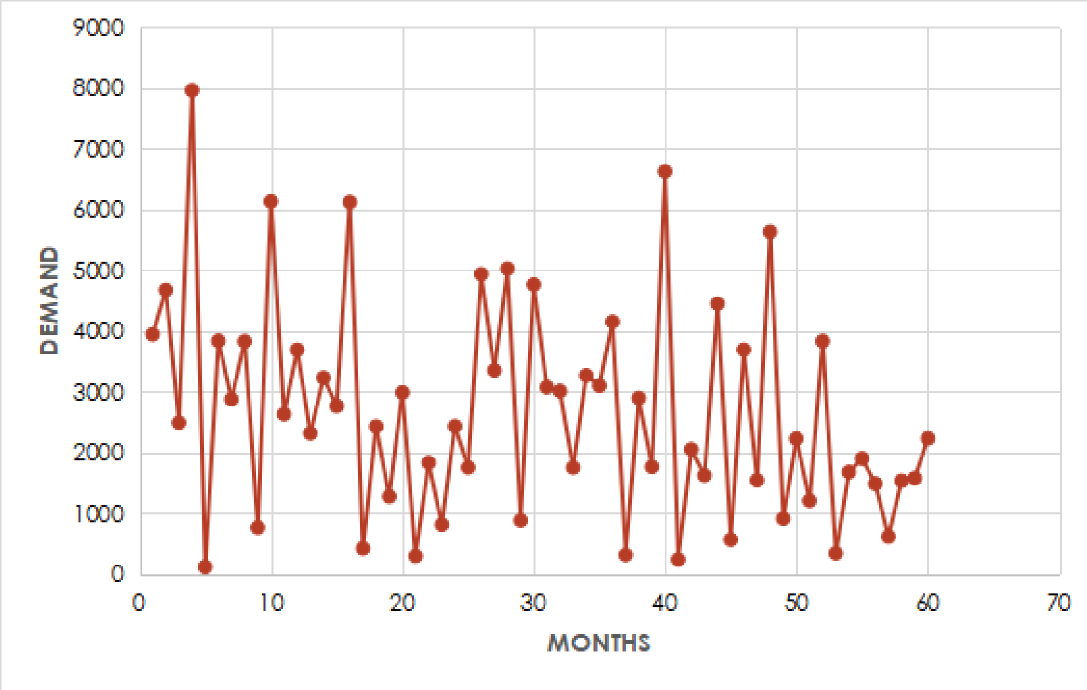

Figure 1.2: Fluctuations and demand uncertainties at MOVIT products (U) Ltd

Figure 1.2 clearly shows the demand uncertainties and fluctuations at Movit products (U) limited and this gives a basis or a need for optimizing the manufacturing lot sizes.

1.3 Objectives of the Study

1.3.1 General objective

To develop an optimization model that predicts optimal manufacturing lot size in production planning (PP) under demand uncertainty.

1.3.2 Specific objectives

1. To characterize the existing production planning system with respect to manufacturing lot sizes at Movit products (U) Ltd
2. Define and formulate the manufacturing lot size problem in PP under demand uncertainty at Movit products(U) Ltd

3. Develop the manufacturing lot-size model (multi-product) under demand uncertainty that predicts optimal manufacturing lot sizes
4. Validate the developed manufacturing lot-size model in part 3 above

1.4 Research Questions

1. What are the distinct nature and features of the production planning system with respect to manufacturing lot sizes at Movit products (U) Ltd?
2. What is the objective function, decision variables, parameters, dependent & independent variables and constraints?
3. What are the optimal manufacturing lot-sizes for finished products under demand uncertainty?
4. What are the optimal manufacturing lot-sizes for finished products under model validation?

1.5 Justification of the study

Preliminary findings have shown that total production costs in terms of; cost of production, holding cost and cost of shortage; constitute the biggest percentage on the total operating costs at Movit products (U) Ltd.

Meanwhile, it has been established that for any manufacturing industry to excel, optimizing manufacturing lot-size as a cost minimization strategy is very important especially for products with stochastic demand. This implies that in today's industrial competitiveness, it is crucial to maintain optimal manufacturing lot-sizes or else manufacturing companies can make losses in the long run. The results of this study therefore provide a model and data that helps in determining the optimal manufacturing lot size at minimum total production and inventory cost, under demand uncertainty.

1.6 Significance of the Study

The research has developed a mathematical model which is optimizing the manufacturing lot size in the production planning considering uncertainties in demand, in effect, overstocking or understocking of products is eliminated as a cost minimization strategy. The model will help in establishing optimal manufacturing lot-sizes that can sustain random demand occurrences; raising interest and awareness to manufacturers, companies and policy makers engaged in production planning and manufacturing lot size of products with demand uncertainty.

1.7 Scope

The research concentrated on modeling and optimization of manufacturing lot-size at Movit products (U) Limited (mainly production department). This research focused on a multi-period, multi-product, lot-sizing system under demand uncertainty.

The study was conducted at Zana-Bunamwaya movit road off Entebbe road Kampala-Uganda, where the main factory is located.

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

This chapter reviewed literature on uncertainties in production environments, production planning and control, optimization of manufacturing lot-sizes, Markov chains, stochastic goal programming and mathematical modelling & optimization.

2.2 Uncertainties in Production Environments

Uncertainties spring an inevitable concern allied with an incessant operation of the manufacturing system. Uncertainty is described as the variance between the amount of data required to carry out a given task and the amount of data already available. It is a condition of insufficient data and this data can be inexact, unreliable, and border with ignorance (Mula et al., 2006).

The forms of uncertainty that affect production processes range from environmental uncertainty to system uncertainty. Environmental uncertainty is described as that which comprises uncertainties outside the production process, including demand and supply uncertainty whereas System uncertainty is described as the one that is associated with uncertainties within the production process, like operation yield uncertainty, quality uncertainty, production lead time uncertainty and failure of production system (Ramaraj, 2017).

Uncertainty in product demand is a common type of uncertainty that characterizes production environments. Manufacturing companies using cost analyses are challenged with the uncertainties of the product demand, because it may impact the manufacturing framework execution subsequently the extreme choice on utilizing the production framework at the starting stages (Vafadar et al., 2017).

Once evaluating the danger associated with a choice, it is of great importance to understand these instabilities and their effects, as they can make it challenging to foresee performance (Assid et al., 2019). The aggregate demand for any product usually comes from a variety of separate customers and therefore the organization has little or no real control over who buys their products or what number buy and in what quantities. Random fluctuations within the number and size of order provides a variable and uncertain demand overall.

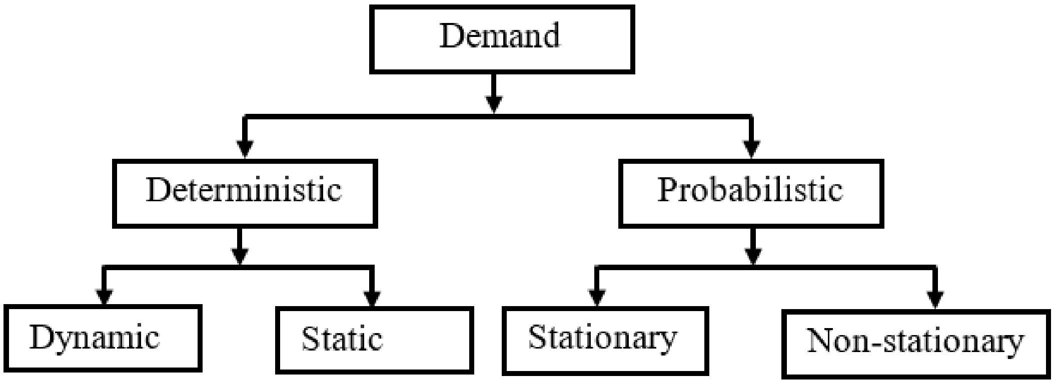

Figure 2.1: Types of demand classification(Systems & Ziukov, 2015)

Types of demand can be categorized as it is shown above in Figure 2.1. Deterministic demand is exactly known, unlike the probabilistic demand which is not known exactly. Deterministic demand can be of two types and one of them is static, which does not have any variation. The amount of demand known or can be computed with certainty. The second type is dynamic, which may vary. This type of demand varies with time, but the way in which the demand varies is known with certainty.

Probabilistic demand can be of two types, that is, stationary and non-stationary. Stationary distribution is with known parameters. This type of demand follows a probability distribution that is known or estimated from historical data. Commonly used distributions include normal, gamma, Poisson.

Non-stationary probabilistic demand type of demand behaves like a random walk that evolves over time, with regular changes in its direction and rate of growth or decline.

2.3 Production Planning

Production planning is that the column of any manufacturing process, having the focal aim of establishing the sum of items to be mass-produced bearing in mind the amount of inventory to be moved from one period to a different. All this can be done through with the target to reduce both the overall “costs of production” and also the stock, meeting the customers’ request (Olanrele et al., 2014).

Often, production planning problems are categorized in line with the ranking framework of strategic, tactical and operational deciding activities (Erenay et al., 2015). Production planning selects the most optimal use of resources that are used in production in a way that satisfies the essential requirements for a given period of time, which is the planning horizon.(Abubakar Yusuf Baba, 2019).

Production planning decisions involve determining the order of the product families, and the manufacturing lot sizes for the items within each product family, having the objective to minimize the total cost. During the planning horizon, demand forecasts and forecast revisions are then considered (Sethi et al., 2002).

Material Requirements Planning (MRP) is one of the most broadly used production planning systems and it aims at converting the Master Production Schedule (MPS) into a production plan for the products with their components.

Based on the demand forecasts over a time horizon, inventory levels, the Bill of Materials (BOM) of products, and lead times, the MRP generates a production plan by determining the quantities and the schedule for the products to be manufactured, to meet the deadlines established by the demand. This technique, however, has got some restrictions since it assumes that there is no capacity constraint, which was later addressed with the introduction of Manufacturing Resource Planning (MRP II) and Enterprise Resource Planning (ERP).

Also, it does not consider the minimum production costs for each production lot, that is, the production cost, inventory cost, and setup cost and yet these limitations are precisely the focus of lot sizing. (Florim et al., 2019).

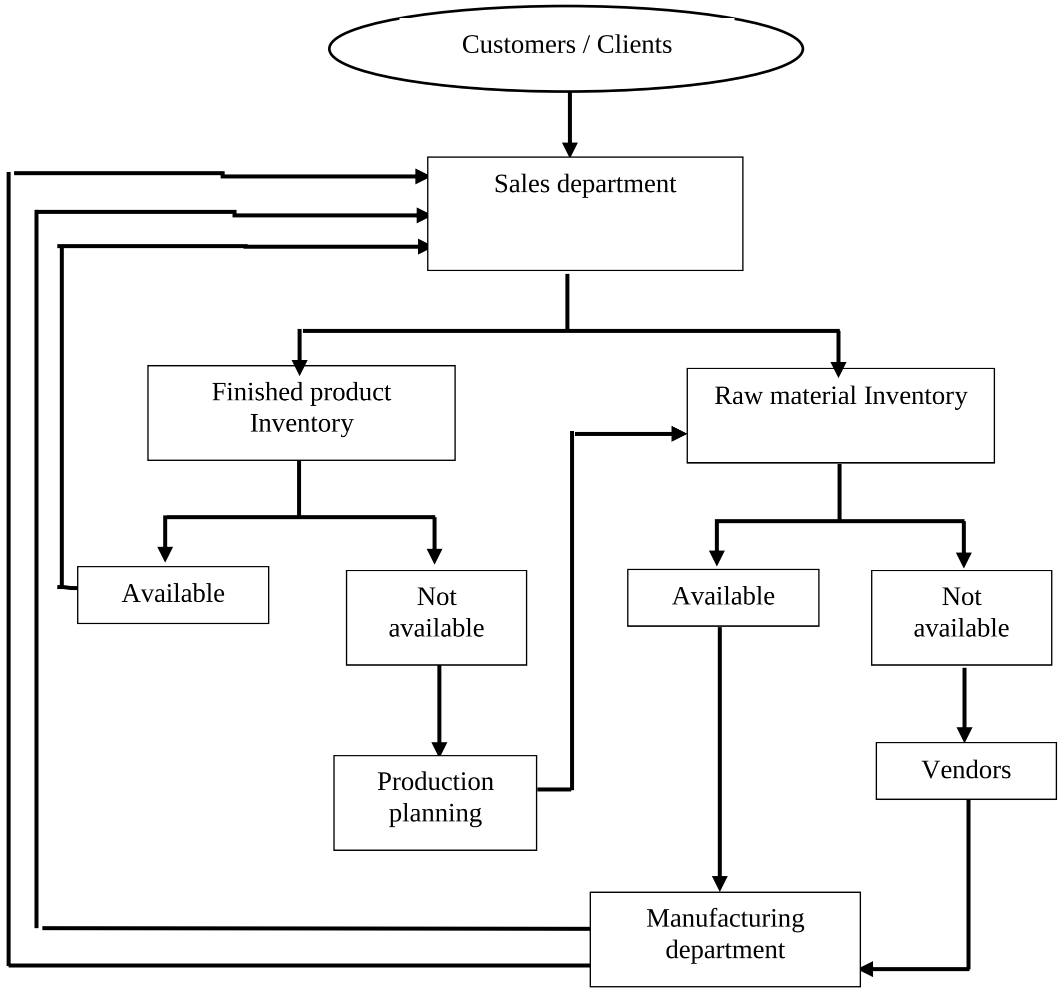

Figure 2.2: Concept design of Enterprise Resource Planning (Elbahri et al., 2019)

Figure 2.2 illustrates the concept design of Enterprise Resource Planning where it starts from the clients or customers to the manufacturing department. It shows the relationship between the sales department, inventory section, production planning and the manufacturing department aiding on how much should be produced optimumly.

The stochastic nature of the demand for the manufactured products makes production planning complex (Naeem et al., 2013).

In the production planning model of the final product, where customer orders need to be managed, the whole problem can be decomposed at the inventory, where it is in control for balancing the process changes between the assembly and machining (Gyulai et al., 2017). In order to optimize production planning, it is essential to establish the whole optimization model based on different level constraints (Wen et al., 2017).

Three essential aspects must be put into consideration in a production plan in a multi-product environment (Juan Alejandro et al., 2013):

1. The type of products to schedule
2. The number of products to produce
3. The time to make the products

Garee et al., 2020 showed innovative ways of dealing with uncertainty in production planning using modern methods in the field of operations research which improved the classical methods and provided valuable information to production managers about accurate predictive models that improve profitability (Garre et al., 2020). Aggregate production planning is considered an appropriate approach as it’s an important tactical level planning in a production management system, usually considered based on some parameters with uncertain values in many manufacturing environments. Aggregate production planning is tactical medium-term capacity planning over a 3 to 18-months planning horizon which determines the optimal production volumes for each planning period (Gholamian et al., 2015).

The purpose of APP is:

1. Determining overall level of each product category to meet fluctuating and uncertain demand in near future,
2. Adopting decisions and policies in regard to hiring, lay off, overtime, backorder, subcontracting, inventory level and available production resources.

Demand patterns that are Seasonal, randomness inherent in quantity and timing of received orders, all make the production planning system uncertain, which then commends applying a decision modelling tool that takes into account of these uncertainties. Due to the dynamic nature of the production planning and the unstable states of real world manufacturing environments, the deterministic models for production planning would lead to unrealistic decisions (Jamalnia et al., 2019).

Figure 2.3 summaries the position of aggregate production planning among other types of production planning and control techniques showing their interconnected relationships.

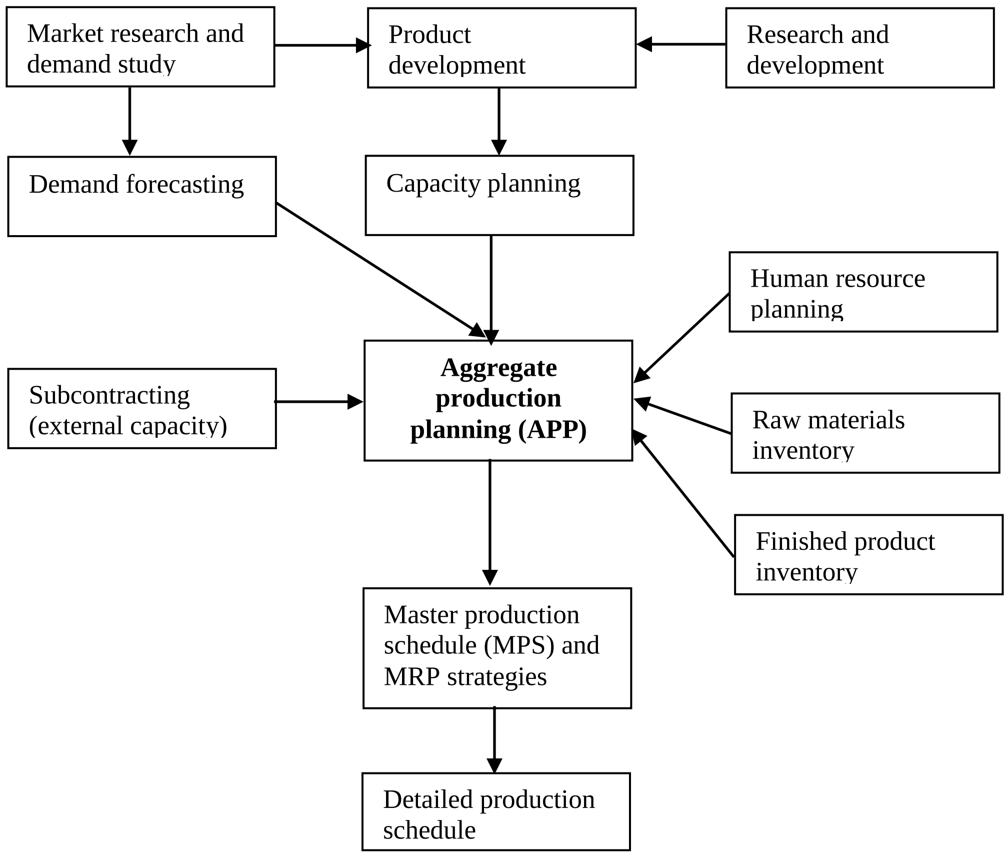

Figure 2.3: APP relationship with other types of PPC activities(Jamalnia et al., 2019)

Figure 2.3 summaries the Aggregate Production Planning position among other types of production planning and control techniques, and their interconnected relationships from a holistic perspective. As it can be seen from the Figure 2.3, in the hierarchy of production planning and control activities,

Aggregate Production Planning falls between long-term strategic planning decisions such as new product development and short term shop floor scheduling practices. “The forecasted demand acts as the driving force of the APP system.

Seasonal demand patterns together with unpredictability inherent in quantity and timing of received orders makes the whole Aggregate Production Planning system uncertain, which in turn recommends utilizing a decision-modelling tool that takes into account of these uncertainties. As such, due to the dynamic nature of Aggregate

Production Planning and instable states of real world industrial environments, the deterministic models for Aggregate Production Planning would lead to unrobust decisions.

Moreover, similar to other production planning family members, Aggregate Production Planning also involves several objectives or criteria in practice".

The role of the production planning layer of the supply chain planning matrix is to transform the orders of the customers into production orders through solving the lot-sizing problems matching the order stream with available capacities, resulting into a production plan (Maxim A. Bushuev, Alfred Guiffrida, M.Y. Jaber, 2015).

2.4 Optimization of Manufacturing Lot-Sizes

Optimization has ended up a standard marvel in most organizations and foundations. Optimization is the method of finding (action of choosing (Afteni & Frumuşanu, 2017) the finest conceivable arrangement to a given issue by looking at a few choices (surveyed after a predefined model) (Ejaz et al., 2019).

This may be done by altering the inputs to or characteristics of an instrument, operation, or try to see the least or greatest yield (Elsheikhi, 2017). The optimization problem contains three fundamental parameters that has need to be considered, that's, the target function, a set of factors, and a bunch of imperatives (Yusoff et al., 2011).

The objective of the "optimization model" depends on definite features of the organization, termed as factors or unknowns to define the values of these factors that enhance the target task, while these factors are regularly limited, or compelled in a way or the opposite. Brahimi et al. classified optimization issues into four groups:

“process planning, layout design, re-configurability and planning, and, scheduling” (Brahimi et al., 2019).

2.4.1 Manufacturing lot-sizing concept

Lot sizing is one of the most common problems in production planning. The number of products that is processed on a production system without interruptions from the processing of other products is known as a lot. One of the key tasks of production planning and control is to determine lot sizes in production areas. Lot sizes need to be determined every time more than one product is to be manufactured on a single resource and setups are required. (Schmidt et al., 2015).

All the “lot sizing” issues have developed an instantaneous outcome on the performance of the system & the efficiency. In the event that a production company desires to compete favorably inside the market, it is a need to form the proper choices in ‘lot-sizing’ issues and this will be a very serious choice for any producer.

The determination of the number of a particular product which is required to be produced during a stated period of time is described as the manufacturing Lot sizing. The narrative of the Manufacturing Lot-Sizing Problem (MLSP) looks at the production and inventory considering their features and decision variables similarly as their effect on the service level. The objective function for the Manufacturing Lot-Sizing Problem is usually to minimize the total cost, that is, the sum of the total holding costs, the stock-out costs, and the other costs which affect the operation of each system (Juan Alejandro et al., 2013).

Usually, lot sizing models are classified (depending on the decision horizon and level of aggregation) as;

1. Tactical models (yearly master production schedule),
2. Operational models (sequencing and loading), and
3. Models between operational and tactical models (monthly lot sizing) (Erenay et al., 2015).

The manufacturing ‘Lot sizing’ issues are generally allied with the capable ‘production planning’ of a particular item. For every production plan, the chief challenge is to establish the manufacturing lot size for every item.

Lot allocation concerns need to be resolved, so as to own proficient production planning, but built on the “demand” that has to be attained & therefore the obtainability of stock reducing the “production costs” by establishing the optimum production amount (Badri et al., 2020).

The more the ‘manufacturing lot size’ is small, the more the amount the “holding” cost is reduced but raising the “ordering” cost while the greater the “manufacturing lot size”, the bigger the “holding” cost but then decreasing the “ordering” cost. Built on the theories of “lean production”, it’s desirable to possess lesser lot size because it avoids the buildup of the stock that arises with “management and holding” costs. The lot-size commended by a mathematical “manufacturing lot size” model will be the most effective because it caters for the balance concerning the prices there in. (Mohammadi & Tap, 2012)

2.4.2 Manufacturing lot sizing variants

Manufacturing Lot sizing decisions ascertain how and when a product should be produced, as well as the optimal level of production, where by the relevant costs (like holding cost and set up) are reduced to an optimal level (Abubakar Yusuf Baba, 2019).

The following are the lot sizing variants:

1. The planning horizon

   This describes the overall length of time in which the production schedule has to be determined. If the demand does not change from period to period then the problem is static and this usually assumes an infinite planning horizon whereas if the demand changes from period to period, the problem is called dynamic (Abubakar Yusuf Baba, 2019).

   The planning horizon is distinguished as either finite or infinite, where by infinite planning horizon is often used with a stationary demand assumption and finite planning horizon is associated usually with dynamic demand (Curcio, 2017).

2. Number of Products

   This is another variant that directly influences the problem complexity. The complexity increases as the number of products increases.

3. Number of levels

   Two main types of production system are considered in lot-sizing problems, that is, single level and multi-level. In single-level lot-sizing only one action is required, only independent demand products are considered and intermediate products are not accounted for producing final products.

   Multi-level lot-sizing problems involves various stages, intermediate items have to be produced in order to be used to produce final products. Single-level lot-sizing problems are simpler and less complex to solve than multi-level lot-sizing problems (Abubakar Yusuf Baba, 2019)(Curcio, 2017).

4. Demand

When demand is static, it means that the demand is equal for any period considered whereas if demand is dynamic, the demand can vary from period to period, but it is known in advance. Random demand is not known in advance and its value is uncertain. Demand can also be categorized as dependent and independent where independent demand refers to the external demand or customer demand for final products and dependent demand is defined by the final products demand and the requirements of intermediate products to produce final products (Abubakar Yusuf Baba, 2019).

Demand uncertainty is addressed in the manufacturing lot sizing model with the intention of bringing in a more realistic perspective to this problem.

5. Set-up cost and time.

Setup costs and times are either sequence independent or sequence dependent. In sequence independent the setup decision of the preceding period does not influence the setup time and costs of the subsequent period whereas in sequence dependent the setup decision of the preceding period influences the setup time and costs of the subsequent period. In sequence dependent the setup costs, different set up times and costs are incurred for different production sequences. There is setup carry-over when the product produced in the previous period is produced in the current period and this happens in both cases and no additional setup is required. Sequence dependent setups are more complex computationally than the sequence independent setups.

6. Capacity constraints

   The capacity of the production system can be limited by resources like manpower, machines and equipment. The lot sizing environment can be characterized by capacitated or un-capacitated problems. An uncapacitated lot-sizing problem is when capacity is not taken into account whereas if capacity is considered, the problem is capacitated. The complexity of the lot sizing problems increases directly by capacitated resources.

7. Backlogging cost

   These are costs that are incurred in circumstances where there are shortages (when the product demand cannot be met on the due date). Backlogging costs increase with the number of periods being late.

2.4.3 Solution to manufacturing lot-sizing problems

Lot-sizing decisions aid the manufacturer in determining the quantity and time to produce a product at a minimum cost which is vital in production planning. The productivity and efficiency of a manufacturing system totally depend on the right choice of lot-sizes hence, developing and improving solution approaches for lot-sizing problems is crucial.

The solution approaches of lot sizing problems include three main areas, that is, Exact methods (useful in exploring the underlying difficulties in solving the lot-sizing problems), Heuristic methods (A strategy designed for solving a problem more quickly when classic methods are too slow, or for finding an approximate solution when classic methods fail to find an exact solution.), and Metaheuristic methods (are not problem-specific but use the domain-specific knowledge in the form of problem-specific heuristics that are controlled by the upper level strategy) (Chowdhury, 2018).

2.5 Markov Chains

Markov chain, known as & after a ‘Russian mathematician Andrey Markov’ in 1907, can be a capable numerical instrument that’s utilized broadly to capture the model of frameworks transitioning in the midst of diverse states (Ye et al., 2019). Markov chains were known quickly for his or her noteworthy control of exemplification and their plausibility of modeling a large variety of actual world issues additionally to the standard of execution directories they produce (Gingu & Zapciu, 2017). When manufacturing frameworks uncover a few irregular behavior, ‘Markov chains’ are often accustomed to carry out execution assessment and modeling (Boteanu & Zapciu, 2017).

A Markov chain, uncommon sort of theoretical account (‘with a Markov property’ (Kiassat et al., 2014)), may be a ‘discrete-time stochastic model’ characterized on a range of states, prepared with ‘transition probabilities’ from one state to a different at the following time stage (Nop et al., 2020).

‘Markov Chains’ have uncovered their quality at modeling ‘stochastic transitions’, from revealing successive designs to straightforwardly modeling choice forms processes (He & McAuley, 2017).

These have gotten uncommon property that probabilities including how the method will advance within the future depend only on the this state of the method, and then are ‘independent of events’ within the past (Ju et al., 2019).

A Markov process may be a theoretical account that fulfills the ‘Markovian property’ (says that the chance of any “future event,” given any “past event” and therefore the “present state X_{t-I}”, is self-governing of the ‘past event’ and influenced only upon the current state (Doubleday & Esunge, 2011), (Otieno et al., 2015)). It’s a sequence of random variables $X_1, X_2, X_3, \dots X_n$ with the Markovian property, namely that, given this state, the long run and past state is independent. Formally,

$$P_r\left(X_{n+r} = \frac{x}{X_1}, = x_1, X_2 = x_2, \dots X_n = x_n\right) = P_r\left(X_{n+r} = \frac{x}{X_n}, = x_n\right), \dots\dots\dots \; 2.1$$

if both conditional probabilities are defined, i.e. if $P_r(X_1 = x_1, \dots X_n = x_n) > 0$ the possible values of X_n form a countable set S called the state space of the chain (Tochukwu & Hyacinth, 2015).

Markov Chains often described by a sequence of directed graphs, where the edges of the graph n labeled by the probabilities of going from one state at time n to another state at time$(\mathrm{n} + 1)$,

$$P_r\left(X_{n+r} = \frac{x}{X_n}, = x_n\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots \; 2.2$$

Nonetheless, ‘Markov Chains’ adopts ‘time-homogenous’ situations, within which case the chart and matrix are self-governing of n and not obtainable as orders (Tochukwu & Hyacinth, 2015).

Markov chains model ‘discrete-time processes’ and Markov processes models ‘continuous-time processes’. They arithmetically model a procedure by presenting how the technique could shift amid dissimilar phases and also the ‘probability’ of generating these ‘transitions’. Markov’s analysis may be characterized diagrammatically as in figure 2.4 which demonstrates a ‘Markov chain model’ of a procedure with two phases A1 and A2, where the ‘probability’ of creating a ‘transition’ from stage *i* to stage *j* is q_{ij} (Leigh et al., 2017).

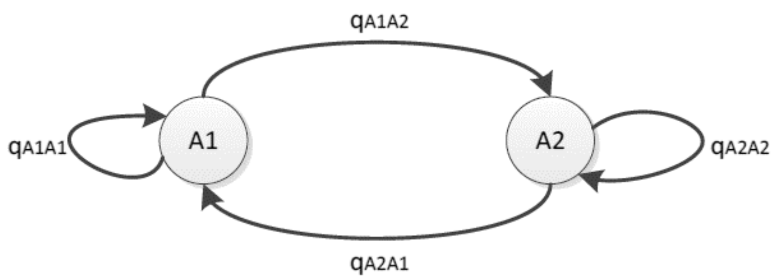

Figure 2.4: Markov Chain Diagram (Leigh et al., 2017)

Figure 2.4 shows a Markov chain model of a process with 2 stages A1 and A2 where the probability of making a transition from stage i to stage j is given by qij.

2.5.1 Markov Chain Model States

The ‘Markov chain’ model may be a chronological procedure which involves numerous stages. For the phases reflected as ‘Markov Chain’ states, they must reverence all the subsequent 3 situations:

1. “State i communicates itself”
2. “If state i communicates with state j, then j communicates with state i.”
3. “If state i communicates with state j, and j communicates with state k, then i communicates with state k.”

Based on (Tochukwu & Hyacinth, 2015), the probability of going from state i to state j in n time steps is given by: $\mathrm{P}_{ij}^{(n)} = P_r\left(X_n = \frac{j}{X_0}, = \mathrm{i}\right)$ and the single step transition is $P_{ij} = P_r\left(X_1 = \frac{j}{X_0}, = \mathrm{i}\right)$

In a ‘time-homogenous’ Markov Chain, the probability is: $\mathrm{P}_{ij}^{(n)} = P_r\left(X_{n+k} = \frac{j}{X_k}, = \mathrm{i}\right)$ and $P_{ij} = P_r\left(X_{k+1} = \frac{j}{X_k}, = \mathrm{i}\right)$. A Markov Chain of order m, where m is finite, is a process satisfying

$$P_r\left(X_n = \frac{x_n}{X_{n-1}}, = x_{n-1}, X_{n+2} = x_{n-2}, \dots X_1 = x_1\right) = P_r\left(X_n = \frac{x_n}{X_{n-1}}, = x_{n-1}, X_{n+2} = x_{n-2}, \dots X_{n-m} = x_{n-m}\right) for\ n > m \quad \dots 2.3$$

In other words, the future state depends on the past m states. It is possible to construct a Chain Y_n from X_n which has the 'classical' Markov property by taking as state-space the ordered m tuples of x values, i.e. $Y_n = (X_n, X_{n-1}, \ldots, X_{n-m+1})$ (Tochukwu & Hyacinth, 2015).

2.5.2 Stochastic Process

Efficient ways of taking uncertainties into account, and to achieve more robust solutions are either applying stochastic models (Naeem et al., 2013) (e.g., by estimating the underlying stochastic processes), or using adaptive and cooperative approaches, which allow prompt responses to changes and disturbances (Gyulai et al., 2017).

A stochastic process is a mathematical model that evolves over time in probabilistic manner (Saad et al., 2014). A stochastic process is a random process (Gingu & Zapciu, 2017), that is, a change in the state of some system over time whose course depends on chance and for which the probability of a particular course is defined. Essentially it is a family of random variables, X (t): t Є T defined on a given probability space, indexed by the time variable t, where t varies over an index set T (Otieno et al., 2015).

A stochastic process may be continuous or discrete. A stochastic process is said to be a discrete time process if set T is finite or countable. That is, if T= (0, 1, 2, 3 ,4…….. , n) resulting in the time process X(0), X(1), X(2), X(3), X(4), …….., X(n) , recorded at time 0,1,2,3,4……,n respectively. On the other hand stochastic processes X (t): t Є T is considered a continuous time process if T is not finite or countable. That is, if T= [0, ∞) or T= [0, k] for some value k.

A state space S is the set of states that a stochastic process can be in. The states can be finite or countable hence the state space S is discrete, that is S=1, 2, 3…, N. Otherwise the space S is continuous (Doubleday & Esunge, 2011).

2.5.3 Transition Probability Matrix

These are 'conditional' probabilities P (X t+1 = j/ X t =i} = P_{ij} organized in the way of a n x n matrix termed as the 'transition probability matrix' given by:

$$\begin{pmatrix} p11 & p12... & p1n \\ p21 & p22... & p2n \\ pn1 & pn2... & pnn \end{pmatrix}$$ which can be denoted as P = P_{ij}

The 'transition' matrix demonstrations the probability of shifting amid the 'row' phase to the 'column' phase. In order to create a 'Markov chain' model the transition probabilities are essential & are solved by the equation 2.4 that defines the probability of creating a 'transition' from stage *i* to stage *j*, and is denoted by P_{ij}. In the equation, m is the overall sum of 'transitions' and n_{ij} is the amount of 'transitions' from *i* to *j* (Leigh et al., 2017).

$$p_{ij} = \frac{n_{ij}}{\sum_{k=1}^{m} n_{i,k}} \quad \text{......................................} \quad 2.4$$

The 'transition probability' matrix has the following properties: (Otieno et al., 2015)

1. $P_{ij} > 0$ used for all i and j.
2. Considering all i and j, addition of the component in every 'row' is equivalent to 1. The addition denotes the overall 'probability' of transition from state i to 'itself' or a different one.
3. The crosswise component signifies transition from one state to same state.

Markov Chain 'models' are beneficial in learning the advancement of organizations over recurrent trials. These recurrent 'trials' are frequently consecutive time periods in that the state of the organization in a specific period can't be established by certainty.

Reasonably, 'transition' probabilities may be applied in describing the manner in which the scheme creates transitions from one period to the next. It aids us in determining the probability of the scheme existing in a specific state at a certain period of time (Vasanthi et al., 2011).

2.6 Stochastic Goal Programming (SGP)

"Stochastic Goal Programming" may be a "multi-criteria" assessment support model which delivers "satisficing" results to a linear structure given an uncertainty situation from the usually probable function perspective (Bravo & Gonzalez, 2009), (Ballestero, 2005).

Contini, presented the primary design of 'Stochastic Goal Programming' in 1968, bearing in mind goals as chance 'variables' possessing numerical distribution & and recommended a model putting under consideration that the expansion of the probability that the choice fits to a vicinity adjoining the random goal. This model persuades an answer that's close to the "random goal" the maximum amount as conceivable (Aouni & La Torre, 2010). The standard design of the 'SGP' model is as follows:

$$\max f(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 2.5$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}i \; (for\ i = 1, 2, \ldots.., p) \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.6$$

$$\underline{x} \geq 0$$

Where $\underline{x}$ signifies an "n-dimensional random vector" of the choice variables, a_{ij} denotes a mxn matrix A of "deterministic coefficients" and $\tilde{b}i$ signifies an "m-dimensional vector" $\underline{b}$ "(stochastic) resource limitations".

Almost real-life 'optimization' issues comprises of numerous imprecise data approximations & goals, contradictory standards. In such circumstances, the "stochastic goal programming" technique proposes a logical structural support in modelling and resolving such issues. "Stochastic goal programming" may accommodate the intrinsic uncertainty and has remained practical in several arenas comprising 'Portfolio selection' (Jones & Tamiz, 2010), 'project selection, resource allocation, Healthcare management' (Li et al., 2014), (Attari & Jami, 2018), transportation (Yang, 2007), marketing (Aouni et al., 2012), cash management (Salas-Molina et al., 2020), wealth management (Kim et al., 2020), economic development, energy consumption, workforce allocation, and greenhouse gas emissions (Jayaraman et al., 2017), forest planning (Eyvindson & Kangas, 2014). Not many applications of "stochastic goal programming" in "production planning" in manufacturing systems are witnessed.

2.6.1 Stochastic Programming (SP)

Stochastic programming is a technique used for modelling optimization problems that involve uncertainty (Gorissen et al., 2015), (Smith & Furse, 2014) ("find an optimal decision in problems involving uncertain data" (Wilson et al., 2011)). "Stochastic programming" models are usually used to optimize probable values or as recursive decisions are required. (Curcio, 2017)

The objective of stochastic programming is to find a policy that:

(i) Is feasible for all or almost all the possible realizations of uncertain parameters of a model, and that

(ii) Optimizes the expectation of some function of the decisions and the random variables. (Xiang, 2019)

Stochastic programming provides a functional tool in which a wide variety of sources of uncertainty can be incorporated into the development of the production plans (Eyvindson & Kangas, 2014). In stochastic programming, uncertainty is modeled through a probability distribution (Salas-Molina et al., 2020).

The fundamental idea behind stochastic programming is its ability to take correct action after the realization of a scenario has taken place (Nejadi, 2016). A number of uses of financial planning and control" (Wilson et al., 2011), portfolio management (Ji et al., 2005). For example 'deterministic' optimization issues may be articulated with well-known factors, real-life issues contain unknown factors at the time a call is formed.

'Stochastic programming' is also practical during a very situation in which a special choice must be taken. The highest mostly practical & deliberated "stochastic programming" models are "two-stage" (linear) series whereby the choice maker performances inside the primary phase, afterwards which a random incident happens disturbing the results of the 'first-stage' choice (Smith & Furse, 2014). The basic stochastic programming problem is:

$$minimize\ F_O(X) = Ef_o(x, w) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 2.7$$

Subject to: $F_i(X) = Ef_i(x, w) \leq 0, i = 1, \ldots\ldots\ldots, m \quad \ldots\ldots\ldots\ldots\ldots\ 2.8$

Here the adjustable value is x, problematic data are f_i , circulation of w If $f_i(x, w)$ are convex in x for every w, F_i are convex henceforth ‘stochastic programming’ issue is convex.

2.6.2 Goal Programming (GP)

Goal programming is a branch of multi-objective optimization (Huang et al., 2017) a generalization of linear programming that strives to reach predefined targets for a set of goals (satisficing philosophy) rather than an optimal solution subject to strict constraints (optimizing philosophy) (Eyvindson & Kangas, 2014). “Abraham Charnes and William W Cooper” presented the primary design of “goal programming” scattering its attraction to present periods (Ballestero, 2001), (Abdelaziz et al., 2007), (Aouni et al., 2014), (Jones & Tamiz, 2010), (Jayaraman et al., 2017).

Goal programming is a sequential optimization procedure that solves multi-objective decision problems whereby it uses priority level of goals rather than importance weights of. Goal programming sequentially optimizes the objectives starting from the highest priority goal and the goals with lower priorities utilize the remaining resources after optimizing the higher goals. The goal programming approach uses input parameters that are simple and more intuitive.(Kim et al., 2020). Goal programming is useful for decision-makers to consider several objectives simultaneously and identify a set of acceptable solutions.(Li et al., 2014).

The goal programming (GP) model is an aggregating procedure and takes into account simultaneously many objectives which can be conflicting whereby the obtained solution represents the best compromise that can be made by the decision marker. (Aouni & La Torre, 2010). This best compromise minimizes the sum of deviations between the achievement and aspiration levels (or targets) of the goals (Aouni et al., 2012). Here the results of the most effective cooperation decreases the overall nonconformities amid the 'achievement' $f_i(x)$ and ambition levels g_i (Aouni et al., 2012).

Goals are overseen by the choice-makers' viewpoint (and may differ with time based on other correlated aspects) (Hop, 2017) , follows a satisfying reason taken by means of objectives and he raises the value of the idea of setting objectives & therefore being openly involved within the growth of additional answers (Eyvindson & Kangas, 2014).

The GP approach may provide more realistic approach because in real application the conflict objectives are needed to consider in the same problem and yet it is impossible to optimize these objectives simultaneously (Abidin Çil et al., 2016).

The underlying idea behind goal programming is that the decision-maker follows a satisfying logic expressed by means of targets. By establishing an achievement objective function, goal programming aims to conciliate the achievement of a set of goals instead of optimizing every goal (Salas-Molina et al., 2020).

Goal programming combines a number of purposes to induce the outcome that reduces in entirety the nonconformities between "achievement" & the "aspiration" levels of the goals. It's crucial to stipulate for every goal g_i, the "aspiration" level or objective $G_i \in R$, with i = 1; 2; : : : ; q presenting positive & negative nonconformity supplementary variables to subordinate goal attainment and aims (Salas-Molina et al., 2020). In terms of important "distance metric", the "goal programming" types are "lexicographic, weighted" (Salas-Molina et al., 2020), (Iskander, 2007), & "Chebyshev" (min-max) goal programming (Britt, 2016) and in terms of the arithmetical nature of the choice variables or aims used are "fuzzy, integer, binary, and fractional" goal programming (Jones & Tamiz, 2010). In "weighted goal programming", every goal is multiplied by a "weight" allocated to that, and also the general "objective" function (archimedian sum of all these), is reduced.

In "lexicographic goal programming", the goals are allocated priorities, and then graded by priority from uppermost to lowermost, then the primary goal is reduced by itself, and a restriction is about after the "optimization" to forestall the following optimization from gaining an inferior outcome, and finally, the process is recurrent for all of the goals. In "min-max goal programming", the most variance amid any goal and its objective is reduced (Britt, 2016). Just like that of a "linear programming model", issue is modelled into a "goal programming model" within the same way, but, the goal programming model has numerous & often opposing incommensurable goals, in a very exact priority order (recognized by positioning or considering numerous goals in accordance with their rank) (Hussain & Kim, 2020).

In the literature, GP models are typically used to (Jayaraman et al., 2017):

1. Determine the required resources to achieve a desired set of goals,
2. Determine the degree of attainment of the goals with the available resources,
3. Provide the best satisfying solution under resource constraints, goal priorities and uncertainties.

The popularity of goal programming is due, in part, to the fact that it is easy to understand and the fact that it easy to apply since it constitutes an extension of linear mathematical programming for which very effective solving algorithms are available (Yahia-Berrouiguet & Tissourassi, 2015)

The broad design of 'goal programming' contains the converting of 'multi-objective programming' as (Ben Abdelaziz et al., 2009):

$$Optimize\ f_i(x) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.9$$

Subject to P1

$$x \in A \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.10$$

In the following form:

$$\min \sum_{i=1}^{n} w_i(\delta_i^+ + \delta_i^+) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.11$$

Subject to

$$f_i(x) + \delta_i^- - \delta_i^+ = \hat{f}_i \quad i = 1,\ldots,n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.12$$

$$x \in A \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.13$$

$$\delta_i^- \text{ and } \delta_i^+ \geq 0 \quad i = 1,\ldots,n$$

Where $f_i(x)$ is the goal function i; $\hat{f}_i$ is the target level of objective i; δ_i^- and δ_i^+ are the negative and positive deviations respectively associated with the objective i from its target; w_i is the weight assigned to the objective i, and A is the set of feasible solutions or system constraints.

2.7 Mathematical Modelling & Optimization

Mathematical modelling is the process of describing real world problems as mathematical equations and using some approaches to solve the mathematical equations as a guide to deconstructing and solving the original problem (Ejaz et al., 2019).

‘Mathematical modelling’ experience means the capability to classify pertinent queries, variables, dealings or rules in a specified real-life condition, to interpret these into arithmetic. This then interprets and authenticate the answer of the subsequent mathematical issue in relation to the specified condition, as well as the capability to examine or associate specified models by scrutinizing the expectations being made, inspecting properties and scope of a specified model (Frejd, 2014).

2.7.1 Optimization

Optimization is the process of finding the best possible solution to a given problem by examining several alternatives. (Ejaz et al., 2019). This may be through regulating the ‘inputs’ to or ‘characteristics’ of a tool, calculation, or trial to seek out the least or greatest yield/result (Elsheikhi, 2017). An optimization model consists of maximizing or minimizing an objective function by systematically choosing input values from within a set that stratifies some constraints and computing the value of the function.

The optimization issue encompasses three elementary factors which need to be well-thought-out, that is, the “objective function”, a group of “variables”, and a group of “constraints” (Yusoff et al., 2011).

The main task of the ‘optimization’ model hinges on specific features of the organization, termed as variables having the goal of defining the values of these variables which enhance the target function, although the variables are frequently limited, or controlled in a technique or the opposite. Brahimi et al. clustered optimization issues in 4 classes: “process planning, layout design, re-configurability and planning, and, scheduling”. In the foundation, “multi-objective optimization” initially advanced from areas comprising “economic equilibrium and welfare theories, game theories, and pure mathematics. Consequently, many terms and fundamental ideas stem from these fields” (Marler & Arora, 2004).

The process identifying objective, variables, and constraints for a given problem is known as modeling or problem formulation. The mathematical formulation can be written by having the following (Elsheikhi, 2017):

1. Defining of the variables (X_1, X_2, X_3... X_i).
2. Objective function: $y=f(x_1, x_2,\ldots\ldots, x_i)$, where f is the objective function as a function of x that needs to be maximized, minimized, or any target value.
3. Identifying the constraints: X_i >, <, =, ≤ or ≥ a certain value based on the nature of the problem

In order to compute optimal or near optimal lot-sizes which minimize total cost, a mathematical model that balances holding costs, ordering costs, and purchasing costs must be used. The mathematical model must also consider safety stock as it reduces the shortage probability in uncertain demand conditions (Mohammadi & Tap, 2012)

2.8 Conceptual Framework

The conceptual framework of modeling and optimizing of the manufacturing lot-size in PP under demand uncertainty consists of inputs, process and outputs, with elements of interest at each stage as shown in figure 2.5. Varying demand was modeled by means of a Markov chain with state transition matrix and optimality of the manufacturing lot size is state-dependent.

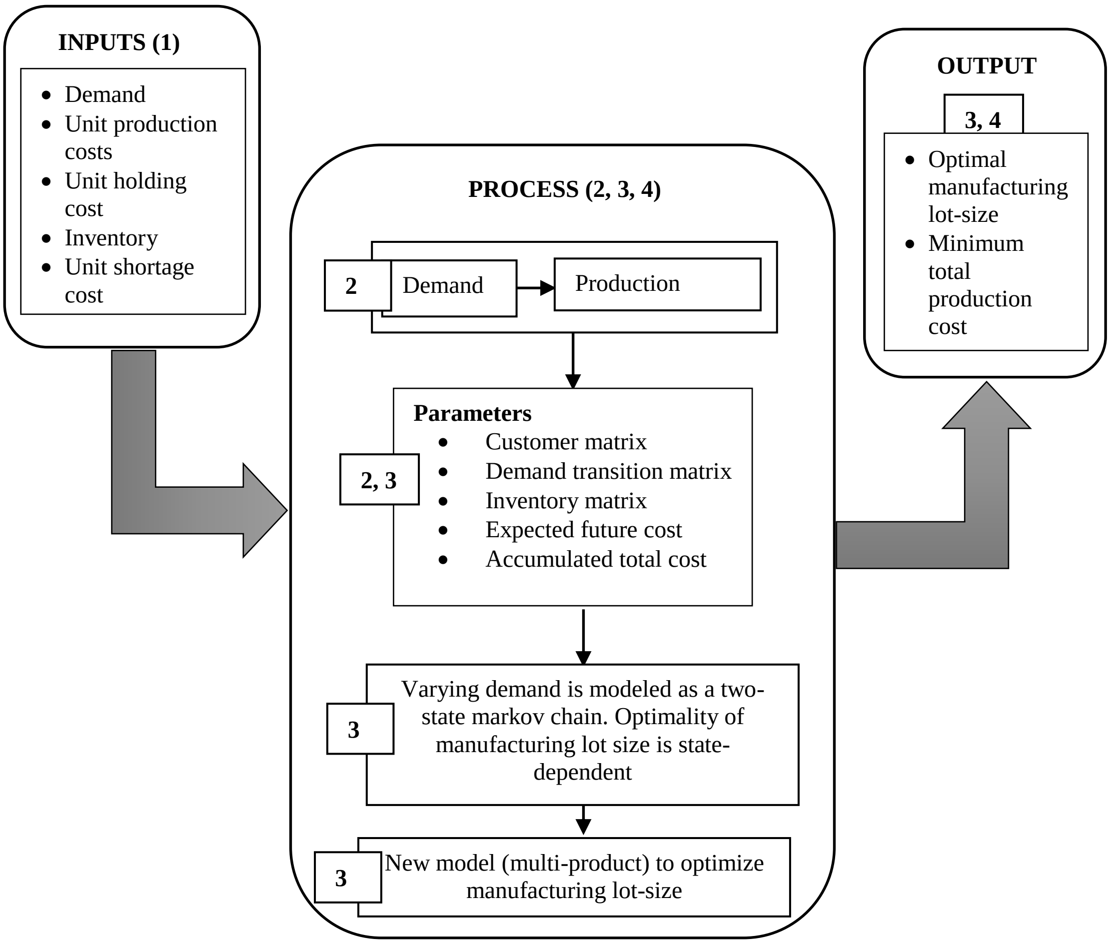

Figure 2.5: Conceptual framework of modeling and optimization of manufacturing lot-size

Figure 2.5 represents the three main stages in this conceptual framework, that is, the input, process and output. It also shows the relationships among these stages and how they relate to each other to obtain the main objective of this study. At the input stage, we have got the demand, unit production costs, unit holding costs, inventory and unit shortage costs.

All this is fed into the process, where production depends on the demand and inventory. The inputs also are used in developing the customer matrix, demand transition matrix, inventory matrix, expected future cost and accumulated total cost. Then Varying demand is modeled as a two-state markov chain and the optimality of manufacturing lot size is state-dependent. A new model optimizing the manufacturing lot size is developed giving the output stage as the optimal manufacturing lot size with minimum total production cost.

2.9 Theory

The theory to be adopted in this study will be the optimal control theory. Optimal control theory is a branch of applied mathematics that deals with finding a control law for a dynamical system over a period of time such that an objective function is optimized. This theory has as its objective the maximization of the return from, or the minimization of the cost of, the operation of physical, social, and economic processes (Weber, 2013), (Lin et al., 2010), (Ghosh, 1987), (Livesey, 1986), (Cherruault & Gallego Medimat, 1985), (Levine, 1972), (Gilbert, 1967). Based on this, in this work the interest was to apply Optimal Control approaches to problems in APP with dynamics in manufacturing lot size levels taking into account the stochastic nature of demand.

2.10 Literature Review Gap

The knowledge gap identified revealed that in the current studies;

- Demand was assumed to be the same for all periods and this could be invalid in different cases, and needs to be taken into account in relevant (models not considering the dynamic nature of demand (consider only the static nature) (Davizón et al., 2015)).
- APP models (Jamalnia et al., 2019), dealt with controlling the in-process inventory in the manufacturing system, that is, doesn’t deal with finished product inventory (Azarskov et al., 2017),
- Models not addressing uncertain events (demand) and their influence on the optimality of the aggregate production planning (Fahimnia et al., 2005) and ,

- Stochastic goal programming is not a common approach to Model development for APP

From some of the literature that has been reviewed in relationship to APP under uncertainty, some of the methods applied include mathematical modeling (Fahimnia et al., 2005), (Azarskov et al., 2017) and statistical analysis (Erenay et al., 2015).

Stochastic mathematical programming, Fuzzy mathematical programming, simulation (Jamalnia et al., 2019), continuous time Markov chain (Yan & Kulkarni, 2008), simulation-based optimization, stochastic programming (Rahdar et al., 2018), linear decision rule, transportation model, dynamic programming, lot sizing model and linear programming (Davizón et al., 2015) are other methods applied.

2.11 Model Validation

Results from decisions that are made in a highly uncertain environment might not be valid by the time the decisions are to be implemented. Decision making might also be happening in a time window through which the new circumstances might occur, whereby it requires the decision to be revised or adapted.

The goal of a model is to make predictions about data. Model validation determines whether the trained model is trustworthy and benefits in discovering more errors, scalability, reducing the costs, flexibility and enhancing the model quality.

Model validation is the task of demonstrating that the model is a reasonable representation of the actual system “that it reproduces system behavior with enough fidelity to satisfy analysis objectives” (Tsioptsias et al., 2016). Model validation must ascertain whether the assumptions which have been made are reasonable with respect to the real system. Model validation is influenced by the objectives of the performance study(Zimmerman, 2000).

In most models, three separate aspects are considered during the process of model validation and this includes, assumptions, input parameter values & distributions and output values & conclusions.

Nonetheless, in practice it may be problematic to attain such a full validation of the model, specifically if the system that is being modelled does not exist yet.

Generally, initial validation attempts will concentrate on the output of the model, and only if that validation suggests a problem will more detailed validation be undertaken.

Generally, there are three approaches to model validation and any combination of them may be applied as appropriate to the different aspects of a particular model. These are, expert intuition, real system measurements and theoretical results/analysis (Pieschacon, 2019).

(i) Expert intuition

This is similar to the use of 'one-step analysis' during model verification. Here, however, the examination of the model should preferably be led by someone other than the modeler, an "expert" with respect to the system, rather than with respect to the model.

'This might be the system designer, service engineers or marketing staff, depending on the stage of the system within its life-cycle'.

'Careful inspection of the model output, and model behavior, will be assisted by one-step analysis, tracing and animation, in the case of simulation models, and the full steady state representation of the state space in the case of Markovian models'. In either case, a model may be fully instrumented, meaning that every possible performance measure is extracted from the model for validation purposes regardless of the objectives of the performance study.

(ii) Real system measurements

Comparison with a real system is the most reliable and preferred way to validate a simulation model. In practice, however, this is often infeasible either because the real system does not exist or because the measurements would be too expensive to carry out. Assumptions, input values, output values, workloads, configurations and system behaviour should all be compared with those observed in the real world. In the case of simulation models, when full measurement data is available it may be possible to use trace-driven simulation to observe the model under exactly the same conditions as the real system(Yin & McKay, 2018).

(iii)Theoretical results or analysis

In the case of detailed Markovian models or simulation models it is sometimes possible to use a more abstract representation of the system to provide a crude validation of the model. In particular, if the results of an operational analysis, based on the operational laws coincide with model output it may be taken as evidence that the model behaves correctly.

Another possible use for the operational laws is to check consistency within a set of results extracted from a simulation model. If a model is behaving correctly we would expect the measures extracted during the evolution of a model to obey the operational laws provided the usual assumptions hold. Failure of the operational laws would suggest that further investigation into the detailed behaviour of the model was necessary. For example, the general residence time law can provide us with a simple validation of the model output values obtained for residence times at individual components if we know their respective visit counts, jobs behave homogeneously and we expect the model to be job flow balanced.

At slightly more detail a simulation model may also be validated by comparing its output with a simple queueing network model of the same system, and conversely, (in academic work) Markovian models are often validated by comparing their outcome with that of a more detailed simulation model.

Validation of models against the results or behaviour of other models is a technique which should be used with care as both may be invalid in the sense that they both may not represent the behaviour of the real system accurately.

Another analytic approach is to determine invariants which must hold in every state of the system. For example, these invariants might capture a mutual exclusion condition or a conservation of work condition. Showing that the model always satisfies such an invariant is one way of increasing confidence in the model, and providing support for its validity. The disadvantage of such an approach is that it can be computationally expensive to carry out the necessary checks regularly within a model.

2.11.1 In-sample and Out-of-sample validation

1) In-sample validation

In-sample validation looks at the “goodness of fit” and is concerned with how well the model fits the data that it has been trained on: its “goodness of fit”.

In-sample validation examines the model fit and is particularly helpful if one is interested in what the current data tell us, e.g., about the relationships between the variables modelled and their effect sizes. By looking at the model coefficients and associated uncertainties of the model one could discover interesting associations, like which types of customer were more likely to respond positively to the campaign (i.e. buy something).

2) Out-of-sample validation

Out-of-sample testing looks at a model’s “predictive performance” and refers to using “new” data which is not found in the dataset used to build the model. This is often considered the best method for testing how good the model is for predicting results on unseen new data: its “predictive performance”.

Usually, out of sample testing refers to cross-validation. This is where the model is first built on a subsection of the data – the “training” set – and then tested on the data which was not used to build it – the “test” or “hold-out” set. This gives a way of looking at how good the model is at predicting results for new data as we can apply the model to what is effectively “unseen” data. Since we are using the hold-out set from the original data we have the advantage of knowing what the true, “real-life” outcome for each data point is, meaning we can assess the accuracy of the model by comparing the predicted outcomes to the actual outcomes.

An advantage of out-of-sample validation over in-sample validation is that it helps to guard against overfitting. Overfitting is when a model is so specifically tuned to one dataset that the chances of it predicting an accurate outcome for the new data are very small.

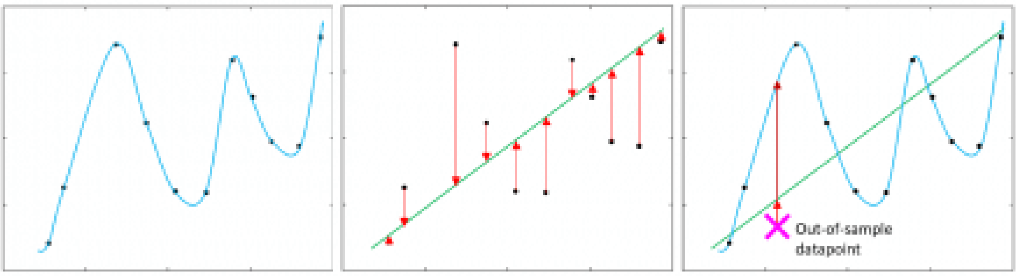

Figure 2.6: Diagram of modelling 11 data points showing an overfit (Approaches to model validation by Sally Hunton)

Figure 2.6 illustrates modelling 11 data points showing an overfit model (blue) vs a less accurate but more useful model (black). The x-axis (horizontal axis) represents the predictor variables and the y-axis (vertical axis) represents the outcome.

The left-hand and middle panels show two different models which could be fitted to the data (black points), with the x-axis (horizonal axis) showing the predictor values and the y-axis (vertical axis) showing the outcome values. The blue model (left hand plot) perfectly fits all the data, whereas the green (central plot) model does not accurately predict all the points. The red arrows in the plots below represent the residuals (how far away the model predictions are from the observed data). Comparing these two models in-sample, it is easily see that the blue model is a better fit of the training data.

CHAPTER THREE: METHODOLOGY

3.1 Introduction

This chapter describes how the research addressed the objectives of the study. It gives an outline of research methods that were used to collect and analyze data. First, the research design that was used to collect data in order to characterize the existing production planning system with respect to manufacturing lot sizes. The approaches for formulating the manufacturing lot size model in PP under demand uncertainty, the techniques for algorithms of the manufacturing lot-size model under demand uncertainty that predicts optimal manufacturing lot size as well as the validation approaches are also described.

3.2 Research Design

The researcher adopted a mixed stream research design approach consisting of both qualitative and quantitative research methods. Qualitative research methods was used in the characterization of the existing production planning system with respect to manufacturing lot sizes at Movit products (U) Ltd whereas the quantitative method was used to define and formulate the ‘manufacturing lot size’ problem, develop the ‘manufacturing lot-size’ model & validate the developed manufacturing lot-size model. The study undertook the description of demand, the manufacturing lot-size and the costs associated with maintaining the optimal production levels (unit production costs, unit holding cost, and unit shortage cost).

3.3 Research Instruments

At Movit products (U) Ltd, demand data, customer data and inventory data for the fast moving (most demanded) products was assessed.

In collecting data, the researcher developed and used appropriate data gathering sheets, interview guide as tools for collecting the data.

3.3.1 Data gathering sheets

The data gathering sheets established the customers, demand, quantity in stock (on hand inventory), states of demand and state transitions of customers demanding the products.

3.3.2 Interview guide

The interview guide was used in the characterization of the existing production planning system at Movit products (U) Ltd in order to obtain the unit production cost, the unit holding cost and the unit shortage cost of selected products.

3.4 Study Population

The study population consisted of Movit products (U) Ltd factory with several products produced and selection was made based on the fast moving (most demanded) products for which the developed manufacturing lot-size model was applied. A list of the different products produced at Movit products (U) ltd with the demand levels, customer levels and inventory levels on quarterly basis was established and data recorded.

3.5 Data Coding

For not disclosing the products investigated in the study, five most demanded products sampled were assigned identifier codes as shown in table 3.1;

Table 3.1: Research Data Coding

Product	Product codes	Description
4000008	A	First Fast moving product at Movit company
4000414	B	Second Fast moving product at Movit company
4000013	C	Third Fast moving product at Movit company
4000217	D	Fourth Fast moving product at Movit company
4000254	E	Fifth Fast moving product at Movit company

Table 3.1 describes the product codes that were assigned to each of the products that were considered in this research study. This was one of the ways of exhibiting confidentiality as requested by the case study not to disclose the exact products investigated.

3.6 Sampling Procedures

A random sampling method was used where by customers at the factory were observed at random. This enabled the researcher to analyze the occurrence of demand for the fast moving (most demanded) finished products. The corresponding quantity demanded and inventory levels were recorded based on the customer demand pattern of the fast moving (most demanded) finished products.

‘Sample size is a research term used for defining the number of individuals included in a research study to represent a population. The sample size references the total number of respondents included in a study, and the number is often broken down into sub-groups by demographics such as age, gender, and location so that the total sample achieves represents the entire population. Determining the appropriate sample size is one of the most important factors in statistical analysis.

If the sample size is too small, it will not yield valid results or adequately represent the realities of the population being studied. On the other hand, while larger sample sizes yield smaller margins of error and are more representative, a sample size that is too large may significantly increase the cost and time taken to conduct the research.'

'At first glance, many pieces of research seem to choose a sample size merely on the basis of what 'looks' about right, or what similar studies have used in the past, or perhaps simply for reasons of convenience: ten seems a bit small, and one hundred would be difficult to obtain, so 40 is a happy compromise!'. Unfortunately, a lot of published research uses precisely this kind of logic. Choosing the correct size of sample is not a matter of preference, it is a crucial element of the research process without which you may well be spending months trying to investigate a problem with a tool which is either completely useless, or over expensive in terms of time and other resources (Fox & Hunn, 2009).

An appropriate sampling procedure was used to calculate the appropriate sample size for the customers from the factory for each of the most consumed product whereby the customers were selected taking into account the heterogeneous nature of the population, selected at random.

The Andrew Fischer's formula below was used to calculate the sample size of the population to participate in the study (Singh & Masuku, 2018).

$$Sample\ size = \frac{(Z^2) * standard\ deviation\ (1 - standard\ deviation)}{confidence\ interval^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

Where:

Z= Normal deviation at the desired confidence interval. In this case it was taken at 95%, that is, the Z value at 95% is 1.96

Standard deviation = 2.6 ≈ 3%

Confidence interval = 5% (Margin of error)

Population proportion = 50%

Population size = 40

Hence, the sample size is 40

Table 3.2: Sample size at the factory

Product codes	Sample size
A	40
B	40
C	40
D	40
E	40
Total	200

Table 3.2 gives the sample size that was used for each product code, that is, A, B, C, D, and E, at Movit products (U) limited (Factory)

3.7 Data Collection

The quantitative technique involved modeling of a ‘real form’ problem into a mathematical form which was solved to arrive at a Solution that would aid the decision makers.

Observations and interview modes of data collection were used whereby data gathering worksheets and interview guides were used for each mode respectively. The data that was collected included the customer levels; quantities demanded and the corresponding quantity in stock, which was used in determining the states of demand, state transitions, and demand transition probabilities.

3.8 Data Analysis

This involved coding and tabulation of the collected data where a quantitative method was used in computation of percentages and totals. Tables were used to present and summarize data for easy interpretation and display of information.

3.8.1 Quantitative Data Analysis

To reduce the data collected to usable dimensions, data editing, processing and analysis is critical with the aim of organizing and interpreting the data generated. Creation of an electronic data base from the raw data source is also essential in order to edit and certify the raw data sources.

Data files were created from the data base generated and then analyzed from frequencies generated and categorical variables tabulated, which is observation of demand by state, inventory levels, and unit production costs.

Matrices were used to establish demand and inventory levels for analysis. The demand pattern of the finished product was examined at chosen epochs, where demand transition probabilities and matrices were determined using probability theory.

The corresponding production cost matrices were similarly determined based on the changing state of demand.

The data obtained was used at a later stage when computing expected production-inventory cost using Markov chain analysis and stochastic goal programming.

3.8.2 Qualitative Data Analysis

Analytical means to describe and explain the social phenomena of the manufacturing lot-size and demand characteristics at factory were critical in this study.

3.9 Ethical Considerations

All the relevant ethical laws of Uganda were followed by the researcher and avoided under all circumstances any conditions that may undermine the production efficiency and future business effectiveness in the production concerns contacted.

3.10 Model development flow chart

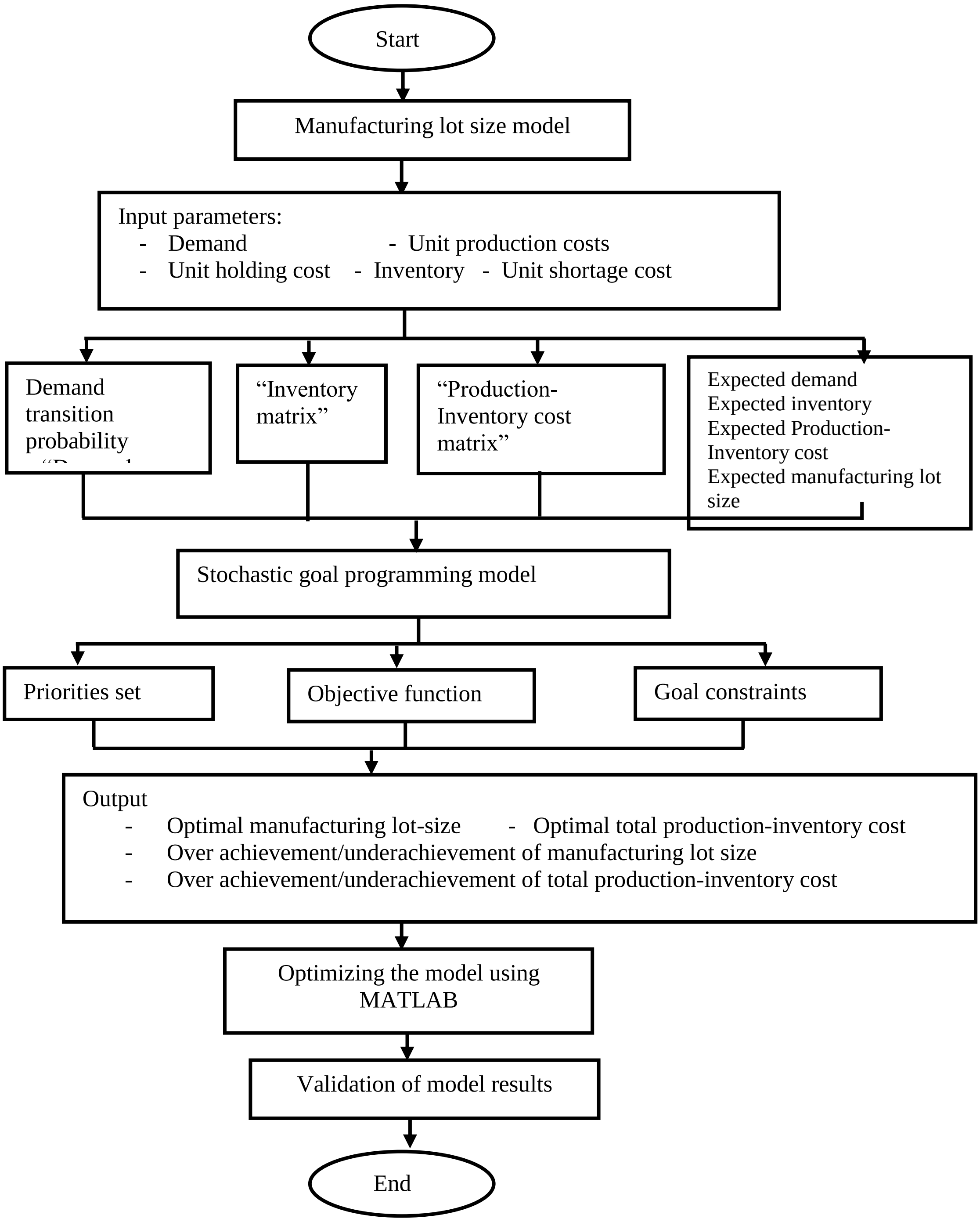

Figure 3.1: Model development flow chart

Figure 3.1 shows the model flow chart that was followed in the development of an optimization model that predicts optimal manufacturing lot size in production planning under demand uncertainty up to model validation.

The following steps were followed to formulate the manufacturing lot-size model:

Step 1: Formulating the demand transition probabilities that generate the demand transition matrices

Step 2: Formulating the production-inventory cost matrix

Step 3: Computation of the expected demand

Step 4: Computation of the expected inventory

Step 5: Computation of the expected production-inventory costs

Step 6: Computation of the expected manufacturing lot-size

Step 7: Formulating the "stochastic goal programming" model by:

- "Setting" priorities
- Describing the target function
- Framing the goal constraints

Step 8: Formulating the stochastic goal programming model for manufacturing lot-size

Step 9: Solve and determine optimum manufacturing lot-size using MATLAB software

Step 10: validation and verification of the mathematical model

3.11 Mathematical Model Formulation

An engineering company making items with changes and instabilities in demand was used. Movit Products (U) Ltd, the firm considered during this study, mainly fabricates, disperses and offers skin care, hair & nail care items. The demand of these items amid each time period over a limited static “planning horizon” was depicted as either “favorable or unfavorable”. The “Markov chain” method was accepted and likewise the states of a “Markov” procedure signify conceivable states of demand for the finished items with the succeeding main symbolizations.

Table 3.3: “Key notations” used in the Markov model

i, j	Established states of demand
F	Favorable demand
U	Unfavorable demand
Q	Demand transition matrix
P	Product
q	Quarter of the year
FF, FU, UF, UU	State transitions
Z	Value of the objective function
P_k	Preemptive priority of the k^{th} goal
d_k^+	Over achievement of the k^{th} goal
d_k^-	Under achievement of the k^{th} goal
	Manufacturing lot-size
$X_{ij}\ (p, q)$	Amount of product p produced in quarter q
C_p	Unit production cost
C_h	Unit holding cost
C_s	Unit shortage cost
N	Customer matrix
D	Demand matrix
V	Inventory matrix
C	Production-Inventory cost matrix

Table 3.3 describes the different codes used in the Markov model and their symbolizations.

Average “on-hand” inventory,

$$V = (B+E)/2 \quad \text{.........................} \quad 3.2$$

The “customer matrix” can be formulated as follows:

$$N(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & N_{FF}(p,q) & N_{FU}(p,q) \\ \boldsymbol{U} & N_{UF}(p,q) & N_{UU}(p,q) \end{matrix} \quad \text{.................................} \quad 3.3$$

The “demand transition probabilities” are then calculated as:

$$Q_{ij}(p, q) = \frac{N_{ij}(p,q)}{N_{if}(p,q)+N_{iu}(p,q)} \quad \text{……………………………………………} 3.4$$

This gives the “demand transition matrix”:

$$Q(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & Q_{FF}(p,q) & Q_{FU}(p,q) \\ \boldsymbol{U} & Q_{UF}(p,q) & Q_{UU}(p,q) \end{matrix} \quad \text{…………………..………… } 3.5$$

The “demand, inventory and production-inventory cost” matrices are then determined as below;

Demand matrix;

$$D(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & D_{FF}(p,q) & D_{FU}(p,q) \\ \boldsymbol{U} & D_{UF}(p,q) & D_{UU}(p,q) \end{matrix} \quad \text{………………….……………… } 3.6$$

Inventory matrix;

$$V(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(p,q) & V_{FU}(p,q) \\ \boldsymbol{U} & V_{UF}(p,q) & V_{UU}(p,q) \end{matrix} \quad \text{………………….……………… } 3.7$$

‘Production-inventory cost matrix’;

As demand exceeds the sum delivered at that point,

$$C_F(p,q) = \begin{bmatrix} C_{r(p)} \\ + \\ C_{h(p)} \\ + \\ C_{s(p)} \end{bmatrix} [D(p,q) - V(p,q)] \quad \text{…………………..……………… } 3.8$$

where C_F –Production-inventory cost when demand is favorable

And as the demand is fewer than the sum created at that point,

$$C_U(p,q) = C_h(p)[V(p,q) - D(p,q)] \text{………………………………… } 3.9$$

where C_U – Production-inventory cost when demand is unfavorable

Hence,

$$C(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(p,q) & C_{FU}(p,q) \\ \boldsymbol{U} & C_{UF}(p,q) & C_{UU}(p,q) \end{matrix} \quad \text{.................................... 3.10}$$

<u>“Expected” demand</u>

“Favorable Demand” $E[D_F(p,q)] = Q_{FF}(p,q)D_{FF}(p,q) + Q_{FU}(p,q)D_{FU}(p,q)$…

3.11

“Unfavorable Demand” $E[D_U(p,q)] = Q_{UF}(p,q)D_{UF}(p,q) + Q_{UU}(p,q)D_{UU}(p,q)$.

3.12

<u>“Expected” inventory</u>

“Favorable Demand” $E[V_F(p,q)] = Q_{FF}(p,q)V_{FF}(p,q) + Q_{FU}(p,q)V_{FU}(p,q)$….

3.13

“Unfavorable Demand” $E[V_U(p,q)] = Q_{UF}(p,q)V_{UF}(p,q) + Q_{UU}(p,q)V_{UU}(p,q)$

3.14

<u>“Expected” production-inventory costs</u>

“Favorable Demand” $E[C_F(p,q)] = Q_{FF}(p,q)C_{FF}(p,q) + Q_{FU}(p,q)C_{FU}(p,q)$….

3.15

“Unfavorable Demand” $E[C_U(p,q)] = Q_{UF}(p,q)C_{UF}(p,q) + Q_{UU}(p,q)C_{UU}(p,q)$

3.16

<u>“Expected” manufacturing lot-size</u>

“Favorable demand”

$$E[M_F(p,q)] = \begin{cases} E[D_F(p,q)] - E[V_F(p,q)] & \text{if } E[D_F(p,q)] > E[V_F(p,q)] \\ \\ 0 & \text{otherwise} \end{cases} \quad \text{...... 3.17}$$

"Unfavorable demand"

$$E[M_U(p,q)] = \begin{cases} E[D_U(p,q)] - E[V_U(p,q)] & if\ E[D_U(p,q)] > E[V_U(p,q)] \\ 0 & otherwise \end{cases} \quad \ldots\ldots 3.18$$

3.12 Goal Programming

The manufacturing lot-size is then tested to determine whether it is achievable using goal programming

Set priorities

P_1: manufacture a lot of $E[M_F(p,q)]$ items as demand is 'favorable'

P_2: manufacture a lot of $E[M_U(p,q)]$ items as demand is "unfavorable"

P_3: "Total production-inventory" cost shouldn't not go above $E[C_F(p,q)]$ as demand is "favorable"

P_4: "Total production-inventory" cost shouldn't go above $E[C_U(p,q)]$ as demand is "unfavorable"

Objective function

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.19$$

Goal constraints

P1: Manufacturing lot-size $E[M_F(p,q)]$ - favorable demand

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \ldots\ldots\ldots\ldots 3.19.1$$

P2: Manufacturing lot-size $E[M_U(p,q)]$ - unfavorable demand

$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \ldots\ldots\ldots\ldots 3.19.2$$

P3: Total production-inventory cost – favorable demand

$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad \ldots\ 3.19.3$$

P4: Total production-inventory cost – unfavorable demand

$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad \ldots\ 3.19.4$$

Manufacturing lot-size model

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \quad \ldots\ 3.20$$

Subject to:

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \quad \ldots\ 3.20.1$$

$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \quad \ldots\ 3.20.2$$

$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad \ldots\ 3.20.3$$

$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad \ldots\ 3.20.4$$

$$X_{FF}(p,q), X_{FU}(p,q), X_{UF}(p,q), X_{UU}(p,q), d_1^-, d_1^+, d_2^-, d_2^+, d_3^+, d_4^+ \geq 0 \quad 3.20.5$$

Model solution

The model was solved by MATLAB using data from first quarter of the year for the five most consumed products selected, that is,

Product A, first quarter: (p = A, q = 1)

Product B, first quarter: (p = B, q = 1)

Product C, first quarter: (p = C, q = 1)

Product D, first quarter: (p = D, q = 1)

Product E, first quarter: (p = E, q = 1)

3.13 Model Validation

For the developed model, data analysis authentication was used to ascertain and approve that the model will work likewise given altered testing circumstances. The developed stochastic goal programming model was then validated using out-of-sample test. This was done by using ‘new’ data which was not found in the dataset that was used to build the model.

In other words, it was used for testing how good the model is for predicting results on unseen new data, that is, predictive performance. Data (for product Y) not used in building the model, was got and used in validating the model to assess how well the model predicts outcomes in new data.

3.14 Methodology Limitations

The limitation to the project methodology was sample bias or selection bias. This is because sampling errors could have occurred due to probability sampling method used to select a sample.

CHAPTER FOUR: RESULTS AND DISCUSSIONS

4.1 Introduction

In this chapter, the results got about each specific objective, using the methods in chapter three are presented and later discussed. The stochastic goal programming model for the five products was developed considering some practical situations where data was collected. The stochastic goal programming model is composed of states of demand (FF, FU, UF, UU), manufacturing lot size and the total production-Inventory costs. The model establishes the amount of the item to manufacture within the given quarter of the year as demand shifts from state i to state j for $i, j \in \{F, U\}$ where total production-Inventory costs are minimized.

4.2 Characterization of the existing PP system at Movit Products (U) Ltd

The existing production planning system at Movit products (U) limited was characterized as;

1. Batch production - identical or similar items produced together for different sized production runs. (mass-production in batches with small to major changes)

   Here, a group of identical products are produced simultaneously (rather than one at a time). It is up to the manufacturer to decide how big the batch will be, and how often these batches will be made. Each batch goes through the separate stages of the manufacturing process together, meaning that another batch can't begin a stage, if the previous one is still within that part of the cycle.

   Each batch can be different, as manufacturers can decide to change the specifications from one group of products to the next.

   Maybe it is necessary to change the colour or size of that particular group (depending on the preferences specified in a particular order).

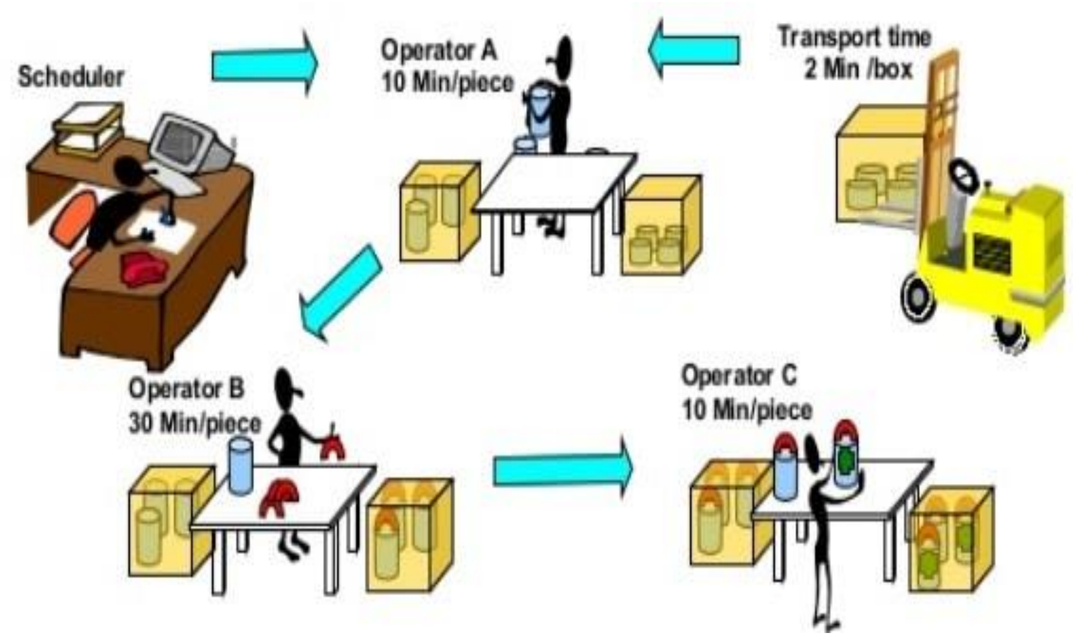

Figure 4.1: Batch production manufacturing system at Movit products (U) limited

Figure 4.1 illustrates the existing production planning system at Movit (U) limited, the case study. It shows the sequence of production from one stage to another and what is done at each stage and the time spent there per piece.

Quality checks can be carried out after each step of the production cycle and machinery can be tested between batches to ensure there are no performance problems (flexibility).

2. Make-to-stock strategy - match inventory with anticipated demand. (requires an accurate forecast of this demand to determine how much stock it produces)

   In this strategy, the production planning and production scheduling are based on forecasted product demand.

   Products made during one production period are used to fulfill orders made in the next production period which means that in make-to-stock production planning, production is triggered prior to and independent of specific customer orders.

Make-to-stock is a push-type operation, which means that supplies, raw materials and supplier-provided components, are "pushed" through the production process, and planning starts with supplies and works forward to the finished product.

Figure 4.2: Make to stock

Figure 4.2 shows Movit factory and the movement to the warehouse, illustrating that after items are produced, they are shipped to the ware house for storage and later taken by the customers upon demand.

A high inventory of finished goods is usually an unacceptable cost burden, imposing the expenses of inventory management, warehousing, spoilage and more. Likewise, inventory shortages are costly because of expediting premiums, overtime, and missed delivery times. Therefore, the ideal of make-to-stock production planning is to match the quantity of finished goods at any given time with customer demand during the next period of time.

3. Standardization of product and process sequence - Continuous Production (Production facilities arranged as per the sequence of production operations from the first operations to the finished product)

   This refers to the process of maintaining uniformity and consistency among the different iterations of a particular good or service. It is made using the same materials and processes, has the same packaging and is marketed under the same name.

   The strategy of product standardization requires the industry or organization to follow certain guidelines in order to maintain the consistency of a product's nature, appearance, and quality.

4. Special purpose machines having higher production capacities and output rates (these are designed according to the requirements of the clients)

4.3 Data classification

The data for the five fast moving products was collected as shown in appendix 5 and after that, diminished to operational measurements as presented by tables 4.1, 4.3, 4.5, 4.7 and 4.9. "Data classification" by state of demand was made, analyzed and utilized to create the scientific model. This was for the first quarter of the year.

<u>Product A</u>

In a specified week, demand was considered as "favorable" (state F) if $N_{ij} > 12$ or else demand was "unfavorable" (state U) if $N_{ij} \leq 12$

Table 4.1: "Data classification" by state of demand for product A

Month	Week	Customers (N)	Demand (D) ($x10^3$)	'On-hand' inventory (V) ($x10^3$)	State of demand (*i*)
1	1	9	3937	6076	U
	2	12	4668	4687	U
	3	8	2485	6306	U
	4	17	7955	10160	F
2	1	1	110	4525	U
	2	15	3832	5681	F
	3	7	2870	4363	U
	4	20	3824	6028	F
3	1	4	758	2018	U
	2	16	6125	4149	F
	3	14	2625	4163	F
	4	17	3685	6279	F

Table 4.1 shows the classification of data by state of demand (as either favorable or unfavorable) of product A for each week for the first quarter of the year.

Overstocking and understocking of product A weekly in the first quarter of the year with the corresponding shortage and holding costs is shown in table 4.2.

Table 4.2: 'Overstocking' & "understocking" with holding & shortage costs for product A

Week	Demand (D) ($x10^3$)	"On-hand" inventory (V) ($x10^3$)	Over/under stocking	Holding/shortage costs (KES)
1	3937	6076	2139	23165.37
2	4668	4687	19	205.77
3	2485	6306	3821	41381.43
4	7955	10160	2205	23880.15
5	110	4525	4415	47814.45
6	3832	5681	1849	20024.67
7	2870	4363	1493	16169.19
8	3824	6028	2204	23869.32
9	758	2018	1260	13645.80
10	6125	4149	-1976	156973.44
11	2625	4163	1538	16656.54
12	3685	6279	2594	28093.02

Table 4.2 shows the overstocking or understocking of product A weekly for the first quarter of the year, with the corresponding holding or shortage costs.

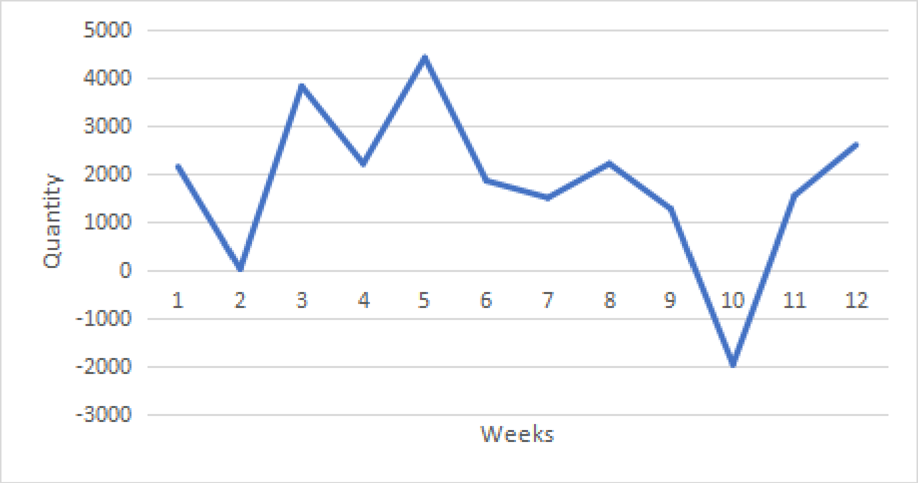

Figure 4.3: Overstocking & understocking of product A

Figure 4.3 shows the graphical representation of overstocking and understocking of product A weekly for the first quarter of the year.

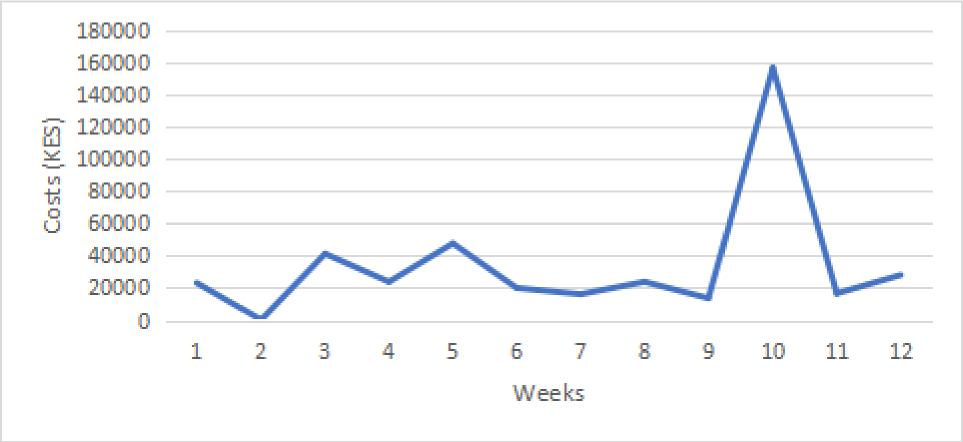

Figure 4.4: Holding & Shortage costs of product A

Figure 4.4 shows the graphical representation of holding and shortage costs of product A weekly for the first quarter of the year.

Product B

In a specified week, demand was considered as ‘favorable’ (state F) if Nij > 25 or else demand was ‘unfavorable’ (state U) if Nij ≤ 25

Table 4.3: “Data classification” by state of demand for product B

Month	**Week**	**Customers (N)**	**Demand (D) (x103)**	**‘On-hand’ inventory (V) (x103)**	**State of demand (i)**
1	1	16	2309	2365	U
	2	34	3224	4459	F
	3	25	2759	3255	U
	4	42	6113	5923	F
2	1	7	414	2095	U
	2	22	2422	2564	U
	3	16	1269	2994	U
	4	36	2981	3372	F
3	1	12	289	2502	U
	2	24	1825	1827	U
	3	30	806	1636	F
	4	33	2426	2992	F

Table 4.3 shows the classification of data by state of demand (as either favorable or unfavorable) of product B for each week for the first quarter of the year.

The overstocking and understocking of product B in the first quarter of the year with the corresponding shortage and holding costs is shown in table 4.4.

Table 4.4: 'Overstocking' & "understocking" with holding & shortage costs for product B

Week	Demand (D) ($x10^3$)	"On-hand" inventory (V) ($x10^3$)	Over/under stocking	Holding/shortage costs (KES)
1	2309	2365	56	2874.48
2	3224	4459	1235	63392.55
3	2759	3255	496	25459.68
4	6113	5923	-190	53642.70
5	414	2095	1681	86285.73
6	2422	2564	142	7288.86
7	1269	2994	1725	88544.25
8	2981	3372	391	20070.03
9	289	2502	2213	113593.29
10	1825	1827	2	102.66
11	806	1636	830	42603.90
12	2426	2992	566	29052.78

Table 4.4 shows the overstocking or understocking of product B weekly for the first quarter of the year, with the corresponding holding or shortage costs.

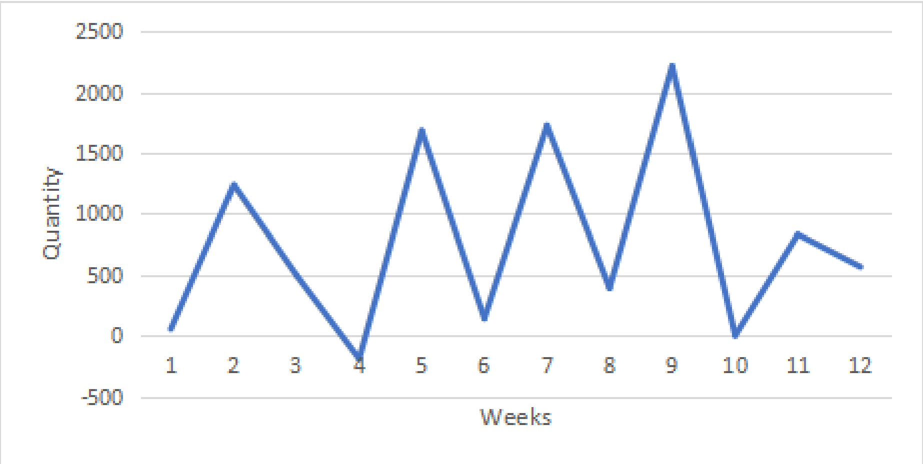

Figure 4.5: Overstocking & understocking of product B

Figure 4.5 shows the graphical representation of overstocking and understocking of product B weekly for the first quarter of the year.

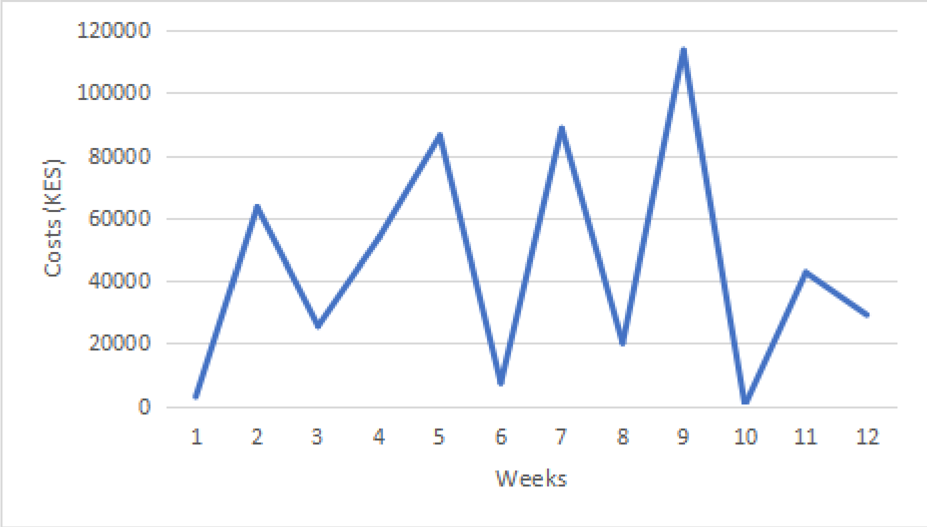

Figure 4.6: Holding & Shortage costs of product B

Figure 4.6 shows the graphical representation of holding and shortage costs of product B weekly for the first quarter of the year.

Product C

In a specified week, demand was considered as ‘favorable’ (state F) if Nij > 27 or else demand was ‘unfavorable’ (state U) if Nij > 27

Table 4.5: “Data classification” by state of demand for product C

Month	**Week**	**Customers (N)**	**Demand (D) ($x10^3$)**	**‘On-hand’ inventory (V) ($x10^3$)**	**State of demand (*i*)**
1	1	16	1746	2581	U
	2	39	4929	4656	F
	3	19	3347	4538	U
	4	37	5020	5514	F
2	1	7	875	2050	U
	2	23	4757	3690	U
	3	19	3068	2687	U
	4	33	3005	4189	F
3	1	16	1745	3309	U
	2	38	3263	3259	F
	3	39	3093	4115	F
	4	33	4146	5177	F

Table 4.5 shows the classification of data by state of demand (as either favorable or unfavorable) of product C for each week for the first quarter of the year.

Overstocking and understocking of product C in the first quarter of the year with the corresponding shortage and holding costs is shown in table 4.6.

Table 4.6: 'Overstocking' & "understocking" with holding & shortage costs for product C

Week	Demand (D) ($x10^3$)	"On-hand" inventory (V) ($x10^3$)	Over/under stocking	Holding/shortage costs (KES)
1	1746	2581	835	6980.60
2	4929	4656	-273	25107.81
3	3347	4538	1191	9956.76
4	5020	5514	494	4129.84
5	875	2050	1175	9823.00
6	4757	3690	-1067	98131.99
7	3068	2687	-381	35040.57
8	3005	4189	1184	9898.24
9	1745	3309	1564	13075.04
10	3263	3259	-4	367.88
11	3093	4115	1022	8543.92
12	4146	5177	1031	8619.16

Table 4.6 shows the overstocking or understocking of product C weekly for the first quarter of the year, with the corresponding holding or shortage costs.

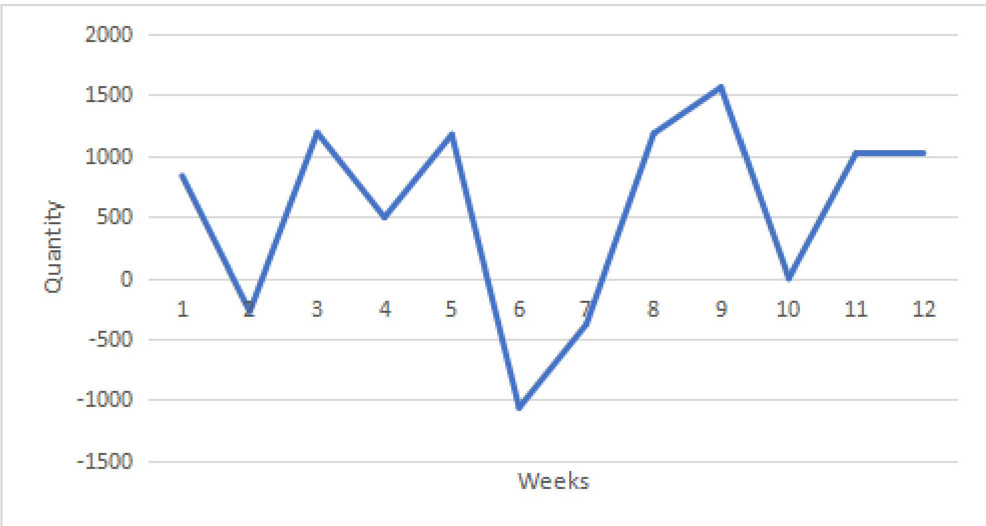

Figure 4.7: Overstocking & understocking of product C

Figure 4.7 shows the graphical representation of overstocking and understocking of product C weekly for the first quarter of the year.

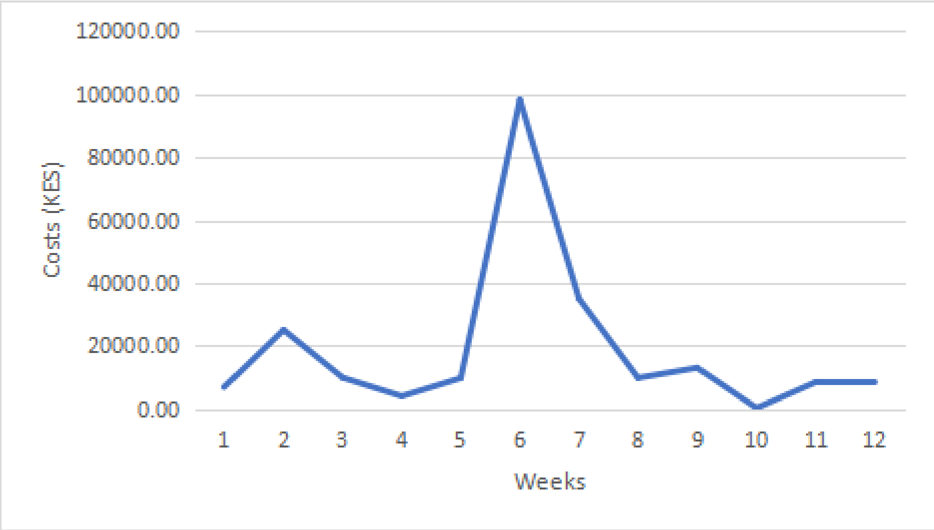

Figure 4.8: Holding & Shortage costs of product C

Figure 4.8 shows the graphical representation of holding and shortage costs of product C weekly for the first quarter of the year.

Product D

In a specified week, demand was considered as "favorable" (state F) if Nij > 26 or else demand was "unfavorable" (state U) if Nij > 26

Table 4.7: "Data classification" by state of demand for product D

Month	Week	Customers (N)	Demand (D) ($x10^3$)	'On-hand' inventory (V) ($x10^3$)	State of demand (*i*)
1	1	15	308	5263	U
	2	29	2891	7337	F
	3	24	1757	7081	U
	4	38	6619	5654	F
2	1	8	231	3525	U
	2	17	2046	6243	U
	3	15	1617	5922	U
	4	45	4443	5951	F
3	1	14	559	3765	U
	2	37	3686	4738	F
	3	28	1537	4980	F
	4	44	5626	5746	F

Table 4.7 shows the classification of data by state of demand (as either favorable or unfavorable) of product D for each week for the first quarter of the year.

The overstocking and understocking of product D in the first quarter of the year with the corresponding shortage and holding costs is shown in table 4.8.

Table 4.8: ‘Overstocking’ & “understocking” with holding & shortage costs for product D

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	Over/under stocking	Holding/shortage costs (KES)
1	308	5263	4955	312165.00
2	2891	7337	4446	280098.00
3	1757	7081	5324	335412.00
4	6619	5654	-965	334372.50
5	231	3525	3294	207522.00
6	2046	6243	4197	264411.00
7	1617	5922	4305	271215.00
8	4443	5951	1508	95004.00
9	559	3765	3206	201978.00
10	3686	4738	1052	66276.00
11	1537	4980	3443	216909.00
12	5626	5746	120	7560.00

Table 4.8 shows the overstocking or understocking of product D weekly for the first quarter of the year, with the corresponding holding or shortage costs.

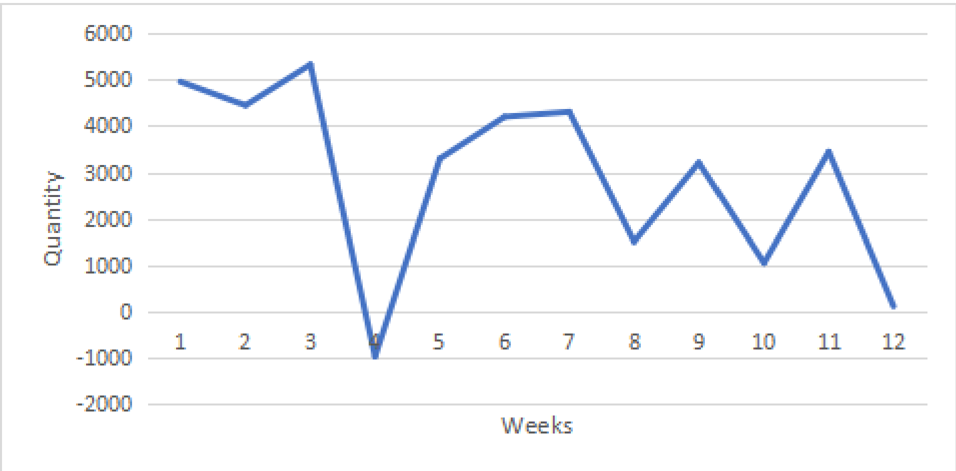

Figure 4.9: Overstocking & understocking of product D

Figure 4.9 shows the graphical representation of overstocking and understocking of product D weekly for the first quarter of the year.

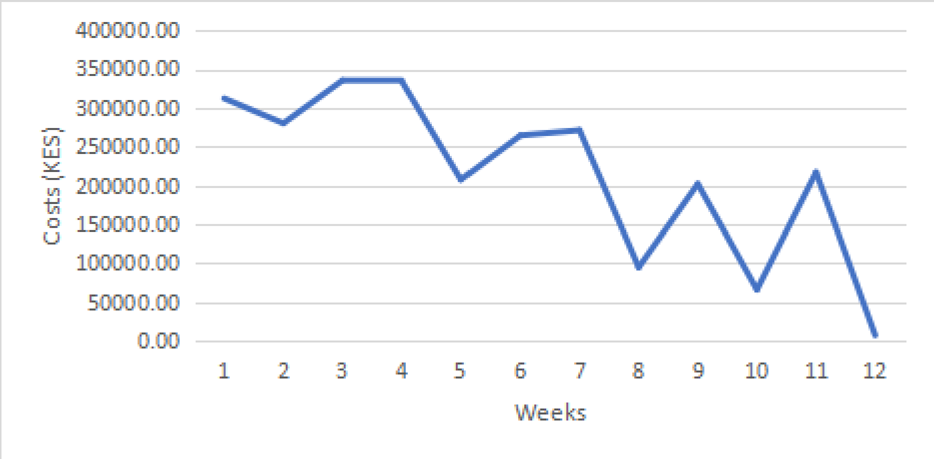

Figure 4.10: Holding & Shortage costs of product D

Figure 4.10 shows the graphical representation of holding and shortage costs of product D weekly for the first quarter of the year.

Product E

In a specified week, demand was considered as 'favorable' (state F) if Nij > 34 or else demand was 'unfavorable' (state U) if Nij > 34.

Table 4.9: "Data classification" by state of demand for product E

Month	**Week**	**Customers (N)**	**Demand (D) ($x10^3$)**	**'On-hand' inventory (V) ($x10^3$)**	**State of demand (*i*)**
1	1	20	904	2333	U
	2	50	2220	4800	F
	3	28	1200	5341	U
	4	58	3827	6400	F
2	1	12	335	2802	U
	2	31	1672	5037	U
	3	24	1893	6102	U
	4	37	1480	5750	F
3	1	17	608	4906	U
	2	39	1528	5433	F
	3	41	1570	5576	F
	4	47	2224	5614	F

Table 4.9 shows the classification of data by state of demand (as either favorable or unfavorable) of product E for each week for the first quarter of the year.

Overstocking and understocking of product E in the first quarter of the year with the corresponding shortage and holding costs is shown in table 4.10.

Table 4.10: 'Overstocking' & "understocking" with holding & shortage costs for product E

Week	Demand (D) ($x10^3$)	"On-hand" inventory (V) ($x10^3$)	Over/under stocking	Holding/shortage costs (KES)
1	904	2333	1429	112533.75
2	2220	4800	2580	203175.00
3	1200	5341	4141	326103.75
4	3827	6400	2573	202623.75
5	335	2802	2467	194276.25
6	1672	5037	3365	264993.75
7	1893	6102	4209	331458.75
8	1480	5750	4270	336262.50
9	608	4906	4298	338467.50
10	1528	5433	3905	307518.75
11	1570	5576	4006	315472.50
12	2224	5614	3390	266962.50

Table 4.10 shows the overstocking or understocking of product E weekly for the first quarter of the year, with the corresponding holding or shortage costs.

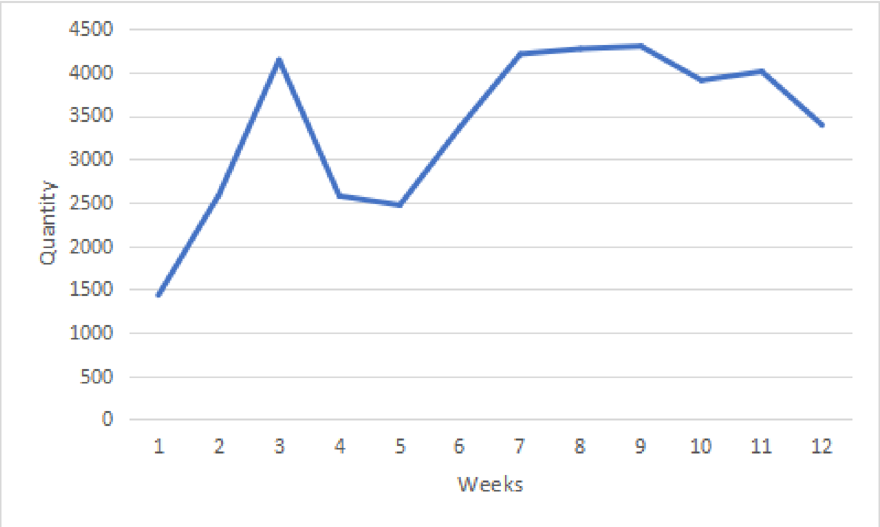

Figure 4.11: Overstocking & understocking of product E

Figure 4.11 shows the graphical representation of overstocking and understocking of product E weekly for the first quarter of the year.

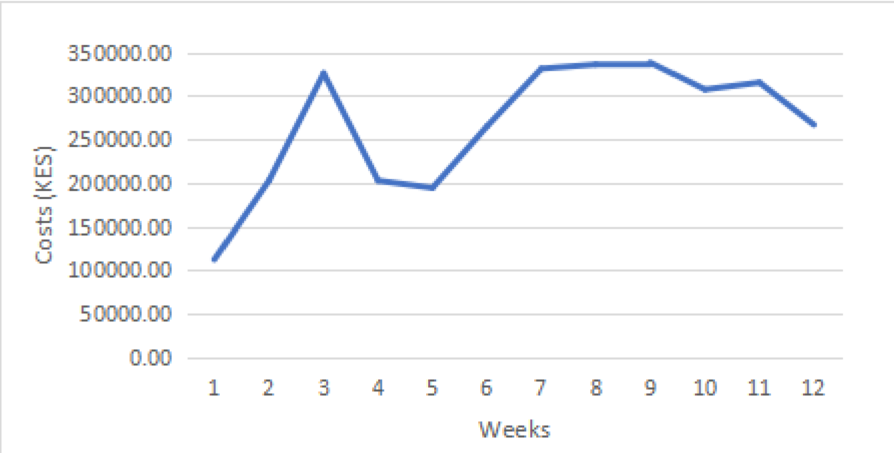

Figure 4.12: Holding & Shortage costs of product E

Figure 4.12 illustrates the graphical representation of holding and shortage costs of product E weekly for the first quarter of the year.

4.4 "State" transitions and "on-hand" inventory

For a specific "state" transition, knowing the starting and finishing stock, as of equation 3.2, the average 'on-hand' inventory was designed for each item as shown from tables 4.11 to 4.15.

Table 4.11: Average 'on-hand' inventory for product A

State transitions (i,j)	Starting inventory (B)	Finishing inventory (E)	Average "on-hand" inventory $V = (B+E)/2$
FF	4163	6279	5221.00
FU	4525	2018	3271.50
UF	10160	4149	7154.50
UU	4687	6306	5496.50

$V_{FF}(A,1) = 5221$ $V_{FU}(A,1) = 3271.5$ $V_{UF}(A,1) = 7154.5$ $V_{UU}(A,1) = 5496.5$

Table 4.11 shows for each state transition, the Average 'on-hand' inventory for product A for the first quarter of the year considering the beginning and ending inventory.

Table 4.12: Average "on-hand" inventory for product B

State transitions (i,j)	Starting inventory (B)	Finishing inventory (E)	Average "on-hand" inventory $V = (B+E)/2$
FF	2992	2992	2992.00
FU	3255	2502	2878.50
UF	4459	1636	3047.50
UU	2564	1827	2195.50

$V_{FF}(B,1) = \mathbf{2992}$ $V_{FU}(B,1) = \mathbf{2878.5}$ $V_{UF}(B,1) = \mathbf{3047.5}$ $V_{UU}(B,1) = \mathbf{2195.5}$

Table 4.12 shows for each state transition, the Average 'on-hand' inventory for product B for the first quarter of the year considering the beginning and ending inventory.

Table 4.13: Average "on-hand" inventory for product C

State transitions (i,j)	Starting inventory (B)	Finishing inventory (E)	Average "on-hand" inventory $V = (B+E)/2$
FF	4115	5177	4646.00
FU	4538	3309	3923.50
UF	4656	3259	3957.50
UU	3690	2687	3188.50

$V_{FF}(C,1) = 4646$ $V_{FU}(C,1) = 3923.5$ $V_{UF}(C,1) = 3957.5$ $V_{UU}(C,1) = 3188.5$

Table 4.13 shows for each state transition, the Average 'on-hand' inventory for product C for the first quarter of the year considering the beginning and ending inventory.

Table 4.14: Average "on-hand" inventory for product D

State transitions (i,j)	**Starting inventory (B)**	**Finishing inventory (E)**	**Average "on-hand" inventory $V=(B+E)/2$**
FF	4980	5746	5363.00
FU	7081	3765	5423.00
UF	7337	4738	6037.50
UU	6243	5922	6082.50

$V_{FF}(D,1) = \mathbf{5363}$ $V_{FU}(D,1) = \mathbf{5423}$ $V_{UF}(D,1) = \mathbf{6037.5}$ $V_{UU}(D,1) = \mathbf{6082.5}$

Table 4.14 shows for each state transition, the Average 'on-hand' inventory for product D for the first quarter of the year considering the beginning and ending inventory.

Table 4.15: Average "on-hand" inventory for product E

State transitions (i,j)	**Starting inventory (B)**	**Finishing inventory (E)**	**Average "on-hand" inventory $V=(B+E)/2$**
FF	5576	5614	5595.00
FU	7081	4906	5993.50
UF	4800	5433	5116.50
UU	5037	6102	5569.50

$V_{FF}(E,1) = \mathbf{5595}$ $V_{FU}(E,1) = \mathbf{5993.5}$ $V_{UF}(E,1) = \mathbf{5116.5}$ $V_{UU}(E,1) = \mathbf{5569.5}$

Table 4.15 shows for each state transition, the Average 'on-hand' inventory for product E for the first quarter of the year considering the beginning and ending inventory.

Table 4.16: Summary of Average "on-hand" inventory and "state" transitions of *products*

State transition	Product A	Product B	Product C	Product D	Product E
FF	5221.00	2992.00	4646.00	5363.00	5595.00
FU	3271.50	2878.50	3923.50	5423.00	5993.50
UF	7154.50	3047.50	3957.50	6037.50	5116.50
UU	5496.50	2195.50	3188.50	6082.50	5569.50

Table 4.16 shows for each state transition, the summary of the Average 'on-hand' inventory for products A, B, C, D, and E for the first quarter of the year.

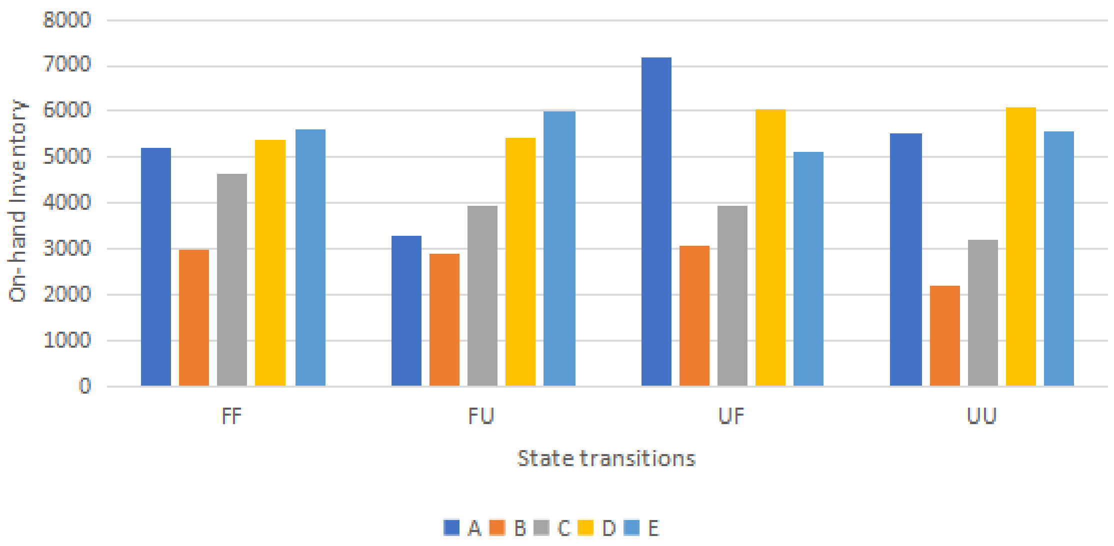

Figure 4.13: Average 'on-hand' inventory and 'state' transitions

Figure 4.13 illustrates the graphical representation of the average 'on-hand' inventory and state transition for products A, B, C, D, and E for the first quarter of the year.

4.5 Demand Transition Probabilities

Data "classification" by "state" transition was carried out as shown from tables 4.17 to 4.21 and then it was utilized to compute the "demand transition probabilities" for each product using equation 3.4.

Table 4.17: Data “classification” by ‘state’ transition for product A

Month	**“State” transition (i,j)**	**Number of customers $N_{ij}(A,1)$**	**Demand $D_{ij}(A,1)$**
1	FF	0	0
	FU	0	0
	UF	25	10440
	UU	41	15758
2	FF	0	0
	FU	22	6702
	UF	43	10636
	UU	0	0
3	FF	61	15060
	FU	0	0
	UF	20	6883
	UU	0	0

Table 4.17 shows for each week, for the first quarter of the year, the number of customers and their corresponding quantity demanded for a particular state transition for product A.

<u>TOTALS</u>

$N_{FF}(A,1) = 61$ $N_{FU}(A,1) = 22$ $N_{UF}(A,1) = 25 + 43 + 20 = 88$ $N_{UU}(A,1) = 41$

$D_{FF}(A,1) = 15060$ $D_{FU}(A,1) = 6702$

$D_{UF}(A,1) = 10440 + 10636 + 6883 = 27959$ $D_{UU}(A,1) = 15758$

Demand transition probabilities

$$Q_{FF}(A,1) = \frac{N_{FF}(A,1)}{N_{FF}(A,1)+N_{FU}(A,1)} = \frac{61}{61+22} = 0.7349$$

$$Q_{FU}(A,1) = \frac{N_{FU}(A,1)}{N_{FF}(A,1)+N_{FU}(A,1)} = \frac{22}{61+22} = 0.2651$$

$$Q_{UF}(A,1) = \frac{N_{UF}(A,1)}{N_{UF}(A,1)+N_{UU}(A,1)} = \frac{88}{88+41} = 0.6822$$

$$Q_{UU}(A,1) = \frac{N_{UU}(A,1)}{N_{UF}(A,1)+N_{UU}(A,1)} = \frac{41}{88+41} = 0.3178$$

Therefore, the 'demand transition matrix' as of equation 3.5 is,

$$Q(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 0.7349 & 0.2651 \\ \boldsymbol{U} & 0.6822 & 0.3178 \end{matrix}$$

Table 4.18: Data "classification" by 'state' transition for product B

Month	"State" transition $(\boldsymbol{i},\boldsymbol{j})$	Number of customers $\boldsymbol{N_{ij}(B,1)}$	Demand $\boldsymbol{D_{ij}(B,1)}$
1	FF	0	0
	FU	59	5983
	UF	117	14405
	UU	0	0
2	FF	0	0
	FU	0	0
	UF	52	4250
	UU	67	6527
3	FF	63	3232
	FU	0	0
	UF	54	2631
	UU	36	2114

Table 4.18 shows for each week, for the first quarter of the year, the number of customers and their corresponding quantity demanded for a particular state transition for product B.

TOTALS

$N_{FF}(B,1) = 63$ $N_{FU}(B,1) = 59$

$N_{UF}(B,1) = 117 + 52 + 54 = 223$ $N_{UU}(B,1) = 67 + 36 = 103$

$D_{FF}(B,1) = 3232$ $D_{FU}(B,1) = 5983$

$D_{UF}(B,1) = 14405 + 4250 + 2631 = 21286$ $D_{UU}(B,1) = 6527 + 2114 = 8641$

Demand transition probabilities

$$Q_{FF}(B,1) = \frac{N_{FF}(B,1)}{N_{FF}(B,1)+N_{FU}(B,1)} = \frac{63}{63+59} = 0.5164$$

$$Q_{FU}(B,1) = \frac{N_{FU}(B,1)}{N_{FF}(B,1)+N_{FU}(B,1)} = \frac{59}{63+59} = 0.4836$$

$$Q_{UF}(B,1) = \frac{N_{UF}(B,1)}{N_{UF}(B,1)+N_{UU}(B,1)} = \frac{223}{223+103} = 0.6840$$

$$Q_{UU}(B,1) = \frac{N_{UU}(B,1)}{N_{UF}(B,1)+N_{UU}(B,1)} = \frac{103}{223+103} = 0.3160$$

Therefore, the “demand transition matrix” as of equation 3.5 is,

$$Q(B,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \quad\quad\quad \boldsymbol{U} \\ \begin{pmatrix} 0.5164 & 0.4836 \\ 0.6840 & 0.3160 \end{pmatrix} \end{matrix}$$

Table 4.19: Data “classification” by “state” transition for product C

Month	“State” transition (i,j)	Number of customers $N_{ij}(C,1)$	Demand $D_{ij}(C,1)$
1	FF	0	0
	FU	58	8276
	UF	111	15042
	UU	0	0
2	FF	0	0
	FU	0	0
	UF	52	6073
	UU	72	13457
3	FF	149	13595
	FU	0	0
	UF	54	5008
	UU	0	0

Table 4.19 shows for each week, for the first quarter of the year, the number of customers and their corresponding quantity demanded for a particular state transition for product C.

TOTALS

$N_{FF}(C,1) = 149$ $N_{FU}(C,1) = 58$

$N_{UF}(C,1) = 111 + 52 + 54 = 217$ $N_{UU}(C,1) = 72$

$D_{FF}(C,1) = 13595$ $D_{FU}(C,1) = 8276$

$D_{UF}(C,1) = 15042 + 6073 + 5008 = 26123$ $D_{UU}(C,1) = 13457$

Demand transition probabilities

$$Q_{FF}(C,1) = \frac{N_{FF}(C,1)}{N_{FF}(C,1)+N_{FU}(C,1)} = \frac{149}{149+58} = 0.7198$$

$$Q_{FU}(C,1) = \frac{N_{FU}(C,1)}{N_{FF}(C,1)+N_{FU}(C,1)} = \frac{58}{149+58} = 0.2802$$

$$Q_{UF}(C,1) = \frac{N_{UF}(C,1)}{N_{UF}(C,1)+N_{UU}(C,1)} = \frac{217}{217+72} = 0.7509$$

$$Q_{UU}(C,1) = \frac{N_{UU}(C,1)}{N_{UF}(C,1)+N_{UU}(C,1)} = \frac{72}{217+72} = 0.2491$$

Therefore, the 'demand transition matrix' as of equation 3.5 is,

$$Q(C,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 0.7198 & 0.2802 \\ \boldsymbol{U} & 0.7509 & 0.2491 \end{matrix}$$

Table 4.20: Data "classification" by "state" transition for product D

Month	**"State" transition (i,j)**	**Number of customers $N_{ij}(D,1)$**	**Demand $D_{ij}(D,1)$**
1	FF	0	0
	FU	53	4648
	UF	106	11575
	UU	0	0
2	FF	0	0
	FU	0	0
	UF	60	6060
	UU	57	5940
3	FF	137	12386
	FU	0	0
	UF	51	4245
	UU	0	0

Table 4.20 shows for each week, for the first quarter of the year, the number of customers and their corresponding quantity demanded for a particular state transition for product D.

TOTALS

$N_{FF}(D,1) = 137$ $N_{FU}(D,1) = 53$

$N_{UF}(D,1) = 106 + 60 + 51 = 217$ $N_{UU}(D,1) = 57$

$D_{FF}(D,1) = 12386$ $D_{FU}(D,1) = 4648$

$D_{UF}(D,1) = 11575 + 6060 + 4245 = 21880$ $D_{UU}(D,1) = 5940$

Demand transition probabilities

$$Q_{FF}(D,1) = \frac{N_{FF}(D,1)}{N_{FF}(D,1)+N_{FU}(D,1)} = \frac{137}{137+53} = 0.7211$$

$$Q_{FU}(D,1) = \frac{N_{FU}(D,1)}{N_{FF}(D,1)+N_{FU}(C,1)} = \frac{53}{137+53} = 0.2789$$

$$Q_{UF}(D,1) = \frac{N_{UF}(D,1)}{N_{UF}(D,1)+N_{UU}(D,1)} = \frac{217}{217+57} = 0.7920$$

$$Q_{UU}(D,1) = \frac{N_{UU}(D,1)}{N_{UF}(D,1)+N_{UU}(D,1)} = \frac{57}{217+57} = 0.2080$$

Therefore, the "demand transition matrix" as of equation 3.5 is,

$$Q(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 0.7211 & 0.2789 \\ \boldsymbol{U} & 0.7920 & 0.2080 \end{matrix}$$

Table 4.21: Data "classification" by "state" transition for product E

Month	'State' transition $(\boldsymbol{i},\boldsymbol{j})$	Number of customers $\boldsymbol{N_{ij}(E,1)}$	Demand $\boldsymbol{D_{ij}(E,1)}$
1	FF	0	0
	FU	78	3420
	UF	156	8151
	UU	0	0
2	FF	0	0
	FU	0	0
	UF	61	3373
	UU	98	5572
3	FF	168	6892
	FU	0	0
	UF	56	2136
	UU	0	0

Table 4.21 shows for each week, for the first quarter of the year, the number of customers and their corresponding quantity demanded for a particular state transition for product E.

<u>TOTALS</u>

$N_{FF}(E,1) = 168$ $\qquad$ $N_{FU}(E,1) = 78$

$N_{UF}(E,1) = 156 + 61 + 56 = 273$ $\qquad$ $N_{UU}(E,1) = 98$

$D_{FF}(E,1) = 6892$ $\qquad$ $D_{FU}(E,1) = 3420$

$D_{UF}(E,1) = 8151 + 3373 + 2136 = 13660$ $\qquad$ $D_{UU}(E,1) = 5572$

Demand transition probabilities

$$Q_{FF}(E,1) = \frac{N_{FF}(E,1)}{N_{FF}(E,1)+N_{FU}(E,1)} = \frac{168}{168+78} = 0.6829$$

$$Q_{FU}(E,1) = \frac{N_{FU}(E,1)}{N_{FF}(E,1)+N_{FU}(E,1)} = \frac{78}{168+78} = 0.3171$$

$$Q_{UF}(E,1) = \frac{N_{UF}(D,1)}{N_{UF}(E,1)+N_{UU}(E,1)} = \frac{273}{273+98} = 0.7358$$

$$Q_{UU}(E,1) = \frac{N_{UU}(E,1)}{N_{UF}(E,1)+N_{UU}(E,1)} = \frac{98}{273+98} = 0.2642$$

Therefore, the ‘demand transition matrix’ as of equation 3.5 is,

$$Q(E,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \quad\quad \boldsymbol{U} \\ \begin{pmatrix} 0.6829 & 0.3171 \\ 0.7358 & 0.2642 \end{pmatrix} \end{matrix}$$

Table 4.22: Demand transition matrix

Product, P	Demand transition matrix, $Q(P,q)$
A	$\begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \quad\quad \boldsymbol{U} \\ \begin{pmatrix} 0.7349 & 0.2651 \\ 0.6822 & 0.3178 \end{pmatrix} \end{matrix}$
B	$\begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \quad\quad \boldsymbol{U} \\ \begin{pmatrix} 0.5164 & 0.4836 \\ 0.6840 & 0.3160 \end{pmatrix} \end{matrix}$
C	$\begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \quad\quad \boldsymbol{U} \\ \begin{pmatrix} 0.7198 & 0.2802 \\ 0.7509 & 0.2491 \end{pmatrix} \end{matrix}$
D	$\begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \quad\quad \boldsymbol{U} \\ \begin{pmatrix} 0.7211 & 0.2789 \\ 0.7920 & 0.2080 \end{pmatrix} \end{matrix}$
E	$\begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \quad\quad \boldsymbol{U} \\ \begin{pmatrix} 0.6829 & 0.3171 \\ 0.7358 & 0.2642 \end{pmatrix} \end{matrix}$

Table 4.22 shows the demand transition matrices got from the demand transition probabilities for each of the five products A, B, C, D, and E as demand transitions from one state to another for the first quarter of the year.

4.6 Demand, Inventory & Production-Inventory Cost Matrix

For all the five fast moving products, the 'demand, inventory and production-inventory cost' matrices are established as follows.

4.6.1 Demand matrix

From equation 3.6, the demand matrix for each of the products becomes;

For product A,

$$D(A,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad\qquad \boldsymbol{U} \\ \begin{pmatrix} D_{FF}(A,1) & D_{FU}(A,1) \\ D_{UF}(A,1) & D_{UU}(A,1) \end{pmatrix} \end{matrix}$$

$$D(A,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad \boldsymbol{U} \\ \begin{pmatrix} 15060 & 6702 \\ 27959 & 15758 \end{pmatrix} \end{matrix}$$

For product B,

$$D(B,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad\qquad \boldsymbol{U} \\ \begin{pmatrix} D_{FF}(B,1) & D_{FU}(B,1) \\ D_{UF}(B,1) & D_{UU}(B,1) \end{pmatrix} \end{matrix}$$

$$D(B,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad \boldsymbol{U} \\ \begin{pmatrix} 3232 & 5983 \\ 21286 & 8641 \end{pmatrix} \end{matrix}$$

For product C,

$$D(C,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad\qquad \boldsymbol{U} \\ \begin{pmatrix} D_{FF}(C,1) & D_{FU}(C,1) \\ D_{UF}(C,1) & D_{UU}(C,1) \end{pmatrix} \end{matrix}$$

$$D(C,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad \boldsymbol{U} \\ \begin{pmatrix} 13595 & 8276 \\ 26123 & 13457 \end{pmatrix} \end{matrix}$$

For product D,

$$D(D,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad\qquad \boldsymbol{U} \\ \begin{pmatrix} D_{FF}(D,1) & D_{FU}(D,1) \\ D_{UF}(D,1) & D_{UU}(D,1) \end{pmatrix} \end{matrix}$$

$$D(D,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \boldsymbol{F} \qquad \boldsymbol{U} \\ \begin{pmatrix} 12386 & 4648 \\ 21880 & 5940 \end{pmatrix} \end{matrix}$$

For product E,

$$D(E,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & D_{FF}(E,1) & D_{FU}(E,1) \\ \boldsymbol{U} & D_{UF}(E,1) & D_{UU}(E,1) \end{matrix}$$

$$D(E,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 6892 & 3420 \\ \boldsymbol{U} & 13660 & 5572 \end{matrix}$$

4.6.2 Inventory matrix

From equation 3.7, the inventory matrix for each of the products becomes;

For product A,

$$V(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(A,1) & V_{FU}(A,1) \\ \boldsymbol{U} & V_{UF}(A,1) & V_{UU}(A,1) \end{matrix}$$

$$V(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 5221 & 3271.5 \\ \boldsymbol{U} & 7154.5 & 5496.5 \end{matrix}$$

For product B,

$$V(B,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(B,1) & V_{FU}(B,1) \\ \boldsymbol{U} & V_{UF}(B,1) & V_{UU}(B,1) \end{matrix}$$

$$V(B,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 2992 & 2878.5 \\ \boldsymbol{U} & 3047.5 & 2195.5 \end{matrix}$$

For product C,

$$V(C,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(C,1) & V_{FU}(C,1) \\ \boldsymbol{U} & V_{UF}(C,1) & V_{UU}(C,1) \end{matrix}$$

$$V(C,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 4646 & 3923.5 \\ \boldsymbol{U} & 3957.5 & 3188.5 \end{matrix}$$

For product D,

$$V(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(D,1) & V_{FU}(D,1) \\ \boldsymbol{U} & V_{UF}(D,1) & V_{UU}(D,1) \end{matrix}$$

$$V(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 5363 & 5423 \\ \boldsymbol{U} & 6037.5 & 6082.5 \end{matrix}$$

For product E,

$$V(E,1) = \begin{matrix} & F & U \\ F & V_{FF}(E,1) & V_{FU}(E,1) \\ U & V_{UF}(E,1) & V_{UU}(E,1) \end{matrix}$$

$$V(E,1) = \begin{matrix} & F & U \\ F & 5595 & 5993.5 \\ U & 5116.5 & 5569.5 \end{matrix}$$

4.6.3 Production-inventory cost matrix

Using equations 3.8, 3.9 and 3.10, the 'production-inventory cost' matrices are then calculated for all the five products as presented below:

For product A,

Unit production cost, $C_p(A) = 722.22$ KES

Unit holding cost, $C_h(A) = 10.83$ KES

Unit shortage cost, $C_s(A) = 79.44$ KES

$$C_{FF}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{FF}(A,1) - V_{FF}(A,1)\right)$$

$$C_{FF}(A,1) = (722.22 + 10.83 + 79.44)(15060 - 5221) = 7994089.11$$

$$C_{FU}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{FU}(A,1) - V_{FU}(A,1)\right)$$

$$C_{FU}(A,1) = (722.22 + 10.83 + 79.44)(6702 - 3271.5) = 2787246.95$$

$$C_{UF}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{UF}(A,1) - V_{UF}(A,1)\right)$$

$$C_{UF}(A,1) = (722.22 + 10.83 + 79.44)(27959 - 7154.5) = 16903448.21$$

$$C_{UU}(A,1) = \left(C_h(A)\right)\left(D_{UU}(A,1) - V_{UU}(A,1)\right)$$

$$C_{UU}(A,1) = (10.83)(15758 - 5496.5) = 111132.05$$

Hence,

$$C(A,1) = \begin{matrix} & F & U \\ F & C_{FF}(A,1) & C_{FU}(A,1) \\ U & C_{UF}(A,1) & C_{UU}(A,1) \end{matrix}$$

$$C(A,1) = \begin{matrix} & F & U \\ F & 7994089.11 & 2787246.95 \\ U & 16903448.21 & 111132.05 \end{matrix}$$

For product B,

Unit production cost, $C_p(B) = 2566.67$ KES

Unit holding cost, $C_h(B) = 51.33$ KES

Unit shortage cost, $C_s(B) = 282.33$ KES

$$C_{FF}(B,1) = \left(C_p(B) + C_h(B) + C_s(B)\right)\left(D_{FF}(B,1) - V_{FF}(B,1)\right)$$

$$C_{FF}(B,1) = (2566.67 + 51.33 + 282.33)(3{,}232 - 2{,}992) = 696079.2$$

$$C_{FU}(B,1) = \left(C_p(B) + C_h(B) + C_s(B)\right)\left(D_{FU}(B,1) - V_{FU}(B,1)\right)$$

$$C_{FU}(B,1) = (2566.67 + 51.33 + 282.33)(5{,}983 - 2{,}878.5) = 9004074.49$$

$$C_{UF}(B,1) = \left(C_p(B) + C_h(B) + C_s(B)\right)\left(D_{UF}(B,1) - V_{UF}(B,1)\right)$$

$$C_{UF}(B,1) = (2566.67 + 51.33 + 282.33)(21{,}286 - 3{,}047.5) = 52897668.71$$

$$C_{UU}(B,1) = \left(C_h(B)\right)\left(D_{UU}(B,1) - V_{UU}(B,1)\right)$$

$$C_{UU}(B,1) = (51.33)(8{,}641 - 2{,}195.5) = 330847.52$$

Hence,

$$C(B,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(B,1) & C_{FU}(B,1) \\ \boldsymbol{U} & C_{UF}(B,1) & C_{UU}(B,1) \end{matrix}$$

$$C(B,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 696079.2 & 9004074.49 \\ \boldsymbol{U} & 52897668.71 & 330847.52 \end{matrix}$$

For product C,

Unit production cost, $C_p(C) = 836.11$ KES

Unit holding cost, $C_h(C) = 8.36$ KES

Unit shortage cost, $C_s(C) = 91.97$ KES

$$C_{FF}(C,1) = \left(C_p(C) + C_h(C) + C_s(C)\right)\left(D_{FF}(C,1) - V_{FF}(C,1)\right)$$

$$C_{FF}(C,1) = (836.11 + 8.36 + 91.97)(13595 - 4646) = 8380201.56$$

$$C_{FU}(C,1) = \left(C_p(C) + C_h(C) + C_s(C)\right)\left(D_{FU}(C,1) - V_{FU}(C,1)\right)$$

$$C_{FU}(C,1) = (836.11 + 8.36 + 91.97)(8276 - 3923.5) = 4075855.1$$

$$C_{UF}(C,1) = \left(C_p(C) + C_h(C) + C_s(C)\right)\left(D_{UF}(C,1) - V_{UF}(C,1)\right)$$

$$C_{UF}(C,1) = (836.11 + 8.36 + 91.97)(26123 - 3957.5) = 20756660.82$$

$$C_{UU}(C,1) = \left(C_h(C)\right)\left(D_{UU}(C,1) - V_{UU}(C,1)\right)$$

$$C_{UU}(C,1) = (8.36)(13457 - 3188.5) = 85844.66$$

Hence,

$$C(C,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(C,1) & C_{FU}(C,1) \\ \boldsymbol{U} & C_{UF}(C,1) & C_{UU}(C,1) \end{matrix}$$

$$C(C,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 8380201.56 & 4075855.1 \\ \boldsymbol{U} & 20756660.82 & 85844.66 \end{matrix}$$

For product D,

Unit production cost, $C_p(D) = 3150$ KES

Unit holding cost, $C_h(D) = 63$ KES

Unit shortage cost, $C_s(D) = 346.5$ KES

$$C_{FF}(D,1) = \left(C_p(D) + C_h(D) + C_s(D)\right)\left(D_{FF}(D,1) - V_{FF}(D,1)\right)$$

$$C_{FF}(D,1) = (3150 + 63 + 346.5)(12386 - 5363) = 24998368.5$$

$$C_{FU}(D,1) = C_h(D)\left(V_{FU}(D,1) - D_{FU}(D,1)\right)$$

$$C_{FU}(D,1) = 63(5423 - 4648) = 48825$$

$$C_{UF}(D,1) = \left(C_p(D) + C_h(D) + C_s(D)\right)\left(D_{UF}(D,1) - V_{UF}(D,1)\right)$$

$$C_{UF}(D,1) = (3150 + 63 + 346.5)(21880 - 6037.5) = 56391378.75$$

$$C_{UU}(D,1) = C_h(D)\left(V_{UU}(D,1) - D_{UU}(D,1)\right)$$

$$C_{UU}(D,1) = (63)(6082.5 - 5940) = 8977.5$$

Hence,

$$C(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(D,1) & C_{FU}(D,1) \\ \boldsymbol{U} & C_{UF}(D,1) & C_{UU}(D,1) \end{matrix}$$

$$C(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 24998368.5 & 48825 \\ \boldsymbol{U} & 56391378.75 & 8977.5 \end{matrix}$$

For product E,

Unit production cost, $C_p(E) = 2625$ KES

Unit holding cost, $C_h(E) = 78.75$ KES

Unit shortage cost, $C_s(E) = 288.75$ KES

$$C_{FF}(E,1) = \left(C_p(E) + C_h(E) + C_s(E)\right)\left(D_{FF}(E,1) - V_{FF}(E,1)\right)$$

$$C_{FF}(E,1) = (2625 + 78.75 + 288.75)(6892 - 5595) = 3881272.5$$

$$C_{FU}(E,1) = C_h(E)\left(V_{FU}(E,1) - D_{FU}(E,1)\right)$$

$$C_{FU}(E,1) = 78.75(5993.5 - 3420) = 202663.13$$

$$C_{UF}(E,1) = \left(C_p(E) + C_h(E) + C_s(E)\right)\left(D_{UF}(E,1) - V_{UF}(E,1)\right)$$

$$C_{UF}(E,1) = (2625 + 78.75 + 288.75)(13660 - 5116.5) = 25566423.75$$

$$C_{UU}(E,1) = \left(C_h(E)\right)\left(D_{UU}(E,1) - V_{UU}(E,1)\right)$$

$$C_{UU}(E,1) = (78.75)(5572 - 5569.5) = 196.875$$

Hence,

$$C(E,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(E,1) & C_{FU}(E,1) \\ \boldsymbol{U} & C_{UF}(E,1) & C_{UU}(E,1) \end{matrix}$$

$$C(E,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 3881272.5 & 202663.13 \\ \boldsymbol{U} & 25566423.75 & 196.875 \end{matrix}$$

Table 4.23: Demand, Inventory and Production-inventory cost matrices

Product, P	Demand matrix, $D(P,q)$	Inventory matrix, $V(P,q)$	Production-inventory cost matrix, $C(P,q)$
A	$\begin{array}{ccc} & F & U \\ F & 15060 & 6702 \\ U & 27959 & 15758 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 5221 & 3271.5 \\ U & 7154.5 & 5496.5 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 7994089.11 & 2787246.95 \\ U & 16903448.21 & 111132.05 \end{array}$
B	$\begin{array}{ccc} & F & U \\ F & 3232 & 5983 \\ U & 21286 & 8641 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 2992 & 2878.5 \\ U & 3047.5 & 2195.5 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 696079.2 & 9004074.49 \\ U & 52897668.71 & 330847.52 \end{array}$
C	$\begin{array}{ccc} & F & U \\ F & 13595 & 8276 \\ U & 26123 & 13457 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 4646 & 3923.5 \\ U & 3957.5 & 3188.5 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 8380201.56 & 4075855.1 \\ U & 20756660.82 & 85844.66 \end{array}$
D	$\begin{array}{ccc} & F & U \\ F & 12386 & 4648 \\ U & 21880 & 5940 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 5363 & 5423 \\ U & 6037.5 & 6082.5 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 24998368.5 & 48825 \\ U & 56391378.75 & 8977.5 \end{array}$
E	$\begin{array}{ccc} & F & U \\ F & 6892 & 3420 \\ U & 13660 & 5572 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 5595 & 5993.5 \\ U & 5116.5 & 5569.5 \end{array}$	$\begin{array}{ccc} & F & U \\ F & 3881272.5 & 202663.13 \\ U & 25566423.75 & 196.875 \end{array}$

Table 4.23 shows the Demand, Inventory and Production-inventory cost matrices for each of the five products A, B, C, D, and E as demand transitions from one state to another for the first quarter of the year.

4.7 Expected Demand

After producing the “demand transition” matrices and defining the “production-inventory cost” matrix, the expected “demand, inventory & production-inventory costs” are calculated for all the items bearing in mind both “favorable & unfavorable” demand.

Using equations 3.11 and 3.12, the expected demand for both ‘favorable & unfavorable’ demand was calculated respectively as below;

For product A

Favorable demand (F)

$$E[D_F(A,1)] = Q_{FF}(A,1) * D_{FF}(A,1) + Q_{FU}(A,1) * D_{FU}(A,1)$$

$$E[D_F(A,1)] = (0.7349 * 15{,}060) + (0.2651 * 6{,}702)$$

$$E[D_F(A,1)] = 12{,}844.3\ units$$

Unfavorable demand (U)

$$E[D_U(A,1)] = Q_{UF}(A,1) * D_{UF}(A,1) + Q_{UU}(A,1) * D_{UU}(A,1)$$

$$E[D_U(A,1)] = (0.6822 * 27{,}959) + (0.3178 * 15{,}758)$$

$$E[D_U(A,1)] = 24{,}081.5\ units$$

For product B

Favorable demand (F)

$$E[D_F(B,1)] = Q_{FF}(B,1) * D_{FF}(B,1) + Q_{FU}(B,1) * D_{FU}(B,1)$$

$$E[D_F(B,1)] = (0.5164 * 3{,}232) + (0.4836 * 5{,}983)$$

$$E[D_F(B,1)] = 4{,}562.3836\ units$$

Unfavorable demand (U)

$$E[D_U(B,1)] = Q_{UF}(B,1) * D_{UF}(B,1) + Q_{UU}(B,1) * D_{UU}(B,1)$$

$$E[D_U(B,1)] = (0.6840 * 21{,}286) + (0.3160 * 8{,}641)$$

$$E[D_U(B,1)] = 17{,}290.18\ units$$

For product C

Favorable demand (F)

$$E[D_F(C,1)] = Q_{FF}(C,1) * D_{FF}(C,1) + Q_{FU}(C,1) * D_{FU}(C,1)$$

$$E[D_F(C,1)] = (0.7198 * 13595) + (0.2802 * 8276)$$

$$E[D_F(C,1)] = 12{,}104.6162\ units$$

Unfavorable demand (U)

$$E[D_U(C,1)] = Q_{UF}(C,1) * D_{UF}(C,1) + Q_{UU}(C,1) * D_{UU}(C,1)$$

$$E[D_U(C,1)] = (0.7509 * 26123) + (0.2491 * 13457)$$

$$E[D_U(C,1)] = 22967.8994\ units$$

For product D

Favorable demand (F)

$$E[D_F(D,1)] = Q_{FF}(D,1) * D_{FF}(D,1) + Q_{FU}(D,1) * D_{FU}(D,1)$$

$$E[D_F(D,1)] = (0.7211 * 12386) + (0.2789 * 4648)$$

$$E[D_F(D,1)] = 11227.8718\, units$$

Unfavorable demand (U)

$$E[D_U(D,1)] = Q_{UF}(D,1) * D_{UF}(D,1) + Q_{UU}(D,1) * D_{UU}(D,1)$$

$$E[D_U(D,1)] = (0.7920 * 21880) + (0.2080 * 5940)$$

$$E[D_U(D,1)] = 18564.48\, units$$

For product E

Favorable demand (F)

$$E[D_F(E,1)] = Q_{FF}(E,1) * D_{FF}(E,1) + Q_{FU}(E,1) * D_{FU}(E,1)$$

$$E[D_F(E,1)] = (0.6829 * 6892) + (0.3171 * 3420)$$

$$E[D_F(E,1)] = 5791.0288\, units$$

Unfavorable demand (U)

$$E[D_U(E,1)] = Q_{UF}(E,1) * D_{UF}(E,1) + Q_{UU}(E,1) * D_{UU}(E,1)$$

$$E[D_U(E,1)] = (0.7358 * 13660) + (0.2642 * 5572)$$

$$E[D_U(E,1)] = 11523.1504\, units$$

4.8 Expected Inventory

Calculation of the “expected inventory” bearing in mind both “favorable & unfavorable” demand for all the five products was done using equations 3.13 and 3.14 respectively as below;

For product A

Favorable demand (F)

$$E[V_F(A,1)] = Q_{FF}(A,1) * V_{FF}(A,1) + Q_{FU}(A,1) * V_{FU}(A,1)$$

$$E[V_F(A,1)] = (0.7349 * 5221) + (0.2651 * 3271.5)$$

$$E[V_F(A,1)] = 4{,}704.2 \; units$$

Unfavorable demand (U)

$$E[V_U(A,1)] = Q_{UF}(A,1) * V_{UF}(A,1) + Q_{UU}(A,1) * V_{UU}(A,1)$$

$$E[V_U(A,1)] = (0.6822 * 7154.5) + (0.3178 * 5496.5)$$

$$E[V_U(A,1)] = 6{,}627.6 \; units$$

For product B

Favorable demand (F)

$$E[V_F(B,1)] = Q_{FF}(B,1) * V_{FF}(B,1) + Q_{FU}(B,1) * V_{FU}(B,1)$$

$$E[V_F(B,1)] = (0.5164 * 2992) + (0.4836 * 2878.5)$$

$$E[V_F(B,1)] = 2{,}937.1114 \; units$$

Unfavorable demand (U)

$$E[V_U(B,1)] = Q_{UF}(B,1) * V_{UF}(B,1) + Q_{UU}(B,1) * V_{UU}(B,1)$$

$$E[V_U(B,1)] = (0.6840 * 3047.5) + (0.3160 * 2195.5)$$

$$E[V_U(B,1)] = 2{,}778.268 \; units$$

For product C

Favorable demand (F)

$$E[V_F(C,1)] = Q_{FF}(C,1) * V_{FF}(C,1) + Q_{FU}(C,1) * V_{FU}(C,1)$$

$$E[V_F(C,1)] = (0.7198 * 4646) + (0.2802 * 3923.5)$$

$$E[V_F(C,1)] = 4{,}443.5555 \; units$$

Unfavorable demand (U)

$$E[V_U(C,1)] = Q_{UF}(C,1) * V_{UF}(C,1) + Q_{UU}(C,1) * V_{UU}(C,1)$$

$$E[V_U(C,1)] = (0.7509 * 3957.5) + (0.2491 * 3188.5)$$

$$E[V_U(C,1)] = 3765.9421 \; units$$

For product D

Favorable demand (F)

$$E[V_F(D,1)] = Q_{FF}(D,1) * V_{FF}(D,1) + Q_{FU}(D,1) * V_{FU}(D,1)$$

$$E[V_F(D,1)] = (0.7211 * 5363) + (0.2789 * 5423)$$

$$E[V_F(D,1)] = 5379.734 \; units$$

Unfavorable demand (U)

$$E[V_U(D,1)] = Q_{UF}(D,1) * V_{UF}(D,1) + Q_{UU}(D,1) * V_{UU}(D,1)$$

$$E[V_U(D,1)] = (0.7920 * 6037.5) + (0.2080 * 6082.5)$$

$$E[V_U(D,1)] = 6046.86 \; units$$

For product E

Favorable demand (F)

$$E[V_F(E,1)] = Q_{FF}(E,1) * V_{FF}(E,1) + Q_{FU}(E,1) * V_{FU}(E,1)$$

$$E[V_F(E,1)] = (0.6829 * 5595) + (0.3171 * 5993.5)$$

$$E[V_F(E,1)] = 5721.3644 \; units$$

Unfavorable demand (U)

$$E[V_U(E,1)] = Q_{UF}(E,1) * V_{UF}(E,1) + Q_{UU}(E,1) * V_{UU}(E,1)$$

$$E[V_U(E,1)] = (0.7358 * 5116.5) + (0.2642 * 5569.5)$$

$$E[V_U(E,1)] = 5236.1826 \; units$$

4.9 Expected Production-Inventory Costs

The “expected production-Inventory” costs are thereafter calculated for all the five products bearing in mind both “favorable & unfavorable” demand using equations 3.15 and 3.16 respectively as presented beneath:

For product A

Favorable demand (F)

$$E[C_F(A,1)] = Q_{FF}(A,1) * C_{FF}(A,1) + Q_{FU}(A,1) * C_{FU}(A,1)$$

$$E[C_F(A,1)] = (0.7349 * 7994089.11) + (0.2651 * 2787246.95)$$

$$E[C_F(A,1)] = 6{,}613{,}755.3 \text{ KES}$$

Unfavorable demand (U)

$$E[C_U(A,1)] = Q_{UF}(A,1) * C_{UF}(A,1) + Q_{UU}(A,1) * C_{UU}(A,1)$$

$$E[C_U(A,1)] = (0.6822 * 16903448.21) + (0.3178 * 111132.05)$$

$$E[C_U(A,1)] = 11{,}566{,}850.1 \text{ KES}$$

For product B

Favorable demand (F)

$$E[C_F(B,1)] = Q_{FF}(B,1) * C_{FF}(B,1) + Q_{FU}(B,1) * C_{FU}(B,1)$$

$$E[C_F(B,1)] = (0.5164 * 696079.2) + (0.4836 * 9004074.49)$$

$$E[C_F(B,1)] = 4{,}713{,}825.72 \text{ KES}$$

Unfavorable demand (U)

$$E[C_U(B,1)] = Q_{UF}(B,1) * C_{UF}(B,1) + Q_{UU}(B,1) * C_{UU}(B,1)$$

$$E[C_U(B,1)] = (0.6840 * 52897668.71) + (0.3160 * 330847.52)$$

$$E[C_U(B,1)] = 36{,}286{,}553.21 \text{ KES}$$

For product C

Favorable demand (F)

$$E[C_F(C,1)] = Q_{FF}(C,1) * C_{FF}(C,1) + Q_{FU}(C,1) * C_{FU}(C,1)$$

$$E[C_F(C,1)] = (0.7198 * 8380201.56) + (0.2802 * 4075855.1)$$

$$E[C_F(C,1)] = 7{,}174{,}123.68 \text{ KES}$$

Unfavorable demand (U)

$$E[C_U(C,1)] = Q_{UF}(C,1) * C_{UF}(C,1) + Q_{UU}(C,1) * C_{UU}(C,1)$$

$$E[C_U(C,1)] = (0.7509 * 20756660.82) + (0.2491 * 85844.66)$$

$$E[C_U(C,1)] = 15{,}607{,}560.51 \text{ KES}$$

For product D

Favorable demand (F)

$$E[C_F(D,1)] = Q_{FF}(D,1) * C_{FF}(D,1) + Q_{FU}(D,1) * C_{FU}(D,1)$$

$$E[C_F(D,1)] = (0.7211 * 24998368.5) + (0.2789 * 48825)$$

$$E[C_F(D,1)] = 18{,}039{,}940.82 \text{ KES}$$

Unfavorable demand (U)

$$E[C_U(D,1)] = Q_{UF}(D,1) * C_{UF}(D,1) + Q_{UU}(D,1) * C_{UU}(D,1)$$

$$E[C_U(D,1)] = (0.7920 * 56391378.75) + (0.2080 * 8977.5)$$

$$E[C_U(D,1)] = 44{,}663{,}839.29 \text{ KES}$$

For product E

Favorable demand (F)

$$E[C_F(E,1)] = Q_{FF}(E,1) * C_{FF}(E,1) + Q_{FU}(E,1) * C_{FU}(E,1)$$

$$E[C_F(E,1)] = (0.6829 * 3881272.5) + (0.3171 * 202663.13)$$

$$E[C_F(E,1)] = 2{,}714{,}785.47 \text{ KES}$$

Unfavorable demand (U)

$$E[C_U(E,1)] = Q_{UF}(E,1) * C_{UF}(E,1) + Q_{UU}(E,1) * C_{UU}(E,1)$$

$$E[C_U(E,1)] = (0.7358 * 25566423.75) + (0.2642 * 196.875)$$

$$E[C_U(E,1)] = 18{,}811{,}826.61 \text{ KES}$$

4.10 Expected Manufacturing Lot Size

Calculation of the "expected manufacturing lot size" bearing in mind both "favorable & unfavorable" demand for all the five products was done using equations 3.17 and 3.18 respectively as below:

For product A

Favorable demand (F)

$$E[M_F(A,1)] = \begin{pmatrix} E[D_F(A,1)] - E[V_F(A,1)] \ if \ E[D_F(A,1)] > E[V_F(A,1)] \\ 0 \qquad\qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_F(A,1)] = E[D_F(A,1)] - E[V_F(A,1)]$$

$$E[M_F(A,1)] = 12{,}844.3 - 4{,}704.2 = 8{,}140.1 \ units$$

Unfavorable demand (U)

$$E[M_U(A,1)] = \begin{pmatrix} E[D_U(A,1)] - E[V_U(A,1)] \ if \ E[D_U(A,1)] > E[V_U(A,1)] \\ 0 \qquad\qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_U(A,1)] = E[D_U(A,1)] - E[V_U(A,1)]$$

$$E[M_U(A,1)] = 24{,}081.5 - 6{,}627.6 = 17{,}453.9 \ units$$

For product B

Favorable demand (F)

$$E[M_F(B,1)] = \begin{pmatrix} E[D_F(B,1)] - E[V_F(B,1)] \ if \ E[D_F(B,1)] > E[V_F(B,1)] \\ 0 \qquad\qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_F(B,1)] = E[D_F(B,1)] - E[V_F(B,1)]$$

$$E[M_F(B,1)] = 4{,}562.3836 - 2{,}937.1114 = 1{,}625.2722 \ units$$

Unfavorable demand (U)

$$E[M_U(B,1)] = \begin{pmatrix} E[D_U(B,1)] - E[V_U(B,1)] \ if \ E[D_U(B,1)] > E[V_U(B,1)] \\ 0 \qquad\qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_U(B,1)] = E[D_U(B,1)] - E[V_U(B,1)]$$

$$E[M_U(B,1)] = 17{,}290.18 - 2{,}778.268 = 14{,}511.912 \ units$$

For product C

Favorable demand (F)

$$E[M_F(C,1)] = \begin{pmatrix} E[D_F(C,1)] - E[V_F(C,1)] \; if \; E[D_F(C,1)] > E[V_F(C,1)] \\ 0 \qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_F(C,1)] = E[D_F(C,1)] - E[V_F(C,1)]$$

$$E[M_F(C,1)] = 12{,}104.6162 - 4{,}443.5555 = 7661.0607 \; units$$

Unfavorable demand (U)

$$E[M_U(C,1)] = \begin{pmatrix} E[D_U(C,1)] - E[V_U(C,1)] \; if \; E[D_U(C,1)] > E[V_U(C,1)] \\ 0 \qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_U(C,1)] = E[D_U(C,1)] - E[V_U(C,1)]$$

$$E[M_U(C,1)] = 22967.8994 - 3765.9421 = 19201.9573 \; units$$

For product D

Favorable demand (F)

$$E[M_F(D,1)] = \begin{pmatrix} E[D_F(D,1)] - E[V_F(D,1)] \; if \; E[D_F(D,1)] > E[V_F(D,1)] \\ 0 \qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_F(D,1)] = E[D_F(D,1)] - E[V_F(D,1)]$$

$$E[M_F(D,1)] = 11227.8718 - 5379.734 = 5848.1378 \; units$$

Unfavorable demand (U)

$$E[M_U(D,1)] = \begin{pmatrix} E[D_U(D,1)] - E[V_U(D,1)] \; if \; E[D_U(D,1)] > E[V_U(D,1)] \\ 0 \qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_U(D,1)] = E[D_U(D,1)] - E[V_U(D,1)]$$

$$E[M_U(D,1)] = 18564.48 - 6046.86 = 12517.62 \; units$$

For product E

Favorable demand (F)

$$E[M_F(E,1)] = \begin{pmatrix} E[D_F(E,1)] - E[V_F(E,1)] \; if \; E[D_F(E,1)] > E[V_F(E,1)] \\ 0 \qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_F(E,1)] = E[D_F(E,1)] - E[V_F(E,1)]$$

$$E[M_F(E,1)] = 5791.0288 - 5721.3644 = 69.6644\ units$$

Unfavorable demand (U)

$$E[M_U(E,1)] = \begin{pmatrix} E[D_U(E,1)] - E[V_U(E,1)]\ if\ E[D_U(E,1)] > E[V_U(E,1)] \\ 0 \qquad\qquad otherwise \end{pmatrix}$$

$$E[M_U(E,1)] = E[D_U(E,1)] - E[V_U(E,1)]$$

$$E[M_U(E,1)] = 11523.1504 - 5236.1826 = 6286.9678\ units$$

Table 4.24: Expected "demand, Inventory, production-inventory" cost & manufacturing lot size

Product	Expected demand		Expected Inventory		Expected 'production-inventory' cost		Expected 'manufacturing lot size'	
	F	U	F	U	F	U	F	U
A	12,844.30	24,081.50	4,704.20	6,627.60	6,613,755.30	11,566,850.10	8,140.10	17,453.90
B	4,562.40	17,290.20	2,937.10	2,778.30	4,713,825.72	36,286,553.21	1,625.30	14,511.90
C	12,104.60	22967.80	4,443.60	3765.90	7,174,123.68	15,607,560.51	7661.10	19201.90
D	11227.90	18564.50	5379.70	6046.90	18,039,940.82	44,663,839.29	5848.10	12517.60
E	5791.00	11523.20	5721.40	5236.20	2,714,785.47	18,811,826.61	69.70	6286.90

Table 4.24 shows the Expected demand, Expected Inventory, Expected production-inventory cost and Expected manufacturing lot size for each of the five products A, B, C, D, and E for each state of demand, that is, either favorable (F) or unfavorable (U).

4.11 Stochastic Goal Programming Model

The "stochastic goal programming" model for each of the five products was expressed by setting priorities, describing the objective function & framing the goal constraints as follows:

For product A

Priorities set

$P_1: Produce\ a\ batch\ of$ 8,140.1 $units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of$ 17,453.9 $units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production\_inventory\ costs\ must\ not\ exceed$ 6,613,755.3 KES $when\ demand\ is\ favorable$

$P_4: Total\ production\_inventory\ costs\ must\ not\ exceed$ 11,566,850.1 KES $when\ demand\ is\ unfavorable$

Objective function

$$Minimize\ Z = \sum_{k=1}^{4}[P_K(A,1)d_k^+ + P_K(A,1)d_k^-]$$

Goal constraints

Manufacturing lot size

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1\ (Favorable\ demand)$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$7994089.11X_{FF}(A,1) + 2787246.95X_{FU}(A,1) - d_3^+ = 6{,}613{,}755.3\ (Favorable\ demand)$$

$$16903448.21X_{UF}(A,1) + 111132.05X_{UU}(A,1) - d_4^+ = 11{,}566{,}850.1\ (Unfavorable\ demand)$$

Non negativity

$$X_{FF}(A,1), X_{FU}(A,1), X_{UF}(A,1), X_{UU}(A,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

For product B

Priorities set

$P_1: Produce\ a\ batch\ of$ 4,562.3836 $units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of$ 17,290.18 $units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production\_inventory\ costs\ must\ not\ exceed$ 4,713,825.72 KES $when\ demand$ $is\ favorable$

$P_4: Total\ production\_inventory\ costs\ must\ not\ exceed$ 36,286,553.21 KES $when\ demand$ $is\ unfavorable$

Objective function

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(B,1)d_k^+ + P_K(B,1)d_k^-]$$

Goal constraints

Manufacturing lot size

$$X_{FF}(B,1) + X_{FU}(B,1) + d_1^- = 4{,}562.3836\ (Favorable\ demand)$$

$$X_{UF}(B,1) + X_{UU}(B,1) + d_2^- = 17{,}290.18\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$696079.2X_{FF}(B,1) + 9004074.49X_{FU}(B,1) - d_3^+ = 4{,}713{,}825.72\ (Favorable\ demand)$$

$$52897668.71X_{UF}(B,1) + 330847.52X_{UU}(B,1) - d_4^+ = 36{,}286{,}553.21(Unfavorable\ demand)$$

Non negativity

$$X_{FF}(B,1), X_{FU}(B,1), X_{UF}(B,1), X_{UU}(B,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

For product C

Priorities set

$P_1: Produce\ a\ batch\ of$ 12,104.6162 $units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of$ 22,967.8994 $units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production\_inventory\ costs\ must\ not\ exceed$ 7,174,123.68 KES $when\ demand$ $is\ favorable$

P_4: $Total\ production\_inventory\ costs\ must\ not\ exceed$ 15,607,560.51 KES $when\ demand$

$is\ unfavorable$

<u>Objective function</u>

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(C,1)d_k^+ + P_K(C,1)d_k^-]$$

<u>Goal constraints</u>

Manufacturing lot size

$$X_{FF}(C,1) + X_{FU}(C,1) + d_1^- = 12{,}104.6162\ (Favorable\ demand)$$

$$X_{UF}(C,1) + X_{UU}(C,1) + d_2^- = 22{,}967.8994\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$8380201.56X_{FF}(C,1) + 4075855.1X_{FU}(C,1) - d_3^+ = 7{,}174{,}123.68\ (Favorable\ demand)$$

$$20756660.82X_{UF}(C,1) + 85844.66X_{UU}(C,1) - d_4^+ = 15{,}607{,}560.51\ (Unfavorable\ demand)$$

Non negativity

$$X_{FF}(C,1), X_{FU}(C,1), X_{UF}(C,1), X_{UU}(C,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

For product D

<u>Priorities set</u>

P_1: $Produce\ a\ batch\ of$ 5848.1378 $units\ when\ demand\ is\ initially\ favorable$

P_2: $Produce\ a\ batch\ of$ 12517.62 $units\ when\ demand\ is\ initially\ unfavorable$

P_3: $Total\ production\_inventory\ costs\ must\ not\ exceed$ 18,039,940.82 KES $when\ demand$

$is\ favorable$

P_4: $Total\ production\_inventory\ costs\ must\ not\ exceed$ 44,663,839.29 KES $when\ demand$

$is\ unfavorable$

Objective function

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(D,1)d_k^+ + P_K(D,1)d_k^-]$$

Goal constraints

Manufacturing lot size

$$X_{FF}(D,1) + X_{FU}(D,1) + d_1^- = 5848.1378\ (Favorable\ demand)$$

$$X_{UF}(D,1) + X_{UU}(D,1) + d_2^- = 12517.62\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$24998368.5X_{FF}(D,1) + 48825X_{FU}(D,1) - d_3^+ = 18{,}039{,}940.82\ (Favorable\ demand)$$

$$56391378.75X_{UF}(D,1) + 8977.5X_{UU}(D,1) - d_4^+ = 44{,}663{,}839.29\ (Unfavorable\ demand)$$

Non negativity

$$X_{FF}(D,1), X_{FU}(D,1), X_{UF}(D,1), X_{UU}(D,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

For product E

Priorities set

P_1: *Produce a batch of* 69.6644 *units when demand is initially favorable*

P_2: *Produce a batch of* 6286.9678 *units when demand is initially unfavorable*

P_3: *Total production _ inventory costs must not exceed* 2,714,785.47 KES *when demand is favorable*

P_4: *Total production _ inventory costs must not exceed* 18,811,826.61 KES *when demand is unfavorable*

<u>Objective function</u>

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(E,1)d_k^+ + P_K(E,1)d_k^-]$$

<u>Goal constraints</u>

Manufacturing lot size

$$X_{FF}(E,1) + X_{FU}(E,1) + d_1^- = 69.6644\ (Favorable\ demand)$$

$$X_{UF}(E,1) + X_{UU}(E,1) + d_2^- = 6286.9678\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$3881272.5X_{FF}(E,1) + 202663.13X_{FU}(E,1) - d_3^+ = 2{,}714{,}785.47\ (Favorable\ demand)$$

$$25566423.75X_{UF}(E,1) + 196.875X_{UU}(E,1) - d_4^+ = 18{,}811{,}826.61\ (Unfavorable\ demand)$$

Non negativity

$$X_{FF}(E,1), X_{FU}(E,1), X_{UF}(E,1), X_{UU}(E,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

4.12 Stochastic Goal Programming Model for Manufacturing Lot Size

The “stochastic goal programming” model for “manufacturing lot size” was thereafter established as below for all the products. The model defines the amount of each item to be manufactured in the first quarter of the year as demand shifts from state i to state j for $i, j \in \{F, U\}$

<u>For product A</u>

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(A,1)d_k^+ + P_K(A,1)d_k^-]$$

Subject to:

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9$$

$$7994089.11X_{FF}(A,1) + 2787246.95X_{FU}(A,1) - d_3^+ = 6{,}613{,}755.3$$

$$16903448.21X_{UF}(A,1) + 111132.05X_{UU}(A,1) - d_4^+ = 11{,}566{,}850.1$$

$$X_{FF}(A,1), X_{FU}(A,1), X_{UF}(A,1), X_{UU}(A,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(A,1)$ – "manufacturing lot size" of product A as originally "favorable" demand remains "favorable"

$X_{FU}(A,1)$– "manufacturing lot size" of product A as primarily "favorable" demand turns to "unfavorable"

$X_{UF}(A,1)$ – "manufacturing lot size" of product A as originally "unfavorable" demand turns to "favorable"

$X_{UU}(A,1)$– "manufacturing lot size" of product A as primarily "unfavorable" demand remains "unfavorable"

For product B

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(B,1)d_k^+ + P_K(B,1)d_k^-]$$

Subject to:

$$X_{FF}(B,1) + X_{FU}(B,1) + d_1^- = 4{,}562.3836$$

$$X_{UF}(B,1) + X_{UU}(B,1) + d_2^- = 17{,}290.18$$

$$696079.2X_{FF}(B,1) + 9004074.49X_{FU}(B,1) - d_3^+ = 4{,}713{,}825.72$$

$$52897668.71X_{UF}(B,1) + 330847.52X_{UU}(B,1) - d_4^+ = 36{,}286{,}553.21$$

$$X_{FF}(B,1), X_{FU}(B,1), X_{UF}(B,1), X_{UU}(B,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(B,1)$ – "manufacturing lot size" of product B as originally "favorable" demand remains "favorable"

$X_{FU}(B,1)$ – "manufacturing lot size" of product B as primarily "favorable" demand turns to "unfavorable"

$X_{UF}(B,1)$ – "manufacturing lot size" of product B as originally "unfavorable" demand turns to "favorable"

$X_{UU}(B,1)$ – "manufacturing lot size" of product B as primarily "unfavorable" demand remains "unfavorable"

<u>For product C</u>

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(C,1)d_k^+ + P_K(C,1)d_k^-]$$

Subject to:

$$X_{FF}(C,1) + X_{FU}(C,1) + d_1^- = 12{,}104.6162$$

$$X_{UF}(C,1) + X_{UU}(C,1) + d_2^- = 22{,}967.8994$$

$$8380201.56X_{FF}(C,1) + 4075855.1X_{FU}(C,1) - d_3^+ = 7{,}174{,}123.68$$

$$20756660.82X_{UF}(C,1) + 85844.66X_{UU}(C,1) - d_4^+ = 15{,}607{,}560.51$$

$$X_{FF}(C,1), X_{FU}(C,1), X_{UF}(C,1), X_{UU}(C,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(C,1)$ – "manufacturing lot size" of product C when initially "favorable" demand remains "favorable"

$X_{FU}(C,1)$ – “manufacturing lot size” of product C when initially “favorable” demand turns to “unfavorable”

$X_{UF}(C,1)$ – “manufacturing lot size” of product C when initially “unfavorable” demand turns to “favorable”

$X_{UU}(C,1)$ – “manufacturing lot size” of product C when initially “unfavorable” demand remains “unfavorable”

<u>For product D</u>

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(D,1)d_k^+ + P_K(D,1)d_k^-]$$

Subject to:

$$X_{FF}(D,1) + X_{FU}(D,1) + d_1^- = 5848.1378$$

$$X_{UF}(D,1) + X_{UU}(D,1) + d_2^- = 12517.62$$

$$24998368.5X_{FF}(D,1) + 48825X_{FU}(D,1) - d_3^+ = 18{,}039{,}940.82$$

$$56391378.75X_{UF}(D,1) + 8977.5X_{UU}(D,1) - d_4^+ = 44{,}663{,}839.29$$

$$X_{FF}(D,1), X_{FU}(D,1), X_{UF}(D,1), X_{UU}(D,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(D,1)$ – “manufacturing lot size” of product D as primarily “favorable” demand remains “favorable”

$X_{FU}(D,1)$ – “manufacturing lot size” of product D as originally “favorable” demand turns to “unfavorable”

$X_{UF}(D,1)$ – “manufacturing lot size” of product D as primarily “unfavorable” demand turns to “favorable”

$X_{UU}(D,1)$ – "manufacturing lot size" of product D as originally "unfavorable" demand remains "unfavorable"

For product E

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(E,1)d_k^+ + P_K(E,1)d_k^-]$$

Subject to:

$$X_{FF}(E,1) + X_{FU}(E,1) + d_1^- = 69.6644$$

$$X_{UF}(E,1) + X_{UU}(E,1) + d_2^- = 6286.9678$$

$$3881272.5X_{FF}(E,1) + 202663.13X_{FU}(E,1) - d_3^+ = 2{,}714{,}785.47$$

$$25566423.75X_{UF}(E,1) + 196.875X_{UU}(E,1) - d_4^+ = 18{,}811{,}826.61$$

$$X_{FF}(E,1), X_{FU}(E,1), X_{UF}(E,1), X_{UU}(E,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(E,1)$ – "manufacturing lot size" of product E as primarily "favorable" demand remains "favorable"

$X_{FU}(E,1)$ – "manufacturing lot size" of product E as originally "favorable" demand turns to "unfavorable"

$X_{UF}(E,1)$ – "manufacturing lot size" of product E as primarily "unfavorable" demand turns to "favorable"

$X_{UU}(E,1)$ – "manufacturing lot size" of product E as originally "unfavorable" demand remains "unfavorable"

4.13 Results of the stochastic goal programming model

In this study, the "stochastic goal programming" model for each of the products in the first quarter of the year was solved using MATLAB™ (in particular the lingprog solver) and the optimum result was achieved having the values as presented in Table 4.25.

Table 4.25: Optimum result from MATLAB

Variables	**Product**				
	A	**B**	**C**	**D**	**E**
X_{FF}	0	6.7720	0	0	0
X_{FU}	2.3729	0	1.7602	369.4800	13.3956
X_{UF}	0	0	0	0	0.6835
X_{UU}	104.0840	109.6800	181.8117	4975.1000	6286.3000
d_1^-	8137.7000	4555.6000	12103.0000	5478.7000	56.2688
d_2^-	17350.0000	17181.0000	22786.0000	7542.5000	0
d_3^+	0	0	0	0	0
d_4^+	0	0	0	0	0
Z	98,702,000	62,715,000	14,378,000	62,316,000	562,630

Table 4.25 shows the optimum results got from the Matlab software, using linprog solver, giving the manufacturing lot sizes, over achievements, under achievements and the total production-inventory cost for all the five products A, B, C, D, and E for the first quarter of the year.

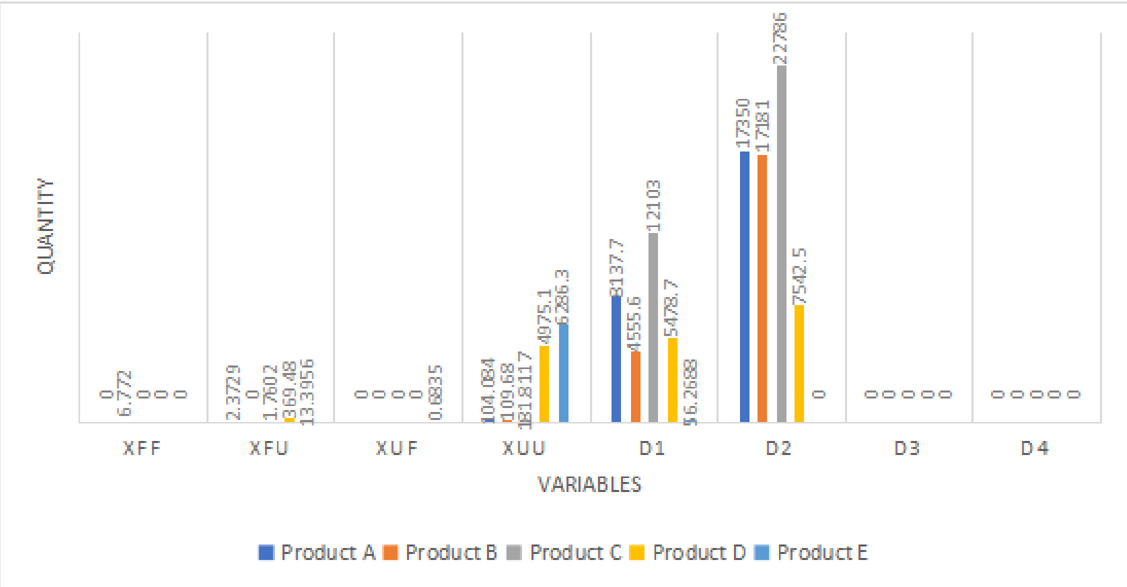

Figure 4.14: Manufacturing lot size with over or under achievement in each state transition

Figure 4.14 illustrates the values of the manufacturing lot size with the over achievement or under achievement in each state transition for all the five products A, B, C, D, and E for the first quarter of the year.

4.14 Discussion of Results

The results were analyzed and discussed for each product basing on the priorities fixed and the optimum values attained from sub section 4.12.

The priorities set for product A are as follows;

$P_1: Produce\ a\ batch\ of\ 8{,}140.1\ units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of\ 17{,}453.9\ units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production_{inventory} costs\ must\ not\ exceed\ 6{,}613{,}755.3\ \text{KES}$

$when\ demand\ is\ favorable$

$P_4: Total\ production_{inventory} costs\ must\ not\ exceed\ 11{,}566{,}850.1\ \text{KES}$

$when\ demand\ is\ unfavorable$

X_{FF} (A,1) = 0;

The “manufacturing lot size” of product A in the first quarter when initially favorable demand remains favorable is 0 units. This means that more products shouldn’t be manufactured but utilize what is existing in stock as it is sufficient to satisfy the demand.

X_{FU} (A,1) = 2.3729;

The manufacturing lot size of product A in the first quarter when initially favorable demand becomes unfavorable is 2.3729 units. This means that more products, 2.3729 units, should be produced to meet demand.

X_{UF} (A,1) = 0;

The manufacturing lot size of product A in the first quarter when initially unfavorable demand becomes favorable is 0 units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

X_{UU} (A,1) = 104.0840;

The manufacturing lot size of product A in the first quarter when initially unfavorable demand remains unfavorable is 104.0840 units. This means that more products, 104.0840 units, should be produced to meet demand.

Goal constraints

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9$$

$$7994089.11X_{FF}(A,1) + 2787246.95X_{FU}(A,1) - d_3^+ = 6{,}613{,}755.3$$

$$16903448.21X_{UF}(A,1) + 111132.05X_{UU}(A,1) - d_4^+ = 11{,}566{,}850.1$$

Priority 1

$P_1: Produce\ a\ batch\ of\ 8{,}140.1\ units\ when\ demand\ is\ initially\ favorable$

When demand is initially favorable (state F), the amount to be produced in the first quarter is;

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1$$

$$0 + 2.3729 + 8137.7 = 8140.07 \text{ units}$$

This is equal to the targeted production level for goal constraint (1). Therefore, Priority 1 may be completely attained. Nonetheless, an "underachievement" of 8137.7 units is however got in the first quarter as demand is originally "favorable" (state F).

<u>Priority 2</u>

$P_2: Produce\ a\ batch\ of$ 17,453.9 $units\ when\ demand\ is\ initially\ unfavorable$

When demand is initially unfavorable (state U) the amount to be produced in the first quarter is;

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9$$

$$0 + 104.0840 + 17350 = 17454.08 \text{ units}$$

Priority 2 may be fully achieved as this value is equal to the targeted production level for goal constraint (2). On the other hand, an "underachievement" of 17350 units is got in the first quarter as demand is primarily 'unfavorable' (state U).

<u>Priority 3</u>

$P_3: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 6,613,755.3KES $when\ demand\ is\ favorable$

When demand is initially favorable (state F) the total "production-inventory" costs in the first quarter is;

$$7994089.11X_{FF}(A,1) + 2787246.95X_{FU}(A,1) - d_3^+ = 6{,}613{,}755.3$$

$$7994089.11(0) + 2787246.95(2.3729) - 0 = 6613858.29$$

This to some extent is greater than the targeted 'production-inventory' costs for goal constraint (3). Priority 3 is therefore partially attained in the first quarter as demand is originally "favorable" (state F).

<u>Priority 4</u>

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 11,566,850.1KES $when\ demand\ is\ unfavorable$

When demand is initially unfavorable (state U) the total "production-inventory" costs in the first quarter is;

$$16903448.21X_{UF}(A,1) + 111132.05X_{UU}(A,1) - d_4^+ = 11{,}566{,}850.1$$

$$16903448.21(0) + 111132.05(104.0840) - 0 = 11567068.29$$

This to some extent is greater than the targeted ‘production-inventory’ costs for goal constraint (4). Priority 4 can therefore be partially attained in the first quarter as demand is originally “unfavorable” (state U).

Table 4.26: Expected goal values for product A, “stochastic” solution with “over & under” achievement

Goals/ priorities	Expected value from Goal	Value of the stochastic solution	Deviation	Over-achievement	Under-achievement
1	8,140.10	8140.07	0.03		8137.70
2	17,453.90	17454.08	0.18		17350.00
3	6,613,755.30	6613858.29	102.99	0	
4	11,566,850.10	11567068.29	218.19	0	

Table 4.26 shows the expected goal values for product A, the value of the stochastic solution giving the deviations with over achievement or under achievement.

The priorities set for product B are as follows;

P_1: *Produce a batch of* 4,562.3836 *units when demand is initially favorable*

P_2: *Produce a batch of* 17,290.18 *units when demand is initially unfavorable*

P_3: *Total production*$_{inventory}$ *costs must not exceed* 4,713,825.72 KES *when demand is favorable*

P_4: *Total production*$_{inventory}$ *costs must not exceed* 36,286,553.21 KES *when demand is unfavorable*

X_{FF} (B,1) = 6.7720;

The manufacturing lot size of product B in the first quarter when initially favorable demand remains favorable is 6.7720 units. This means that more products, 6.7720 units, should be produced to meet demand.

X_{FU} (B,1) = 0;

The manufacturing lot size of product B in the first quarter when initially favorable demand becomes unfavorable is 0 units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

X_{UF} (B,1) = 0;

The manufacturing lot size of product B in the first quarter when initially unfavorable demand becomes favorable is 0 units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

X_{UU} (B,1) = 109.68;

The manufacturing lot size of product B in the first quarter when initially unfavorable demand remains unfavorable is 109.68 units. This means that more products, 109.68 units, should be produced to meet demand.

Goal constraints;

$$X_{FF}(B,1) + X_{FU}(B,1) + d_1^- = 4{,}562.3836$$

$$X_{UF}(B,1) + X_{UU}(B,1) + d_2^- = 17{,}290.18$$

$$696079.2X_{FF}(B,1) + 9004074.49X_{FU}(B,1) - d_3^+ = 4{,}713{,}825.72$$

$$52897668.71X_{UF}(B,1) + 330847.52X_{UU}(B,1) - d_4^+ = 36{,}286{,}553.21$$

<u>Priority 1</u>

$P_1: Produce\ a\ batch\ of$ 4,562.3836 $units\ when\ demand\ is\ initially\ favorable$

When demand is initially favorable (state F), the amount to be produced in the first quarter is;

$$X_{FF}(B,1) + X_{FU}(B,1) + d_1^- = 4{,}562.3836$$

$$6.7720 + 0 + 4555.6 = 4562.372 \text{ units}$$

Priority 1 can be achieved as this value is equal to the targeted production level for goal constraint (1). On the other hand, an "underachievement" of 4555.6 units is however got in the first quarter as demand is originally "favorable" (state F)

<u>Priority 2</u>

$P_2: Produce\ a\ batch\ of$ 17,290.18 $units\ when\ demand\ is\ initially\ unfavorable$

When demand is initially unfavorable (state U) the amount to be produced in the first quarter is;

$$X_{UF}(B,1) + X_{UU}(B,1) + d_2^- = 17{,}290.18$$

$$0 + 109.68 + 17181 = 17290.68 \text{ units}$$

This is equal to the targeted production level for goal constraint (2). Therefore, Priority 2 may be completely attained. On the other hand, an "underachievement" of 17255 units is got in the first quarter as demand is originally 'unfavorable' (state U).

<u>Priority 3</u>

$P_3: Total\ production_{inventory} costs\ must\ not\ exceed$ 4,713,825.72 KES

$when\ demand\ is\ favorable$

When demand is initially favorable (state F) the total "production-inventory" costs in the first quarter is;

$$696079.2X_{FF}(B,1) + 9004074.49X_{FU}(B,1) - d_3^+ = 4{,}713{,}825.72$$

$$696079.2(6.7720) + 9004074.49(0) - 0 = 4713848.34$$

This is a little greater than the targeted 'production-inventory' costs for goal constraint (3). Priority 3 is therefore partially achieved in the first quarter as demand is originally "favorable" (state F).

Priority 4

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 36,286,553.21 KES $when\ demand\ is\ unfavorable$

When demand is initially unfavorable (state U) the total "production-inventory" costs in the first quarter is;

$$52897668.71X_{UF}(B,1) + 330847.52X_{UU}(B,1) - d_4^+ = 36{,}286{,}553.21$$

$$52897668.71(0) + 330847.52(109.68) - 0 = 36287355.99$$

This is a little greater than the targeted 'production-inventory' costs for goal constraint (4). Priority 4 can therefore be partially attained in the first quarter as demand is primarily "unfavorable" (state U).

Table 4.27: Expected goal values for product B, "stochastic" solution with "over & under" achievement

Goals/ priorities	Expected value from Goal	Value of the stochastic solution	Deviation	Over-achievement	Under-achievement
1	4,562.3836	4562.372	0.0116		4555.6
2	17,290.18	17290.68	0.5		17181
3	4,713,825.72	4713848.34	22.62	0	
4	36,286,553.21	36287355.99	802.78	0	

Table 4.27 shows the expected goal values for product B, the value of the stochastic solution giving the deviations with over achievement or under achievement.

The priorities set for product C are as follows;

$P_1: Produce\ a\ batch\ of$ 12,104.6162 $units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of$ 22,967.8994 $units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 7,174,123.68 KES

$when\ demand\ is\ favorable$

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 15,607,560.51 KES

$when\ demand\ is\ unfavorable$

$X_{FF}(C,1) = 0$;

The manufacturing lot size of product C in the first quarter when initially favorable demand remains favorable is 0 units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

$X_{FU}(C,1) = 1.7602$;

The manufacturing lot size of product C in the first quarter when initially favorable demand becomes unfavorable is 1.7602 units. This means that more products, 1.7602 units, should be produced to meet demand.

$X_{UF}(C,1) = 0$;

The manufacturing lot size of product C in the first quarter when initially unfavorable demand becomes favorable is 0 units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

$X_{UU}(C,1) = 181.8117$;

The manufacturing lot size of product C in the first quarter when initially unfavorable demand remains unfavorable is 181.8117 units. This means that more products, 181.8117 units, should be produced to meet demand.

<u>Goal constraints;</u>

$$X_{FF}(C,1) + X_{FU}(C,1) + d_1^- = 12{,}104.6162$$

$$X_{UF}(C,1) + X_{UU}(C,1) + d_2^- = 22{,}967.8994$$

$$8380201.56X_{FF}(C,1) + 4075855.1X_{FU}(C,1) - d_3^+ = 7{,}174{,}123.68$$

$$20756660.82X_{UF}(C,1) + 85844.66X_{UU}(C,1) - d_4^+ = 15{,}607{,}560.51$$

<u>Priority 1</u>

$P_1: Produce\ a\ batch\ of\ 12{,}104.6162\ units\ when\ demand\ is\ initially\ favorable$

When demand is initially favorable (state F), the amount to be produced in the first quarter is;

$$X_{FF}(C,1) + X_{FU}(C,1) + d_1^- = 12{,}104.6162$$

$$0 + 1.7602 + 12103 = 12104.7602 \text{ units}$$

Priority 1 can be achieved as this value is equal to the targeted production level for goal constraint (1). Nonetheless, an “underachievement” of 12103 units is however got in the first quarter as demand is originally “favorable” (state F).

<u>Priority 2</u>

$P_2: Produce\ a\ batch\ of\ 22{,}967.8994\ units\ when\ demand\ is\ initially\ unfavorable$

When demand is initially unfavorable (state U) the amount to be produced in the first quarter is;

$$X_{UF}(C,1) + X_{UU}(C,1) + d_2^- = 22{,}967.8994$$

$$0 + 181.8117 + 22786 = 22967.8117 \text{ units}$$

This is equal to the targeted production level for goal constraint (2). Therefore, Priority 2 may be completely attained. Nonetheless, an “underachievement” of 22786 units is got in the first quarter as demand is originally ‘unfavorable’ (state U).

Priority 3

$P_3: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 7,174,123.68 KES

$when\ demand\ is\ favorable$

When demand is initially favorable (state F) the total "production-inventory" costs in the first quarter is;

$$8380201.56X_{FF}(C,1) + 4075855.1X_{FU}(C,1) - d_3^+ = 7{,}174{,}123.68$$

$$8380201.56(0) + 4075855.1(1.7602) - 0 = 7174320.15$$

This is a little greater than the targeted 'production-inventory' costs for goal constraint (3). Priority 3 is therefore partially achieved in the first quarter as demand is originally "favorable" (state F).

Priority 4

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 15,607,560.51 KES

$when\ demand\ is\ unfavorable$

When demand is initially unfavorable (state U) the total "production-inventory" costs in the first quarter is;

$$20756660.82X_{UF}(C,1) + 85844.66X_{UU}(C,1) - d_4^+ = 15{,}607{,}560.51$$

$$20756660.82(0) + 85844.66(181.8117) - 0 = 15607563.57$$

This is equal to the targeted production-inventory costs for goal constraint (4). Priority 4 can therefore be fully attained in the first quarter as demand is originally "unfavorable" (state U).

Table 4.28: Expected goal values for product C, "stochastic solution with over & under" achievement

Goals/ priorities	Expected value from Goal	Value of the stochastic solution	Deviation	Over-achievement	Under-achievement
1	12,104.6162	12104.7602	0.144		12103
2	22,967.8994	22967.8117	0.0877		22786
3	7,174,123.68	7174320.15	196.47	0	
4	15,607,560.51	15607563.57	3.06	0	

Table 4.28 shows the expected goal values for product C, the value of the stochastic solution giving the deviations with over achievement or under achievement.

The priorities set for product D are as follows;

$P_1: Produce\ a\ batch\ of$ 5848.1378 $units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of$ 12517.62 $units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 18,039,940.82 KES $when\ demand\ is\ favorable$

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 44,663,839.29 KES $when\ demand\ is\ unfavorable$

$X_{FF}(D,1) = 0;$

The manufacturing lot size of product D in the first quarter when initially favorable demand remains favorable is 0 units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

$X_{FU}(D,1) = 369.48;$

The manufacturing lot size of product D in the first quarter when initially favorable demand becomes unfavorable is 369.48 units. This means that more products, 369.48 units, should be produced to meet demand.

$X_{UF}(D,1) = 0$;

The manufacturing lot size of product D in the first quarter when initially unfavorable demand becomes favorable is 0 units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

$X_{UU}(D,1) = 4975.1$;

The manufacturing lot size of product D in the first quarter when initially unfavorable demand remains unfavorable is 4975.1 units. This means that more products, 4975.1 units, should be produced to meet demand.

Goal constraints;

$$X_{FF}(D,1) + X_{FU}(D,1) + d_1^- = 5848.1378$$

$$X_{UF}(D,1) + X_{UU}(D,1) + d_2^- = 12517.62$$

$$24998368.5X_{FF}(D,1) + 48825X_{FU}(D,1) - d_3^+ = 18{,}039{,}940.82$$

$$56391378.75X_{UF}(D,1) + 8977.5X_{UU}(D,1) - d_4^+ = 44{,}663{,}839.29$$

Priority 1

$P_1: Produce\ a\ batch\ of\ 5848.1378\ units\ when\ demand\ is\ initially\ favorable$

When demand is initially favorable (state F), the amount to be produced in the first quarter is; $X_{FF}(D,1) + X_{FU}(D,1) + d_1^- = 5848.1378$

$$0 + 369.48 + 5478.7 = 5848.18 \text{ units}$$

Priority 1 can be achieved as this value is equal to the targeted production level for goal constraint (1). Nonetheless, an "underachievement" of 5478.7 units is however got in the first quarter as demand is originally "favorable" (state F).

Priority 2

$P_2: Produce\ a\ batch\ of$ 12517.62 $units\ when\ demand\ is\ initially\ unfavorable$

When demand is initially unfavorable (state U) the amount to be produced in the first quarter is;

$$X_{UF}(D,1) + X_{UU}(D,1) + d_2^- = 12517.62$$

$$0 + 4975.1 + 7542.5 = 12517.6 \text{ units}$$

This is equal to the targeted production level for goal constraint (2). Priority 2 can therefore be completely attained in the first quarter as demand is originally “unfavorable” (state U).

Priority 3

$P_3: Total\ production_{inventory} costs\ must\ not\ exceed$ 18,039,940.82 KES $when\ demand\ is\ favorable$

When demand is initially favorable (state F) the total “production-inventory” costs in the first quarter is;

$$24998368.5X_{FF}(D,1) + 48825X_{FU}(D,1) - d_3^+ = 18{,}039{,}940.82$$

$$24998368.5(0) + 48825(369.48) - 0 = 18039861$$

This is slightly lower than the targeted production-inventory costs for goal constraint (3). Priority 3 can therefore be partially achieved in the first quarter as demand is primarily “favorable” (state F).

Priority 4

$P_4: Total\ production_{inventory} costs\ must\ not\ exceed$ 44,663,839.29 KES $when\ demand\ is\ unfavorable$

When demand is initially unfavorable (state U) the total “production-inventory” costs in the first quarter is;

$$56391378.75X_{UF}(D,1) + 8977.5X_{UU}(D,1) - d_4^+ = 44{,}663{,}839.29$$

$56391378.75(0) + 8977.5(4975.1) - 0 = 44663960.25$

This is a little greater than the targeted "production-inventory" costs for goal constraint (4) and priority 4 is partially attained in the first quarter as demand is originally "unfavorable" (state U).

Table 4.29: Expected goal values for product D, "stochastic solution with over & under" achievement

Goals/ priorities	Expected value from Goal	Value of the stochastic solution	Deviation	Over-achievement	Under-achievement
1	5848.1378	5848.18	0.0422		5478.7
2	12517.62	12517.6	0.02		7542.5
3	18,039,940.82	18039861	79.82	0	
4	44,663,839.29	44663960.25	120.96	0	

Table 4.29 shows the expected goal values for product D, the value of the stochastic solution giving the deviations with over achievement or under achievement.

The priorities set for product E are as follows;

$P_1: Produce\ a\ batch\ of\ 69.6644\ units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of\ 6286.9678\ units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production_{inventory}\ costs\ must\ not\ exceed\ 2{,}714{,}785.47$ KES

$when\ demand\ is\ favorable$

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed\ 18{,}811{,}826.61$ KES

$when\ demand\ is\ unfavorable$

$X_{FF}(E,1) = 0$;

The manufacturing lot size of product E in the first quarter when initially favorable demand remains favorable is 0units. This means that more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand.

$X_{FU}(E,1) = 13.3956$

The manufacturing lot size of product E in the first quarter when initially favorable demand becomes unfavorable is 13.3956units. This means that more products, 13.3956units, should be produced to meet demand.

$X_{UF}(E,1) = 0.6835$

The manufacturing lot size of product E in the first quarter when initially unfavorable demand becomes favorable is 0.6835units. This means that more products, 0.6835units, should be produced to meet demand.

$X_{UU}(E,1) = 6286.3$

The manufacturing lot size of product E in the first quarter when initially unfavorable demand remains unfavorable is 6286.3units. This means that more products, 6286.3units, should be produced to meet demand.

Goal constraints;

$$X_{FF}(E,1) + X_{FU}(E,1) + d_1^- = 69.6644$$

$$X_{UF}(E,1) + X_{UU}(E,1) + d_2^- = 6286.9678$$

$$3881272.5X_{FF}(E,1) + 202663.13X_{FU}(E,1) - d_3^+ = 2{,}714{,}785.47$$

$$25566423.75X_{UF}(E,1) + 196.875X_{UU}(E,1) - d_4^+ = 18{,}811{,}826.61$$

Priority 1

P_1: *Produce a batch of* 69.6644 *units when demand is initially favorable*

When demand is initially favorable (state F), the amount to be produced in the first quarter is;

$$X_{FF}(E,1) + X_{FU}(E,1) + d_1^- = 69.6644$$

$$0 + 13.3956 + 56.2688 = 69.6644 \text{ unit}$$

This is equal to the targeted production level for goal constraint (1). Therefore priority 1 can be achieved. Underachievement of 56.2688 units is however got in the first quarter as demand is originally “favorable” (state F)

Priority 2

$P_2: Produce\ a\ batch\ of\ 6286.9678\ units\ when\ demand\ is\ initially\ unfavorable$

When demand is initially unfavorable (state U) the amount to be produced in the first quarter is;

$$X_{UF}(E,1) + X_{UU}(E,1) + d_2^- = 6286.9678$$

$$0.6835 + 6286.3 + 0 = 6286.9835 \text{ units}$$

This is equal to the targeted production level for goal constraint (2). Priority 2 can consequently be completely attained in the first quarter as demand is originally “unfavorable” (state U).

Priority 3

$P_3: Total\ production_{inventory}\ costs\ must\ not\ exceed\ 2{,}714{,}785.47\ \text{KES}$

$when\ demand\ is\ favorable$

When demand is initially favorable (state F) the total “production-inventory” costs in the first quarter is;

$$3881272.5X_{FF}(E,1) + 202663.13X_{FU}(E,1) - d_3^+ = 2{,}714{,}785.47$$

$$3881272.5(0) + 202663.13(13.3956) - 0 = 2714794.22$$

This is somewhat greater than the targeted ‘production-inventory’ costs for goal constraint (3). Priority 3 can consequently be partly attained in the first quarter as demand is originally “favorable” (state F).

Priority 4

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 18,811,826.61 KES

$when\ demand\ is\ unfavorable$

When demand is initially unfavorable (state U) the total "production-inventory" costs in the first quarter is;

$$25566423.75X_{UF}(E,1) + 196.875X_{UU}(E,1) - d_4^+ = 18,811,826.61$$

$$25566423.75(0.6835) + 196.875(6286.3) - 0 = 18712265.95$$

This is lower than the targeted production-inventory costs for goal constraint (4). Priority 4 is partly attained in the first quarter as demand is originally "unfavorable" (state U)

Table 4.30: Expected goal values for product E, "stochastic" solution with "over & under" achievement

Goals/ priorities	"Expected" value from Goal	Value of the "stochastic" solution	Deviation	"Over-achievement"	"Under-achievement"
1	69.6644	69.6644	0		56.2688
2	6286.9678	6286.9835	0.0157		0
3	2,714,785.47	2714794.22	8.75	0	
4	18,811,826.61	18712265.95	99560.66	0	

Table 4.30 shows the expected goal values for product E, the value of the stochastic solution giving the deviations with over achievement or under achievement.

4.15 Model validation

Data for product Y not used in building the model, was got and used in validating the model to assess how well the model predicts outcomes in new data.

For a specified week, demand is considered to be "favorable" (state F) if Nij > 21 or else demand will be taken as 'unfavorable' (state U) if Nij ≤ 21

Table 4.31: Data "classification" by state of demand

Month	**Week**	**Customers (N)**	**Demand (D) ($x10^3$)**	**On hand inventory (V) ($x10^3$)**	**State of demand (*i*)**
1	1	14	2664	3674	U
	2	28	4274	4601	F
	3	17	2864	4700	U
	4	32	6363	7199	F
2	1	5	466	2890	U
	2	20	3670	3978	U
	3	14	2402	3348	U
	4	30	3270	4530	F
3	1	11	931	2610	U
	2	26	3738	3078	F
	3	28	2175	3305	F
	4	28	3419	4816	F

Table 4.31 shows the classification of data by state of demand (as either favorable (F) or unfavorable (U)) of product Y for each week for the first quarter of the year.

'Overstocking' and "understocking" of product Y with the matching shortage and holding costs in the first quarter of the year is shown in table 4.32.

Table 4.32: "Overstocking & understocking" with holding and "shortage" costs for product Y

Week	**Demand (D) ($x10^3$)**	**"On hand" inventory (V) ($x10^3$)**	**"Over/under" stocking**	**"Holding/shortage" costs (KES)**
1	2664	3674	1010	23744.40
2	4274	4601	327	7687.50
3	2864	4700	1836	43163.00
4	6363	7199	836	19653.70
5	466	2890	2424	56986.40
6	3670	3978	308	7240.90
7	2402	3348	946	22239.80
8	3270	4530	1260	29621.70
9	931	2610	1679	39472.00
10	3738	3078	-660	99825.00
11	2175	3305	1130	26565.50
12	3419	4816	1397	32842.40

Table 4.32 shows the overstocking or understocking of product Y weekly for the first quarter of the year, with the corresponding holding or shortage costs.

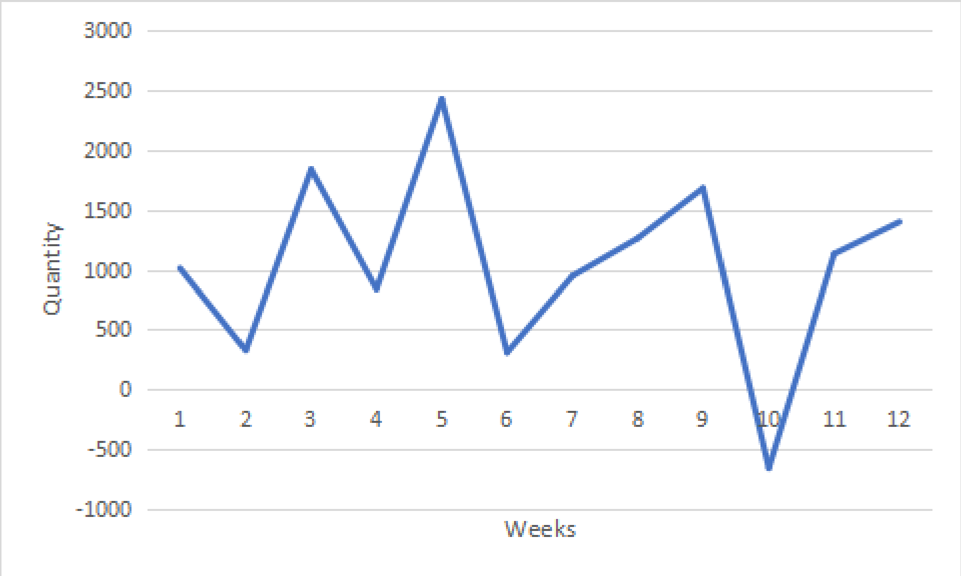

Figure 4.15: Overstocking & understocking of product Y

Figure 4.15 shows the graphical representation of overstocking and understocking of product Y weekly for the first quarter of the year.

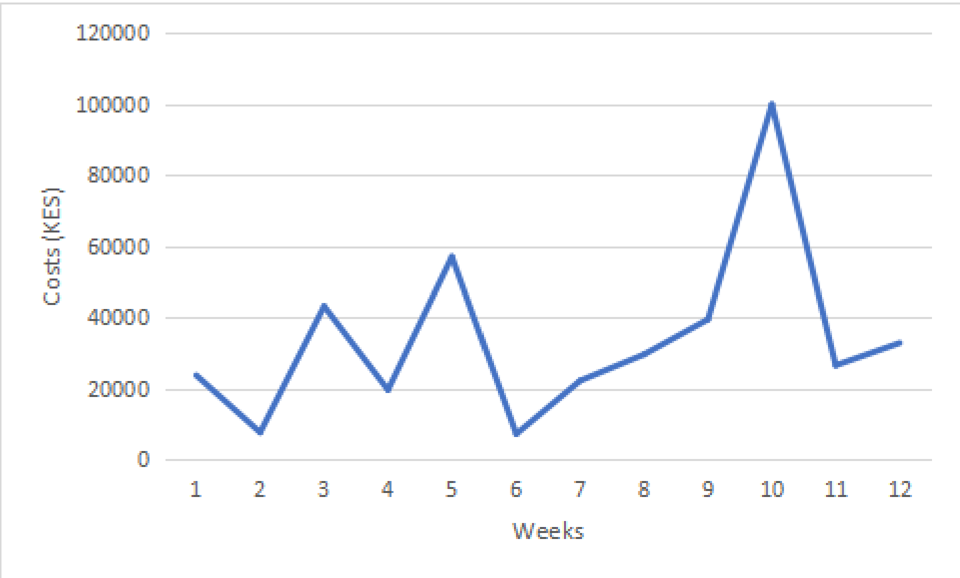

Figure 4.16: Holding & Shortage costs of product Y

Figure 4.16 shows the graphical representation of holding and shortage costs of product Y weekly for the first quarter of the year.

State "transitions" and "on-hand" inventory

For every state "transition", using equation 3.2, given the beginning & ending inventory, the average "on-hand" inventory was calculated as shown in table 4.33.

Table 4.33: Average "on-hand" inventory

State "transitions" (i, j)	"Beginning" inventory (B)	"Ending" inventory (E)	Average "on-hand" inventory $V = (B + E)/2$
FF	3305	4816	4060.5
FU	4700	2610	3655
UF	4601	3078	3839.5
UU	3978	3348	3663

$V_{FF}(Y,1) = 4060.5$ $V_{FU}(Y,1) = 3655$ $V_{UF}(Y,1) = 3839.5$ $V_{UU}(Y,1) = 3663$

Table 4.33 shows for each state transition, the Average 'on-hand' inventory for product Y for the first quarter of the year considering the beginning and ending inventory.

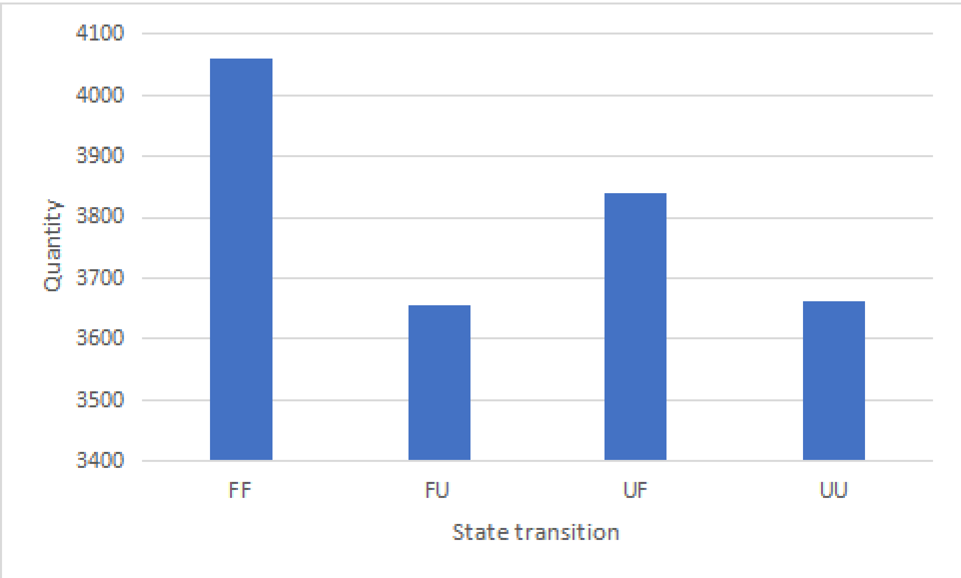

Figure 4.17: Average "on-hand" inventory and state "transitions"

Figure 4.17 illustrates the graphical representation of the average 'on-hand' inventory and state transition for product Y for the first quarter of the year.

Demand "transition" probabilities

Classification of data by "state-transition" was then carried out as showed from table 4.34 and then using equation 3.4 the demand transition probabilities was calculated.

Table 4.34: Data classification by state-transition

Month	**"State" transition** $(\boldsymbol{i},\boldsymbol{j})$	**"Number" of customers** $\boldsymbol{N_{ij}(Y,1)}$	**Demand** $\boldsymbol{D_{ij}(Y,1)}$
1	FF	0	0
	FU	45	7138
	UF	91	16165
	UU	0	0
2	FF	0	0
	FU	0	0
	UF	44	5672
	UU	59	10208
3	FF	110	11507
	FU	0	0
	UF	37	4669
	UU	0	0

Table 4.34 shows for each week, for the first quarter of the year, the number of customers and their corresponding quantity demanded for a particular state transition for product Y.

TOTALS

$N_{FF}(Y,1) = 110$ $N_{FU}(Y,1) = 45$ $N_{UF}(Y,1) = 91 + 44 + 37 = 172$ $N_{UU}(Y,1) = 59$

$D_{FF}(Y,1) = 11507$ $D_{FU}(Y,1) = 1738$

$D_{UF}(Y,1) = 16165 + 5672 + 4669 = 26506$ $D_{UU}(Y,1) = 10208$

Demand transition probabilities

$$Q_{FF}(Y,1) = \frac{N_{FF}(Y,1)}{N_{FF}(Y,1)+N_{FU}(Y,1)} = \frac{110}{110+45} = 0.7097$$

$$Q_{FU}(Y,1) = \frac{N_{FU}(Y,1)}{N_{FF}(Y,1)+N_{FU}(Y,1)} = \frac{45}{110+45} = 0.2903$$

$$Q_{UF}(Y,1) = \frac{N_{UF}(Y,1)}{N_{UF}(Y,1)+N_{UU}(Y,1)} = \frac{172}{172+59} = 0.7446$$

$$Q_{UU}(Y,1) = \frac{N_{UU}(Y,1)}{N_{UF}(Y,1)+N_{UU}(Y,1)} = \frac{59}{172+59} = 0.2554$$

Hence,

$$Q(Y,1) = \begin{matrix} F \\ U \end{matrix}\begin{bmatrix} 0.7097 & 0.2903 \\ 0.7446 & 0.2554 \end{bmatrix}$$

From equation 3.6, the demand matrix for product Y becomes;

$$D(Y,1) = \begin{matrix} F \\ U \end{matrix}\begin{bmatrix} 11507 & 1738 \\ 26506 & 10208 \end{bmatrix}$$

From equation 3.7, the inventory matrix for product Y becomes;

$$V(Y,1) = \begin{matrix} F \\ U \end{matrix}\begin{bmatrix} 4060.5 & 3655 \\ 3839.5 & 3663 \end{bmatrix}$$

Using equations 3.8, 3.9 and 3.10, the “production-inventory cost” matrices are then calculated for product Y as presented below:

Production-inventory cost matrix

Unit production cost, $C_p(Y) = 1375.00$ KES

Unit holding cost, $C_h(Y) = 23.51$ KES

Unit shortage cost, $C_s(Y) = 151.25$ KES

$$C_{FF}(Y,1) = \left(C_p(Y) + C_h(Y) + C_s(Y)\right)\left(D_{FF}(Y,1) - V_{FF}(Y,1)\right)$$

$$= (1375.00 + 23.51 + 151.25)(11507 - 4060.5) = 11540287.84$$

$$C_{FU}(Y,1) = C_h(Y)\left(V_{FU}(Y,1) - D_{FU}(Y,1)\right)$$

$$= 23.51(3655 - 1738) = 45068.67$$

$$C_{UF}(Y,1) = \left(C_p(Y) + C_h(Y) + C_s(Y)\right)\left(D_{UF}(Y,1) - V_{UF}(Y,1)\right)$$

$$C_{UF}(Y,1) = (1375.00 + 23.51 + 151.25)(26506 - 3839.5) = 35127635.04$$

$$C_{UU}(Y,1) = \left(C_p(Y) + C_h(Y) + C_s(Y)\right)\left(D_{UU}(Y,1) - V_{UU}(Y,1)\right)$$

$$= (1375.00 + 23.51 + 151.25)(10208 - 3663) = 10143179.2$$

Hence,

$$C(Y,1) = \begin{matrix} F \\ U \end{matrix}\begin{bmatrix} 11540287.84 & 45068.67 \\ 35127635.04 & 10143179.2 \end{bmatrix}$$

Using equations 3.11 and 3.12, the predictable demand for both "favorable" and "unfavorable" demand was calculated correspondingly as below;

"Favorable" demand (F)

$$E[D_F(Y,1)] = Q_{FF}(Y,1) * D_{FF}(Y,1) + Q_{FU}(Y,1) * D_{FU}(Y,1)$$

$$E[D_F(Y,1)] = (0.7097 * 11507) + (0.2903 * 1738)$$

$$E[D_F(Y,1)] = 8671.1\ units$$

Unfavorable demand (U)

$$E[D_U(Y,1)] = Q_{UF}(Y,1) * D_{UF}(Y,1) + Q_{UU}(Y,1) * D_{UU}(Y,1)$$

$$E[D_U(Y,1)] = (0.7446 * 26506) + (0.2554 * 10208)$$

$$E[D_U(Y,1)] = 22343.4\ units$$

By means of equations 3.13 and 3.14, the predictable inventory bearing in mind both "favorable" and "unfavorable" demand for product Y was calculated respectively as below;

'Favorable' demand (F)

$$E[V_F(Y,1)] = Q_{FF}(Y,1) * V_{FF}(Y,1) + Q_{FU}(Y,1) * V_{FU}(Y,1)$$

$$E[V_F(Y,1)] = (0.7097 * 4060.5) + (0.2903 * 3655)$$

$$E[V_F(Y,1)] = 3942.7\ units$$

Unfavorable demand (U)

$$E[V_U(Y,1)] = Q_{UF}(Y,1) * V_{UF}(Y,1) + Q_{UU}(Y,1) * V_{UU}(Y,1)$$

$$E[V_U(Y,1)] = (0.7446 * 3839.5) + (0.2554 * 3663)$$

$$E[V_U(Y,1)] = 3794.4\ units$$

The probable "production-Inventory" costs were then and there calculated for product Y bearing in mind both 'favorable' and 'unfavorable' demand using equations 3.15 and 3.16 respectively as presented below:

Favorable demand (F)

$$E[C_F(Y,1)] = Q_{FF}(Y,1) * C_{FF}(Y,1) + Q_{FU}(Y,1) * C_{FU}(Y,1)$$

$$E[C_F(Y,1)] = (0.7097 * 11540287.84) + (0.2903 * 45068.67)$$

$$E[C_F(Y,1)] = 8203225.71\ \text{KES}$$

Unfavorable demand (U)

$$E[C_U(Y,1)] = Q_{UF}(Y,1) * C_{UF}(Y,1) + Q_{UU}(Y,1) * C_{UU}(Y,1)$$

$$E[C_U(Y,1)] = (0.7446 * 35127635.04) + (0.2554 * 10143179.2)$$

$$E[C_U(Y,1)] = 28746605.02\ \text{KES}$$

By means of equations 3.17 and 3.18, the probable 'manufacturing lot size' bearing in mind both 'favorable' & 'unfavorable' demand for product Y was calculated correspondingly as below:

"Favorable demand" (F)

$$E[M_F(Y,1)] = \begin{pmatrix} E[D_F(Y,1)] - E[V_F(Y,1)]\ if\ E[D_F(Y,1)] > E[V_F(Y,1)] \\ 0 \qquad\qquad otherwise \end{pmatrix}$$

$$E[M_F(Y,1)] = E[D_F(Y,1)] - E[V_F(Y,1)]$$

$$E[M_F(Y,1)] = 8671.1 - 3942.7 = 4728.4\ units$$

Unfavorable demand (U)

$$E[M_U(Y,1)] = \begin{pmatrix} E[D_U(Y,1)] - E[V_U(Y,1)]\ if\ E[D_U(Y,1)] > E[V_U(Y,1)] \\ 0 \qquad\qquad otherwise \end{pmatrix}$$

$$E[M_U(Y,1)] = E[D_U(Y,1)] - E[V_U(Y,1)]$$

$$E[M_U(Y,1)] = 22343.4 - 3794.4 = 18{,}549\ units$$

<u>"Stochastic goal programming" model</u>

Through fixing priorities, describing the "objective" function & framing the "goal constraints", the "stochastic goal programming" model was then expressed as follows:

Priorities fixed

$P_1: Produce\ a\ batch\ of\ 4728.4\ units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of\ 18{,}549\ units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production_{inventory}\ costs\ must\ not\ exceed\ 8{,}203{,}225.71$ KES

$when\ demand\ is\ favorable$

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed\ 28{,}746{,}605.02$ KES

$when\ demand\ is\ unfavorable$

<u>Objective function</u>

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(Y,1)d_k^+ + P_K(Y,1)d_k^-]$$

<u>Goal constraints</u>

Manufacturing lot size

$$X_{FF}(Y,1) + X_{FU}(Y,1) + d_1^- = 4728.4\ (Favorable\ demand)$$

$$X_{UF}(Y,1) + X_{UU}(Y,1) + d_2^- = 18{,}549\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$11540287.84X_{FF}(Y,1) + 45068.67X_{FU}(Y,1) - d_3^+ = 8{,}203{,}225.71\ (Favorable\ demand)$$

$$35127635.04X_{UF}(Y,1) + 10143179.2X_{UU}(Y,1) - d_4^+ = 28746605.02\ (Unfavorable\ demand)$$

Non negativity

$$X_{FF}(Y,1), X_{FU}(Y,1), X_{UF}(Y,1), X_{UU}(Y,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

<u>"Stochastic goal programming" model for the "manufacturing lot size"</u>

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(Y,1)d_k^{+} + P_K(Y,1)d_k^{-}]$$

Subject to:

$$X_{FF}(Y,1) + X_{FU}(Y,1) + d_1^{-} = 4728.4$$

$$X_{UF}(Y,1) + X_{UU}(Y,1) + d_2^{-} = 18{,}549$$

$$11540287.84X_{FF}(Y,1) + 45068.67X_{FU}(Y,1) - d_3^{+} = 8{,}203{,}225.71$$

$$35127635.04X_{UF}(Y,1) + 10143179.2X_{UU}(Y,1) - d_4^{+} = 28746605.02$$

$$X_{FF}(Y,1), X_{FU}(Y,1), X_{UF}(Y,1), X_{UU}(Y,1), d_1^{-}, d_2^{-}, d_3^{+}, d_4^{+} \geq 0$$

Where:

d_1^{-}, d_2^{-} = slack variables

d_3^{+}, d_4^{+} = surplus variables

$X_{FF}(Y,1)$ – "manufacturing lot size" of product Y as originally "favorable" demand remains favorable

$X_{FU}(Y,1)$ – "manufacturing lot size" of product Y while at first "favorable" demand becomes "unfavorable"

$X_{UF}(Y,1)$ – "manufacturing lot size" of product Y as primarily "unfavorable" demand becomes "favorable"

$X_{UU}(Y,1)$ – "manufacturing lot size" of product Y while originally "unfavorable" demand remains "unfavorable"

<u>Optimization of the model</u>

Using MATLAB™, the "stochastic goal programming model" for the product was then solved whereby the figures were placed in and using the linprog solver the results were obtained. An optimum result was established and the figures are as follows:

Table 4.35: Optimal solution from MATLAB

Variables	$X_{FF}(Y,1)$	$X_{FU}(Y,1)$	$X_{UF}(Y,1)$	$X_{UU}(Y,1)$	d_1^-	d_2^-	d_3^+	d_4^+	Z
Values	0	182.02	0	2.8341	4546.38	18546.17	0	0	63,987,000

Table 4.35 shows the optimum results got from the Matlab software, using linprog solver, giving the manufacturing lot sizes, over achievements, under achievements and the total production-inventory cost for product Y for the first quarter of the year.

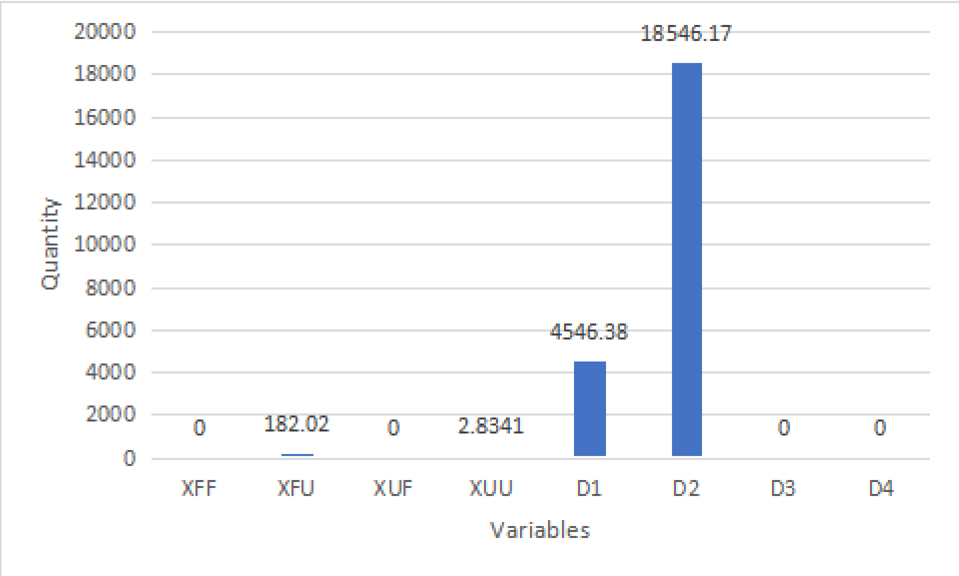

Figure 4.18: Manufacturing lot size with over or under achievement in each state transition

Figure 4.18 illustrates the values of the manufacturing lot size with the over achievement or under achievement in each state transition for product Y for the first quarter of the year.

The priorities set for product Y are as follows;

P_1: *Produce a batch of* 4728.4 *units when demand is initially favorable*

P_2: *Produce a batch of* 18,549 *units when demand is initially unfavorable*

P_3: *Total production$_{inventory}$ costs must not exceed* 8,203,225.71 KES *when demand is favorable*

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 28,746,605.02 KES

$when\ demand\ is\ unfavorable$

X_{FF} (Y,1) = 0;

The “manufacturing lot size” of product Y in the “first quarter” when initially favorable demand remains favorable is 0 units. This means that no extra products must be produced however utilize that which is at present in stock because it’s adequate to satisfy the demand.

X_{FU} (Y,1) = 182.02;

The “manufacturing lot size” of product Y in the “first quarter” when initially favorable demand becomes unfavorable is 182.02 units. This means that additional products, 182.02 units, must be produced to satisfy demand.

X_{UF} (Y,1) = 0;

The “manufacturing lot size” of product Y in the ‘first quarter’ when initially unfavorable demand becomes favorable is 0 units. This means that no additional products must be produced but utilize that which is at present in stock because it’s sufficient to satisfy the demand.

X_{UU} (Y,1) = 2.8341;

The “manufacturing lot size” of product Y in the “first quarter” when initially unfavorable demand remains unfavorable is 2.8341units. This means that more products, 2.8341units, should be produced to meet demand.

<u>Goal constraints;</u>

$$X_{FF}(Y,1) + X_{FU}(Y,1) + d_1^- = 4728.4$$

$$X_{UF}(Y,1) + X_{UU}(Y,1) + d_2^- = 18,549$$

$$11540287.84X_{FF}(Y,1) + 45068.67X_{FU}(Y,1) - d_3^+ = 8,203,225.71$$

$$35127635.04X_{UF}(Y,1) + 10143179.2X_{UU}(Y,1) - d_4^+ = 28746605.02$$

Priority 1

P_1: *Produce a batch of* 4728.4 *units when demand is initially favorable*

When demand is initially favorable (state F), the amount to be produced in the first quarter is;

$$X_{FF}(Y,1) + X_{FU}(Y,1) + d_1^- = 4728.4$$

$$0 + 182.02 + 4546.38 = 4728.4 \text{ units}$$

This is equal to the targeted production level for goal constraint (1). Therefore, Priority 1 is completely attained. On the other hand, an "underachievement" of 4546.38 units is still got in the first quarter as demand is originally "favorable" (state F)

Priority 2

P_2: *Produce a batch of* 18,549 *units when demand is initially unfavorable*

When demand is initially unfavorable (state U) the amount to be produced in the first quarter is;

$$X_{UF}(Y,1) + X_{UU}(Y,1) + d_2^- = 18{,}549$$

$$0 + 2.8341 + 18546.17 = 18549 \text{ units}$$

Priority 2 can be fully achieved as this value is equal to the targeted production level for goal constraint (2). Nevertheless, an "underachievement" of 18546.17 units is got in the first quarter as demand is originally unfavorable (state U).

Priority 3

P_3: *Total production*$_{inventory}$ *costs must not exceed* 8,203,225.71 KES *when demand is favorable*

When demand is initially favorable (state F) the total "production-inventory costs" in the first quarter is;

$$11540287.84X_{FF}(Y,1) + 45068.67X_{FU}(Y,1) - d_3^+ = 8{,}203{,}225.71$$

$$11540287.84(0) + 45068.67(182.02) - 0 = 8203399.31$$

This value is somewhat greater than the targeted ‘production-inventory costs’ for goal constraint (3) and therefore priority 3 can be partially attained in the first quarter while demand is originally favorable (state F).

Priority 4

$P_4: Total\ production_{inventory}\ costs\ must\ not\ exceed$ 28,746,605.02 KES

$when\ demand\ is\ unfavorable$

When demand is initially unfavorable (state U) the total ‘production-inventory costs’ in the half-moon is;

$$35127635.04X_{UF}(Y,1) + 10143179.2X_{UU}(Y,1) - d_4^+ = 28746605.02$$

$$35127635.04(0) + 10143179.2(2.8341) - 0 = 28746784.1$$

This value is somewhat greater than the targeted ‘production-inventory costs’ for goal constraint (4). Priority 4 can consequently be partly attained in the half-moon as demand is originally unfavorable (state U).

Table 4.36: Expected goal values for product Y, “stochastic solution with over & under achievement”

“Goals”/ priorities	“Expected” value from Goal	“Value” of the stochastic solution	“Deviation”	“Over-achievement”	“Under-achievement”
1	4728.4	4728.4	0		4546.38
2	18,549	18549	0		18546.17
3	8,203,225.71	8203399.31	173.6	0	
4	28746605.02	28746784.17	179.15	0	

Table 4.36 shows the expected goal values for product Y, the value of the stochastic solution giving the deviations with over achievement or under achievement.

4.16 Discussion

During the research, after developing the “stochastic goal programming” model for the products, it was then resolved by the use of applied mathematics “linprog” solver in “MATLAB” obtaining an optimum result having the values as seen in Table 4.25.

The outcomes give the optimum values of the “manufacturing lot size” of the products within the half-moon of the year as demand shifts from one state to a different. The solutions were examined and deliberated basing on the priorities fixed and also the optimum values that were attained. The expansion of the result from this study was determining the “over-achievement” and “under- achievement” of the “manufacturing lot size” priorities wanted in the course of production planning.

During the study, an extension is incorporating in “Markov chains” considering shifts form one state to a different. As shown in Table 4.25, product A, as originally favorable demand remains “favorable” and initially “unfavorable” demand remains favorable, no additional items must be produced but utilize that which is at present available because it’s sufficient to satisfy the “demand” as the model forecasts 0 “manufacturing lot size” of product A within the half-moon of the year.

The developed model likewise forecasts the “manufacturing lot size” of product A as 2.3729 units and 104.0840 units while originally “favorable” demand becomes unfavorable and “unfavorable” demand remains unfavorable correspondingly. This means that the above amount of products must be manufactured to fulfill the demand. Having the fixed priorities and probable values as of every goal, the outcomes as in Tables 4.26 to 4.30 display the significance of using the existing sources of data while making a strategy. As seen in Table 4.26, both priority 1 & 2 stand completely attained but, an “under-achievement” of 8137.7 units & 17350 units correspondingly is got within the half-moon while demand is originally “favorable” (state F) and “unfavorable” (state U) correspondingly. This suggests that the values are fewer than the target values in priorities 1 and 2. The real stochastic result of Priority 3 is somewhat beyond the probable goal value targeted “production-inventory costs” within the half-moon when demand is originally favorable (state F), having it partially achieved.

Also priority 4 is completely attained within the half-moon when demand is originally “unfavorable” (state U). Both priority 3 and 4 haven’t any “over-achievement”, meaning that the values are adequate to the target values in priorities 3 and 4.

As this model helps in establishing optimum manufacturing lot-sizes that can sustain random demand occurrences, it is of significance to optimization as it eliminates overstocking or understocking of products as a cost minimization strategy.

CHAPTER FIVE: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

For any manufacturer that wants its firm to compete favorably within the market, it's very critical for him or her to make the right decisions concerning the "manufacturing lot-sizing" problems.

The study was observing demand uncertainty of the "manufacturing lot size" in "aggregate production planning" of finished products and an approach using "Markov chains" in combination with "stochastic goal programming" was adopted.

- ❖ The existing production planning system at Movit products (U) Ltd was characterized as batch production, using make-to-stock strategy with Standardization of product & process sequence and special purpose machines having higher production capacities and output rates.
- ❖ The manufacturing lot size problem was defined and formulated for determining the optimal manufacturing lot size minimizing the total production-inventory cost. Varying demand was modelled as a two-state markov chain where the optimality of the manufacturing lot size was state dependent. Goal constraints, deviation variables, priorities & objective function were defined to establish the over-achievement or under achievement of the manufacturing lot size priorities desired.
- ❖ An optimization model, that is, Stochastic goal programming, that predicts optimal manufacturing lot size in production planning under demand uncertainty was developed and validated (using out of sample testing).
- ❖ The developed model was solved using Linprog Solver in MATLAB software where results indicated optimal manufacturing lot size levels as demand changes

from one state to another to obtain the over-achievement or underachievement of the manufacturing lot size priorities desired.

5.2 Scientific Contribution

Through this study, more information has been added to the group of knowledge and that is of stochastic goal programming in aggregate production planning.

This is the first application of operations research techniques in particular “Markov chains” in combination with “stochastic goal programming” in Uganda to establish the optimum manufacturing lot size considering uncertainty in demand. This research is of significance as it will raise interest and awareness to manufacturers, companies and policy makers engaged in production planning of manufacturing lot-sizes of products with demand uncertainty.

5.3 Recommendations

In this thesis, it has been established that for any manufacturing industry to excel, optimizing manufacturing lot-size as a cost minimization strategy is very important especially for products with stochastic demand.

The research has developed a mathematical model that optimizes the manufacturing lot size in aggregate production planning under demand uncertainty in effect, overstocking or understocking of products is eliminated as a cost minimization strategy.

The model helps in establishing and maintaining optimal manufacturing lot-sizes that can sustain random demand occurrences.

Movit products (U) Ltd should consider adopting the stochastic goal programming model as a strategy to optimize the manufacturing lot size of the finished product.

This is also applicable to any other manufacturing companies, manufacturers, and policy makers engaged in production planning of products with demand uncertainty (stochastic demand).

5.3.1 Further Research

Several possibilities for more investigation are recommended considering the accomplishments throughout this study, as below:

- The suggested developed model should be spread so as consider numerous finished items with uncertainties in price and demand
- Analyzing situations of non-stationary demand for the products where demand transition probabilities change over time should also be considered.
- In addition, “weighted goal programming” can also be introduced to improve computational proficiency but managing “pre-emptive priorities” of the products.

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APPENDICES

Appendix 1, Paper 1: "Application of Markov chains in manufacturing systems: A review"

Maureen Nalubowa Ssempijja, Saul Namango, Jerry Ochola, & Paul Kizito Mubiru (2021). "Application of Markov chains in manufacturing systems: A review". *International Journal of Industrial Engineering and Operational Research*, *3*(1), 1-13. ISSN: 2676-3311. Retrieved from http://bgsiran.ir/journal/ojs-3.1.1-4/index.php/IJIEOR/article/view/26

Application of Markov chains in manufacturing systems: A review

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Abstract
Manufacturing is an essential aspect to the global economy and prosperity. Many Manufacturing systems operate in an uncertain environment which affects the system performance. Production planning is very key in improving the overall manufacturing system performance. Systems that apply production planning approaches not considering uncertainties yield inferior planning decisions as compared to those that explicitly account for the uncertainty. Markov chains can be used to capture the transition probabilities as changes occur. Some existing literature on application of Markov chains in manufacturing systems has been reviewed. The objective is to give the reader beginning points about uncertainty modelling in manufacturing systems using Markov chains.

Keywords:
Manufacturing systems, Uncertainties, Production planning, Markov chains

1. Introduction

Manufacturing is described as the procedure of using raw materials, components or sub-components to produce finished products that meet the customers' requirements [1]. Characterization of manufacturing systems, like many other systems, can be dynamic or static, stationary (time-invariant)[2] or non-stationary (time-varying), linear or non-linear, discrete-state (time) or continuous-state (time), event-driven or time-driven, and stochastic or deterministic [3]

Manufacturing companies are facing a growing and rapid change where trends like globalization, customer orientation and increasing market dynamics have led to a move in both managerial and manufacturing principles which calls for more flexibility, fast and effectiveness [4].

Product demand uncertainty is one of the challenges faced by manufacturing companies [5] and influences the performance of the manufacturing system and the final decision on utilizing the manufacturing system [6].

The criteria of performance like manufacturing lead times, inventory costs, customer satisfaction, machine utilization, meeting due dates, and quality of products all dependent on how efficiently the jobs are scheduled in the system [7]. Therefore it becomes increasingly important to develop effective production planning approaches that help in achieving the desired objectives.

Production planning has an important role in the manufacturing system. The more variety of products, increased number of orders, increased number and size of workshops and expansion of factories have all made production planning more complicated, making the traditional methods of optimization unable to solve them [4]

Production planning in manufacturing systems is affected by a number of uncertainties which need to be considered in order to generate better planning decisions. [8]

Markov chain is a powerful mathematical tool that is extensively used to capture the stochastic process of systems transitioning among different states [9].

When manufacturing systems reveal some random behavior (breakdowns, random time to process a part), markov chains can be used for modeling and performance evaluation [10]. Companies' model manufacturing processes for many reasons, including predicting cost, predicting resource and material demand and running optimization studies. Basing future business simulations on these markov chains can give a more reliable representation of the business which reduces the risk of modelling inaccuracies and can help to predict future outcomes and run optimization more accurately [1].

To gain a better understanding of the application of Markov chains in manufacturing systems and to provide a basis for future research, a broad review of some existing research on the topic has been presented.

2. Basic concepts

The concepts and theory applied in this study are presented in the section below. This study was centered on the theory of Markov Chains focusing on their application in manufacturing systems. A Markov chain may be a special sort of model. Manufacturing systems, the concepts of stochastic process, Markov chain, types of Markov chains, Markov chain model states, transition probability matrix, properties of Markov chains, classification of states and application areas are presented in sub sections as outlined below.

2.1 Manufacturing systems

A manufacturing system may be a network of interacting parts. Managing the network of interacting parts is as important as managing individual parts, if not so more. In manufacturing systems research, a lot of interesting fields come to mind, such as design, analysis, modeling, optimization and control [11].

Manufacturing systems contain a number of several system factors among which exists work environment, physical structure, performance measurements, work organization, market & strategy, and manufacturing development process.[12]

Most of the manufacturing companies are large, complex systems characterized by a number of decision subsystems, like finance, personnel, marketing, operations and operates in an uncertain environment.[13]

A manufacturing system is an objective oriented network of processes through which entities flow with an objective of improving throughput or flow time. It also contains processes that are not only physical, but can include support of direct manufacturing (e.g., order entry, maintenance). Due to variability in manufacturing systems, values of performance measures fluctuate, resulting in complexity. Therefore, models are required to imitate behavior of manufacturing systems. Together with variability, the evolution of manufacturing systems leads to a need for predicting behavior of the manufacturing systems [11]

2.2 Stochastic process

A stochastic process may be a mathematical model that evolves over time in probabilistic manner [14]. A stochastic process is a random process [10], that is, a change in the state of some system over time whose course depends on chance and for which the probability of a particular course is defined. Essentially it is a family of random variables, X (t): t Є T defined on a given probability space, indexed by the time variable t, where t varies over an index set T [15].
A stochastic process may be continuous or discrete. A stochastic process is claimed to be a discrete time process if set T is finite or countable. That is, if T= (0, 1, 2, 3 ,4…….. , n) resulting in the time process X(0), X(1), X(2), X(3), X(4), …….., X(n) , recorded at time 0,1,2,3,4……,n respectively [16]. On the other hand stochastic processes X (t): t Є T is considered a continuous time process if T is not finite or countable. That is, if T= [0, ∞) or T= [0, k] for some value k.
A state space S is the set of states that a stochastic process can be in. The states can be finite or countable hence the state space S is discrete, that is S=1, 2, 3…, N. Otherwise the space S is continuous [17].

2.3 Markov chain

Markov chain, named after a Russian mathematician Andrey Markov in 1907, is a powerful mathematical tool that is used widely to capture the stochastic process of systems transitioning among different states [9]. Markov chains were recognized rapidly for their significant power of representation and their possibility of modeling a wide range of real life problems in addition to the quality of performance indices they give [10]. When manufacturing systems reveal some random behavior, Markov chains can be used to carry out performance evaluation and modeling [18].
A Markov chain, special type of stochastic process (with a Markov property [19]), is a discrete-time stochastic model defined on a space of states, equipped with transition probabilities from one state to another at the next time stage [20].
Markov Chains have revealed their strength at modeling stochastic transitions, from uncovering sequential patterns to directly modeling decision processes [21]. These have got a special property that probabilities involving how the process will evolve in the future depend only on the present state of the process, and so are independent of events in the past [22].
A Markov process is a stochastic process that satisfies the Markovian property (says that the conditional probability of any future "event," given any past "event" and the present state Xt-i, is independent of the past event and depends only upon the present state[17], [15]). It is a sequence of random variables $X_1, X_2, X_3, \dots X_n$ with the Markovian property, namely that, given the present state, the future and past state is independent. Formally[23],

$$P_r\left(X_{n+r} = \frac{x}{X_1},\ = x_1, X_2 = x_2, \dots X_n = x_n\right) = P_r\left(X_{n+r} = \frac{x}{X_n},\ = x_n\right), \quad (1)$$

if both conditional probabilities are defined, i.e. if $P_r(X_1 = x_1, \ldots X_n = x_n) > 0$ the possible values of X_n form a countable set S called the state space of the Chain [4].
Markov Chains often described by a sequence of directed graphs, where the edges of the graph n labeled by the probabilities of going from one state at time n to another state at time$(n+1)$,

$$P_r\left(X_{n+r} = \frac{x}{X_n},\ = x_n\right) \qquad (2)$$

However, Markov Chains assumes time-homogenous scenarios[24], in which case the graph and matrix are independent of n and not presented as sequences [4].

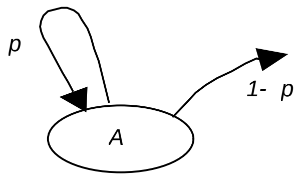
2.3.1 Types of Markov chains

There are two differing types when approaching Markov chains which is, discrete-time Markov chains and continuous-time Markov chains. This means that there are scenarios where the changes happen at specific states and others where the changes are continuous [25].

Discrete-Time Markov Chains (DTMC)

These are Markov chains that are observed only at discrete points in time (e.g., the end of each day) rather than continuously. Each time it is observed, the Markov chain can be in any one of a number of states [26].

Fig. 1. Discrete-Time Markov Chains [27]



P {system stays in state A for N time units | as long as the system is currently in state A} = p^N
P {system stays in state A for N time units before exiting from state A} = $p^N(1-p)$

State changes are pre-ordained to occur only at the integer points 0, 1, 2, ..., n (that is at the time points t_0, t_1, t_2,......, t_n)[28] [29]
The sequence of random variables X_1, X_2, ... forms a Markov Chain if for all n (n = 1, 2, ...) and all possible values of the random variables, giving;

$$P\left\{\frac{X_n=j}{X_1=i_1 \_\_\_\_\_ X_{n-1}=i_{n-1}}\right\} = P\left\{\frac{X_n=j}{X_{n-1}=i_{n-1}}\right\} \qquad (3)$$

Continuous-time Markov Chains (CTMC)

A continuous-time Markov chain changes at any time (State changes may occur anywhere in time) [26].
A Markov chain with continuous time is a stochastic process with Markov characteristics whose future state conditional probability, depends on present state which have no relation to past state of process [30].

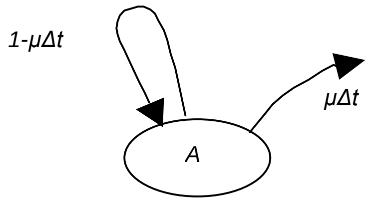


Fig. 2. Continuous -Time Markov Chains [27]

P{system in state A for time T | system currently in state A}

$$= (1-\mu\Delta t)^{\frac{T}{\Delta t}} \rightarrow e^{-\mu T} \qquad \Delta t \rightarrow 0 \tag{4}$$

2.3.2 Markov chains exploration

Markov chains model discrete-time processes and Markov processes models continuous-time processes. They mathematically model a process by showing how the method can move between different stages and therefore the probability of creating these transitions. Markov's analysis can be represented diagrammatically as in figure 1 which shows a Markov chain model of a process with two stages A1 and A2, where the probability of making a transition from stage *i* to stage *j* is q_{ij} [1].

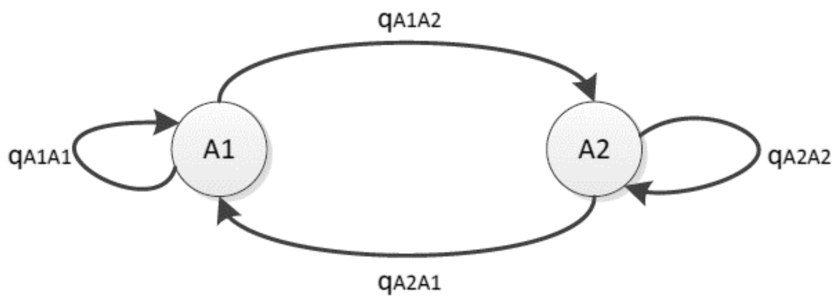


Fig. 3. Markov Chain Diagram [1]

2.4 Markov Chain Model States

The Markov chain model is a sequential process that consists of many steps. For those steps considered as Markov Chain states, they should respect all the following three conditions:

1. "State i communicates itself"
2. "If state i communicates with state j, then j communicates with state i."
3. "If state i communicates with state j, and j communicates with state k, then i communicates with state k."

According to [4], the probability of going from state i to state j in n time steps is given by: $P_{ij}^{(n)} = P_r\left(X_n = \frac{j}{X_0}, = \mathrm{i}\right)$ and the single step transition is $P_{ij} = P_r\left(X_1 = \frac{j}{X_0}, = \mathrm{i}\right)$

For a time-homogenous Markov Chain, the probability is: $P_{ij}^{(n)} = P_r\left(X_{n+k} = \frac{j}{X_k}, = \mathrm{i}\right)$ and $P_{ij} = P_r\left(X_{k+1} = \frac{j}{X_k}, = \mathrm{i}\right)$. A Markov Chain of order m, where m is finite, may be a process satisfying

$$P_r\left(X_n = \frac{x_n}{X_{n-1}}, = x_{n-1}, X_{n+2} = x_{n-2}, \ldots X_1 = x_1\right)$$
$$= P_r\left(X_n = \frac{x_n}{X_{n-1}}, = x_{n-1}, X_{n+2} = x_{n-2}, \ldots X_{n-m} = x_{n-m}\right) \; for\; n > m$$

In other words, the future state depends on the past m states. It is possible to construct a Chain Y_n from X_n which has the 'classical' Markov property by taking as state-space the ordered m tuples of x values, i.e. $Y_n = (X_n, X_{n-1}, \ldots, X_{n-m+1})$ [4]

2.5 Transition Probability Matrix

Transition probabilities are conditional probabilities P (X t+1 = j/ X t =i} = P_{ij} arranged in the form of a n x n matrix called the transition probability matrix given by:

$$\begin{pmatrix} p11 & p12\ldots & p1n \\ p21 & p22\ldots & p2n \\ pn1 & pn2\ldots & pnn \end{pmatrix}$$ which can be denoted as P = P_{ij}

The transition matrix shows the probability of transitioning between the row stage to the column stage. To form a Markov chain model the transition probabilities are required and are calculated using the equation below which determines the probability of making a transition from stage i to stage j, which is represented by P_{ij}. Where m is the total number of transitions and n_{ij} is the number of transitions from i to j [1].

$$p_{ij} = \frac{n_{ij}}{\sum_{k=1}^{m} n_{i,k}} \qquad (5)$$

The Transition Probability Matrix has the following properties: [15]

1. $P_{ij} > 0$ for all i and j.
2. For all i and j, sum of the element in each row is equal to 1. The sum represents total probability of transition from state i to itself or the other state.
3. The diagonal element represents transition from one state to same state.

Markov Chain models are useful in studying the evolution of systems over repeated trials. The repeated trials are often successive time periods where the state of the system in any particular period cannot be determined with certainty. Rather, transition probabilities are used to describe the way during which the system makes transitions from one period to subsequent. It helps us to determine the probability of the system being in a particular state at a given period of time [31].

2.6 Properties of Markov chains

Periodicity

A state i has period k if any return to state i must occur in multiple of k time steps. Formally, the period of a state is defined as:

$$\mathrm{K} = \gcd\left\{n: P_r\left({X_n=i}/{X_0=i}\right) > 0\right\} \qquad (6)$$

(where "gcd" is the greatest common division). Note that even though a state has period k, it may not be possible to succeed in the state in k steps. For example, suppose it is possible to return to the state in {6, 8, 10, 12….} time steps; k would be 2, even though 2 does not appear in this list.

If k = 1, then the state is claimed to be a periodic: returns to state i can occur at irregular times, in other words, a state i is aperiodic if there exists n such that for all $n^1 \geq 0$

$$P_r\left(X_n^1=i/_{X_0=i}\right) > 0 \quad (7)$$

Otherwise$(k > 1)$, the state is said to be periodic with period k. a Markov Chain is aperiodic if every state is aperiodic. An irreducible Markov Chain only needs one aperiodic state to imply all states are aperiodic. Every state of a bi partite graph has an even period. [4], [15].

Recurrence

A state i is said to be transient if, given that the system start in state i, there is a non-zero probability that the system will never return to i formally, but the random variable Ti be the first return time to state i (the "hitting time"): [4]

$$T_i = inf\left\{n \geq 1: X_n=i/_{X_0=i}\right\} \quad (8)$$

The number $f_{ii}^{(n)} = P_r(T_i = n)$ is the probability that state is returned to for the first time after n steps. Therefore, state i is transient if

$$P_r(T_i < \propto) = \sum_{n=1}^{\infty} f_{ii}^{(n)} < 1 \quad (9)$$

State i is recurrent if it is not transient. Recurrent states are guaranteed to have a finite hitting time [15].

Ergodicity

A state *i* is said to be ergodic if it is periodic and positive recurrent. In other words, a state has a period of 1 and it has finite mean recurrence time. If all states in an irreducible Markov chain are ergodic, then the chain is claimed to be ergodic. It can be shown that a finite state irreducible Markov chain is ergodic if it's a periodic state.
A model has the ergodic property if there's a finite number such that any state can be reached from any other state in exactly N steps. In case of a fully connected transition matrix where all transitions have a non-zero probability, this condition is fulfilled with N = 1. That is a Markov chain is ergodic if there exists some finite k such that;

$$P\left\{X(t+k)=j/_{X(t)=i}\right\} > 0$$ for all i and j [15]

A model with more than one state and just one out transition per state cannot be ergodic.

Reducibility

A state *j* is claimed to be accessible from a state a system started in state *i* has a non-zero probability of transitioning into state *j* at some point. Formally, state accessible from state *i* if there exists an integer $n_{ij} \geq 0$ such that

$$P_r(X_n=j/X_0=i) = p_{ij}^{n_{ij}} > 0 \quad (10)$$

This integer is allowed to vary for every pair of states, hence the subscripts in nij. Allowing *n* to be zero means that every state is defined to be accessible from itself. A state i is said to communicate with state *j* (written $i \leftrightarrow j$) if both $i \rightarrow j$ and $j \rightarrow i$. A set of states *C* may be a communicating class if every pair of states in *C* communicates with each other, and no state in *C* communicates with any state not in *C*. It may be shown that communication in this sense is an equivalence relation and thus that communicating classes are the equivalence classes of this relation. A communicating class is closed if the probability of leaving the category is zero, namely that if *i* is in C but j isn't, then *j* isn't accessible from *i*.

A state *i* is claimed to be essential or final if for all j such that $i \rightarrow j$ it's also true that $j \rightarrow i$. A state *i* is inessential if it's not essential. A Markov chain is claimed to be irreducible if its state space may be a single communicating class; in other words, if it's possible to get to any state from any state [15].

2.7 Classification of states of a Markov chain

Recurrent States

A state is claimed to be a **recurrent state** if, upon entering this state, the method definitely will return to the present state again. Therefore, a state is recurrent if and as long as it's not transient.

Since a recurrent state definitely will be revisited after each visit, it will be visited infinitely often if the process continues forever [32].

If the method enters a particular state then stays during this state at the subsequent step, this is often considered a return to the present state. Hence, the following kind of state is a special type of recurrent state [26].

Transient States

A state is claimed to be a **transient state** if, upon entering this state, the method may never return to the present state again. Therefore, state i is transient if and as long as there exists a state j (j≠ i) that's accessible from state i but not the other way around, that is, state i is not accessible from state j [33].

Thus, if state i is transient and the process visits this state, there is a positive probability (perhaps even a probability of 1) that the process will later move to state j and so will never return to state i. Consequently, a transient state will be visited only a finite number of times[26]. When starting in state i, another possibility is that the process definitely will return to this state [34].

The Markov process is transient if the state can only be visited a finite number of times otherwise, the state is recurrent[11].

Absorbing states

In an absorbing Markov chain model, the Markov chain may include circles and it theoretically allows an infinite number of circulations among certain process states [35]

A state is claimed to be an **absorbing state** if, upon entering this state, the process never will leave this state again. Therefore, state i is an absorbing state if and only if $P_{ii} = 1$ [33]

A Markov chain with one or more absorbing states is understood as absorbing Markov chain. An absorbing state is, because the name implies, one that endures. In other words, when a work-part reaches such a state, it never leaves the state [36].

2.8 Areas of application of Markov chains

Markov chains are used in a variety of situations since they can be considered to model many real-world processes. These fields include, to mention but a few, quality management [37], system performance (reliability & availability)[38], electronics [35], condition monitoring [39], physics, chemistry, computer science, queuing theory, economics, games, and sports [23].

3. Markov models in manufacturing

In an effort to gain a better understanding of the markov chains and its application in manufacturing, and to provide a basis for future research, a broad review of some existing research on the subject has been presented.

Table 1 gives a summary of citations on Markov models in manufacturing. A complete of 39 citations on Markov models in manufacturing were reviewed. The majority of the citations were found in journals (76.92%), proceedings, conferences and others (12.82%), books (5.13%) and published PhD Thesis (5.13%).

Table 1: summary of citations on Markov models in manufacturing

Source	Number of citations	% total
Journal of Industrial Engineering	1	2.564
Procedia Manufacturing	2	5.128
International Journal of computer science issues	1	2.564
Conference proceedings	5	12.821
Thesis	2	5.128
Journal of Mathematics and Statistics	1	2.564
Book	2	5.128
UPB Scientific Bulletin, Series D: Mechanical Engineering	2	5.128
Periodica Polytechnica Social and Management Sciences	1	2.564
Nuclear Engineering and Design	1	2.564
Journal of Advanced Mechanical Design, Systems and Manufacturing	1	2.564
Journal of the Operational Research Society	1	2.564
International Journal of Engineering Research & Technology	1	2.564
Advances in Science and Technology Research Journal	1	2.564
International Journal of Production Economics	2	5.128
Journal of Cleaner Production	2	5.128
International Journal of Current Research	2	5.128
Journal of Banking Financial	1	2.564
Computers and Chemical Engineering	1	2.564
Computers & Industrial Engineering	2	5.128
Quality Engineering	1	2.564
Manufacturing and Service Operations Management	1	2.564
Journal of Industrial Mathematics	1	2.564
Applied Sciences (Switzerland)	1	2.564
IJISET-International Journal of Innovative Science, Engineering & Technology	1	2.564
Acta Mathematica Scientia	1	2.564
Annals of the Academy of Romanian Scientists Series on Engineering Sciences	1	2.564
Total	**39**	**100**

Table 2 provides a summary of the classification scheme of Markov models in manufacturing giving the research topic, nature of uncertainty, research approach and conclusions drawn.

Table 2: Classification scheme of Markov models in manufacturing

Author(s)	**Research topic**	**Uncertainty**	**Approach detail**	**Conclusion**
Leigh et al., 2017	Modelling manufacturing processes	Human interaction with very variable products	Radio Frequency Identification (RFID)	Created a Markov chain model used to predict future product paths for use in discrete event simulation
Tochukwu et al., 2015	Agent Based Markov Chain for Job Shop Scheduling and Control	Dynamic market changes	Scheduling algorithms	Developed an agent based model where all information of the dynamics of the model was formulated as a Markov chain
Kiassat et al., 2013	Effects of operator learning on production output	Operator learning	Proportional hazards model	Developed a Markov chain approach to forecast production output of a human-machine system, considering HR factors and operator learning.
Gingu & Zapciu, 2017	Synchronizing the manufacturing production rate with real market demand	Market demand	Markov chains and decomposition method, C++	Offered a solution, by avoiding intermediary stocks at the same time, and a predictable market demand of these products (balancing between demand and production)
Ye et al., 2019	Modeling for reliability optimization of system design and maintenance based on Markov chain theory	System failures and repairs	Continuous-time Markov chain	Proposed a non-convex MINLP model
Chatys, 2020	Application of the Markov Chain Theory in Estimating the Strength of Fiber-Layered Composite Structures with Regard to Manufacturing Aspects	Static strength and fatigue life	Vacuum bag method (mathematical model)	MM can be used for “predicting” the S-N curve, taking into account the maximum volume share of reinforcement in the composite and manufacturing technology
Sastri et al., 2001	Markov chain approach to failure cost Estimation in batch manufacturing	Failure cost estimation (repair/rework)	Markov chain approach,	Showed how a markov chain model is used to estimate a fore mentioned activity based failure costs
Santhi, 2019	Markov decision process in supply chain management	Inventory levels	Markov decision process	Determined the service rates to be employed as a function of the number of customers in the queue and the amount of inventory on hand so that

				the long-run expected cost rate is minimized
Mubiru, 2013	An EOQ Model For Multi-Item Inventory With Stochastic Demand	Demand	Markov decision process	Demonstrates the existence of an optimal state dependent EOQ, produces optimal ordering policies and the corresponding total inventory costs for items.
Boteanu & Zapciu, 2017	Modeling and simulation of manufacturing flows for optimizing the number of work pieces on buffers from manufacturing systems	Failures, demand modifications, breakdown	Markov chains, Decomposition method, C++ programme, discrete event simulation (analytical approach)	Dynamic adaptation of the production rate by optimizing the buffers according to the effective demand or estimated demand of the market.
Janicijevic et al., 2014	Using a markov chain for product quality Improvement simulation	Customer requirements	Simulation	Modelled the stochastic processes of a system of quality management and selection of the optimum set of FIPQ.
Sharma & Vishwakarma, 2014	Application of Markov Process in Performance Analysis of Feeding System of Sugar Industry	Systeme performance (failures)	Markov modelling	The system can be analyzed easily by concerning the process as Markov process and it helps the system design analyst or plant personnel to select the most appropriate structural components. (high performance measures for maximum duration of time)
Pillai & Chandrasekharan, 2008	An absorbing Markov chain model for production systems with rework and scrapping	Scrapping and reworking	Probabilistic model	Identifies production system parameters under scrapping and reworking, and accurately estimates the quantity of raw materials required.
Afrinaldi, 2020	Exploring product lifecycle using Markov chain	Behavior of the product	Markov chain	The number of trips and duration of stay of a product in a particular lifecycle stage, number of products visiting a specific lifecycle stage, probability of a product being discarded, and the expected total environmental impact of the product are predicted

Sobaszek et al., 2020	Predictive Scheduling with Markov Chains and ARIMA Models	Machine failure	Markov process	Inclusion of machine failure in the production schedule results in the extension of the performance indicators, mean flow time, mean job completion time, and the central criterion describing the performance of the production system
Strachan et al., 2009	A Hidden Markov Model for Condition Monitoring of a manufacturing drilling process.	Tool wear and impending failure	Algorithm; hidden Markov model	presented an algorithm for the condition monitoring of a manufacturing drilling process that will be able to detect tool wear and impending failure
Jónás et al., 2014	Application of Markov Chains for Modeling and Managing Industrial Electronic Repair Processes	Repairs	Absorbing markov chain	Modeling repair, manufacturing and business processes as acyclic absorbing Markov chains can ground for many process management activities which enable managers to determine the probability distribution of lead time of any repairing process.
Beijsens & Rooda, 2005	Markov based modeling of manufacturing systems dynamics	Manufacturing system properties	Markov theory	Control a discrete manufacturing system with a continuous controller. And the continuous model validated with a discrete-event model
Karim & Nakade, 2020	A Markovian production-inventory system with consideration of random quality disruption	Product quality disruption	Stochastic model	Under the situation of production time constraint, the integration of safety stock in an interruption prone production–inventory system, assists in improving the average cost function.
Abedi et al., 2009	Using Markov Chain to Analyze Production Lines Systems with Layout Constraints	Layout constraints	Hybrid model (Markov chain in queue theory)	Developed a queuing model by analyzing a real queuing system with layout limitations in specific conditions and applying Markov chain concepts

From the reviewed literature, there a number of uncertainties that affect the performance in manufacturing companies. From table 2 it is seen that system failure and repairs (45%) is the most researched nature of uncertainty affecting

manufacturing, then market /customer requirements or demand (25%), inventory levels (5%), product quality (5%), and others (20%)
It is also seen that both Discrete-Time Markov Chains (DTMC) and Continuous-time Markov Chains (CTMC) approaches are used although Discrete-Time Markov Chains was used more.

Conclusion

This paper has presented an extensive literature survey about the application of Markov chains in manufacturing systems. Markov chain is an established concept in operations research and probability theory and it has been applied to many areas in manufacturing including quality management, system performance (reliability & availability), supply chain, electronics, condition monitoring, queuing theory, economics, to mention but a few.
As a basis for decision making, Markov Chain prediction method is no exception and a combination of results from using Markov Chain to predict with other factors can be more useful.

More research should be done on development of models in the context of Continuous Time Markov Chains (CTMC) [5].
Models should further be developed to be applied for products having components and modules, the logistics operation behind the transition needs to be modeled so that the accuracy of the model is improved and, the economic aspects should be included in the model, to aid policymakers in making a comprehensive decision[23] .

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Appendix 2, Paper 2: "Manufacturing Lot Size Optimization under Demand Uncertainty: A Stochastic Goal Programming Approach"

"Maureen Nalubowa S, Paul Kizito Mubiru, Jerry Ochola, Saul Namango. Manufacturing Lot Size Optimization Under Demand Uncertainty: A Stochastic Goal Programming Approach. *International Journal of Academic Engineering Research (IJAER)* ISSN: 2643-9085 Vol. 5 Issue 12, December - 2021, Pages: 45-55"

Manufacturing lot size optimization under demand uncertainty: A stochastic goal programming approach

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Abstract
Optimization has become a standard phenomenon in the majority of organizations and establishments. Many Manufacturing companies operate under uncertainties which affect the system performance. Product demand is one of the common kinds of uncertainty that characterizes production environments. One of the challenges faced by manufacturing companies that use cost analyses is product demand uncertainty that often affects the manufacturing system performance and decision making. Manufacturing Lot size problems are normally related to proficient production planning of a given product. If a manufacturing firm wants to compete within the market, it must make the right decisions regarding lot-sizing problems and this can be a critical decision for any manufacturer. In this paper, an optimization model for the manufacturing lot size was developed using Markov chains in conjunction with stochastic goal programming. The goal constraints, deviation variables, priorities and objective function were defined to determine the over-achievement or underachievement of the manufacturing lot size for aggregate production planning, the different states of demand for the product being represented by states of a Markov chain. The model was solved using the linear programming solver in MATLAB™ to determine the quantity of product plan for manufacturing within the first quarter of the year when demand changes from one state to another.

Keywords
Optimization, manufacturing lot size, demand uncertainty, production planning, goals

1. Introduction

Uncertainties present an unavoidable concern associated with a continuous operation of the manufacturing system, a state of insufficient information, and this can be seen in three forms: inexactness, unreliability, and border with ignorance [1].
One of the challenges faced by manufacturing companies that use cost analyses is product demand uncertainty, as it may influence the manufacturing system performance hence the final decision on utilizing a manufacturing system at the initial

stages [2]. When assessing the risk related to a decision, understanding these uncertainties and their impacts, which can make it difficult to predict performance, are of major concern [3].
Production planning is the pillar of any manufacturing operation, with the main aim of determining the amount of products to be manufactured considering the level of inventory to be shifted from one period to another with the objective to minimize both the total costs of production and the inventory, meeting the customers' demand [4].
In production planning, making the right decisions about the lot size is very important as it directly affects the system performance and productivity [5] and this is key for any manufacturing firm that wants to compete in the Market.
Lot sizing problems have got a direct effect on the system performance and productivity. Manufacturing Lot sizing can be defined as determining the quantity of a given product that needs to be manufactured in a specified period of time. Manufacturing Lot sizing problems are normally associated with proficient production planning of a given product. Each production plan has got the main problem of determining the manufacturing lot size for each product. In order to have efficient production planning lot allocation issues must be solved based on the demand that needs to be achieved and the availability of inventory stock minimizing production costs by determining the optimal production quantity[6].
The smaller the manufacturing lot size, the less the holding cost but raises the ordering cost whereas the larger the manufacturing lot size, the more the holding cost but reducing the ordering cost. Based on the concepts of lean production, it is preferable to have a small lot size as it prevents the accumulation of inventory which comes with management and holding costs. The lot size recommended by a mathematical manufacturing lot size model would be the best as it accounts for the tradeoff between the costs involved [5].
Optimization is the process of finding (activity of choosing [7]) the best possible solution to a given problem by examining several alternatives (assessed after a predefined criterion) [8] and can be done by adjusting the inputs to or characteristics of a device, mathematical process, or experiment to find the minimum or maximum output [9].
The optimization problem contains three basic parameters needed to be considered, that is, the objective function, a set of variables and a set of constraints [10].
The objective of the optimization model depends on certain characteristics of the system, called variables or unknowns with the goal of finding the values of these variables that optimize the objective, although these variables are often restricted, or constrained in one way or the other. Brahimi et al. grouped optimization problems into four categories: process planning, layout design, reconfigurability and planning and scheduling.
Manufacturing lot size is in the category of planning and scheduling. Manufacturing companies must have the ability to adjust scalable production capacities and to respond rapidly to market demands making planning and scheduling become complex in such a dynamic environment [11].
Stochastic analysis and goal programming are introduced into the framework to handle uncertainties in real-world manufacturing systems.
Stochastic Goal Programming is a multi-criteria decision support model that gives "satisficing" solutions to a linear system under an uncertainty case from the normally expected utility viewpoint [12], [13]. Most real-world optimization problems consist of various inexact information estimates and goals, conflicting criteria. In such

situations, the stochastic goal programming method suggests an analytical structure aid in modelling and solving such problems.

Stochastic goal programming can deal with the inherent uncertainty and has been applied in different fields including Portfolio selection, project selection, resource allocation, Healthcare management, transportation, marketing [14], cash management [15], wealth management [16], economic development, energy consumption, workforce allocation, and greenhouse gas emissions[17], forest planning [18]. Not many applications are seen in production planning in manufacturing systems hence the need for manufacturing lot size optimization under demand uncertainty. This can be considered as a guideline for production planners and practitioners used to solve specific decision-making problems (optimal manufacturing lot size). Manufacturing companies will minimize on overproduction when demand is actually low or under-producing when demand is actually high.

Due to the fluctuating and uncertainties in demand, manufacturing companies over and over again face the challenge of establishing optimal manufacturing lot sizes in production planning systems. Manufacturing companies are continuously looking for efficiency to overcome the challenges associated with the market dynamics. One of the common types of uncertainty that characterizes production environments is uncertainty in product demand. It is therefore important that these uncertain parameters be considered in the production planning process when developing a robust production plan because when neglected, production efficiency and system performance will be affected [19].

Manufacturing industries establish their production plans based on external demands with the core aim of determining the quantity (lot size) to be produced given each period while satisfying the demands and minimizing total costs [20]. In production planning, making the right decisions about the lot size is very important as it directly affects the system performance and productivity [5] and this is key for any manufacturing firm that wants to compete on market.

As this is complex as well as important, it has been highly studied although, there is still a gap about showing the contributions to clarify the suitability of those methods used concerning each kind of underlying manufacturing environment (regarding variations in demand and peaks of seasonality) [21].

Therefore the present study aimed at developing an optimization model for the manufacturing lot size under demand uncertainty, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired for aggregate production planning.

2. Mathematical model formulation

A manufacturing company producing products with fluctuations and uncertainties in demand was considered. The demand for these products during each time period over a finite fixed planning horizon was described as either favorable or unfavorable.

The Markov chain approach ([22], [23], [24], [25], [26]) in conjunction with stochastic goal programming ([13], [27], [14], [28], [18], [15]) was adopted and the states of a Markov chain represent possible states of demand for the finished products with the notations shown in Table 1.

Table 1: Notations used in the Markov models

i, j	Set of states of demand	M	Manufacturing lot-size
F	Favorable demand	$X_{ij}(p, q)$	Quantity of product p to be manufactured in quarter q
U	Unfavorable demand	N	Customer matrix
Q	Demand transition matrix	C_p	Unit production cost
p	Product	C_h	Unit holding cost
q	Quarter of the year	C_s	Unit shortage cost
FF, FU, UF, UU	State transitions	D	Demand matrix
Z	Value of the objective function	V	Inventory matrix
P_k	Preemptive priority of the k^{th} goal	C	Production-Inventory cost matrix
d_k^+	Over achievement of the k^{th} goal	B	Beginning Inventory
d_k^-	Under achievement of the k^{th} goal	E	Ending Inventory

Average on-hand inventory, $V = (B+E)/2$ (1)

Consider the customer matrix:

$$N(p, q) = \begin{bmatrix} N_{FF}(p, q) & N_{FU}(p, q) \\ N_{UF}(p, q) & N_{UU}(p, q) \end{bmatrix} \tag{2}$$

2.1 Demand transition probability

As demand changes from state *i* to state *j* for $i, j \in \{F, U\}$, the associated demand transition probabilities are calculated as:

$$Q_{ij}(p, q) = \frac{N_{ij}(p,q)}{N_{if}(p,q)+N_{iu}(p,q)} \tag{3}$$

This yields the demand transition matrix:

$$Q(p, q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & Q_{FF}(p, q) & Q_{FU}(p, q) \\ \boldsymbol{U} & Q_{UF}(p, q) & Q_{UU}(p, q) \end{matrix} \tag{4}$$

Then the demand matrix, the inventory matrix and the production-inventory cost matrix.

Demand matrix;

$$D(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & D_{FF}(A,1) & D_{FU}(A,1) \\ \boldsymbol{U} & D_{UF}(A,1) & D_{UU}(A,1) \end{matrix} \tag{5}$$

Inventory matrix;

$$V(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(A,1) & V_{FU}(A,1) \\ \boldsymbol{U} & V_{UF}(A,1) & V_{UU}(A,1) \end{matrix} \tag{6}$$

Production-inventory cost matrix;

When demand outweighs the amount produced then,

$$C(p, q) = \begin{bmatrix} C_p \\ + \\ C_h \\ + \\ C_s \end{bmatrix} [D(p, q) - V(p, q)] \tag{7}$$

Similarly, when the demand is less than the amount produced then,

$$C(p,q) = C_h[\,V(p,q) - D(p,q)] \quad (8)$$

Hence, as demand changes from state *i* to state *j* ($i, j \in \{F, U\}$)

$$C(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(p,q) & C_{FU}(p,q) \\ \boldsymbol{U} & C_{UF}(p,q) & C_{UU}(p,q) \end{matrix} \quad (9)$$

where $C(p,q) =$ production-inventory cost matrix.

2.2 Expected demand, inventory, production-inventory costs and manufacturing lot-size

Expected demand

Favorable Demand $E[D_F(p,q)] = Q_{FF}(p,q)D_{FF}(p,q) + Q_{FU}(p,q)D_{FU}(p,q)$ (10)

Unfavorable Demand $E[D_U(p,q)] = Q_{UF}(p,q)D_{UF}(p,q) + Q_{UU}(p,q)D_{UU}(p,q)$ (11)

Expected inventory

Favorable Demand $E[V_F(p,q)] = Q_{FF}(p,q)V_{FF}(p,q) + Q_{FU}(p,q)V_{FU}(p,q)$ (12)

Unfavorable Demand $E[V_U(p,q)] = Q_{FU}(p,q)V_{UF}(p,q) + Q_{UU}(p,q)V_{UU}(p,q)$ (13)

Expected production-inventory costs

Favorable Demand $E[C_F(p,q)] = Q_{FF}(p,q)C_{FF}(p,q) + Q_{FU}(p,q)C_{FU}(p,q)$ (14)

Unfavorable Demand $E[C_U(p,q)] = Q_{UF}(p,q)C_{UF}(p,q) + Q_{UU}(p,q)C_{UU}(p,q)$ (15)

Expected manufacturing lot-size

Favorable demand

$$E[M_F(p,q)] = \begin{cases} E[D_F(p,q)] - E[V_F(p,q)] & if\ E[D_F(p,q)] > E[V_F(p,q)] \\ \\ 0 & otherwise \end{cases} \quad (16)$$

Unfavorable demand

$$E[M_U(p,q)] = \begin{cases} E[D_U(p,q)] - E[V_U(p,q)] & if\ E[D_U(p,q)] > E[V_U(p,q)] \\ \\ 0 & otherwise \end{cases} \quad (17)$$

2.3 Stochastic goal programming formulation

The stochastic goal programming model was formulated by setting priorities, defining the objective function and formulating the goal constraints as follows:

Set priorities

P_1: Produce a batch of $E[M_F(p,q)]$ units when demand is favorable

P_2: Produce a batch of $E[M_U(p,q)]$ units when demand is unfavorable

P_3: Total production-inventory cost must not exceed $E[C_F(p,q)]$ when demand is favorable

P_4: Total production-inventory cost must not exceed $E[C_U(p,q)]$ when demand is unfavorable

Objective function

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \quad (18)$$

Goal constraints

P1: Manufacturing lot-size $E[M_F(p,q)]$ - favorable demand

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \quad (18.1)$$

P2: Manufacturing lot-size $E[M_U(p,q)]$ - unfavorable demand

$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \quad (18.2)$$

P3: Total production-inventory cost – favorable demand

$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad (18.3)$$

P4: Total production-inventory cost – unfavorable demand

$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad (18.4)$$

2.4 Stochastic goal programming model for manufacturing lot-size

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \quad (19)$$

Subject to:

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \quad (19.1)$$

$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \quad (19.2)$$

$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad (19.3)$$

$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad (19.4)$$

$$X_{FF}(p,q), X_{FU}(p,q), X_{UF}(p,q), X_{UU}(p,q), d_1^-, d_1^+, d_2^-, d_2^+, d_3^+, d_4^+ \geq 0 \quad (19.5)$$

3. Case study

In this section, a real case application from Movit Products Uganda limited was used to demonstrate the applicability of the proposed mathematical models. The manufacturing industry manufactures, distributes and sells skincare, hair & nail care products. The numerical illustration contains real data for the first quarter of the year, which was collected and then reduced to usable dimensions as shown in Table 2. Data classification by state of demand was made, analyzed and used in the proposed mathematical model.

Considering a product A, for a given week, demand is favorable (state F) if $N_{ij} > 12$ otherwise demand is unfavorable (state U) if $N_{ij} \leq 12$ as shown in Table 2.

Table 2: Data classification by state of demand for product A

Month	Week	Customers (N)	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	State of demand (*i*)
1	1	9	3937	6076	U
	2	12	4668	4687	U
	3	8	2485	6306	U
	4	17	7955	10160	F
2	1	1	110	4525	U
	2	15	3832	5681	F
	3	7	2870	4363	U
	4	20	3824	6028	F
3	1	4	758	2018	U
	2	16	6125	4149	F
	3	14	2625	4163	F
	4	17	3685	6279	F

Table 3a, 3b and 3c shows the over stocking or under stocking of product A with the corresponding holding or shortage costs in the first quarter of the year.

Table 3a: Overstocking and understocking with holding and shortage costs for 1st month

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortag e costs
1	3937	6076	2139	231.6537
2	4668	4687	19	2.0577
3	2485	6306	3821	413.8143
4	7955	10160	2205	238.8015

Table 3b: Overstocking and understocking with holding and shortage costs for 2nd month

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	110	4525	4415	478.1445
2	3832	5681	1849	200.2467
3	2870	4363	1493	161.6919
4	3824	6028	2204	238.6932

Table 3c: Overstocking and understocking with holding and shortage costs for 3rd month

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	758	2018	1260	136.458
2	6125	4149	-1976	1569.734
3	2625	4163	1538	166.5654
4	3685	6279	2594	280.9302

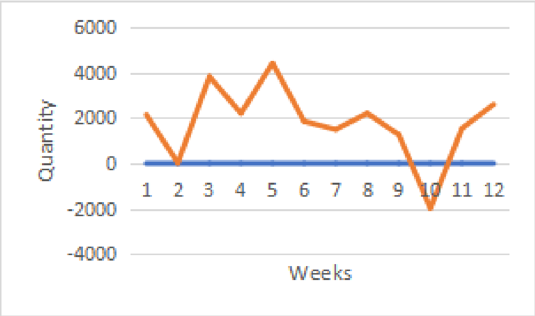

Figure 2: Over stocking and under stocking of product A

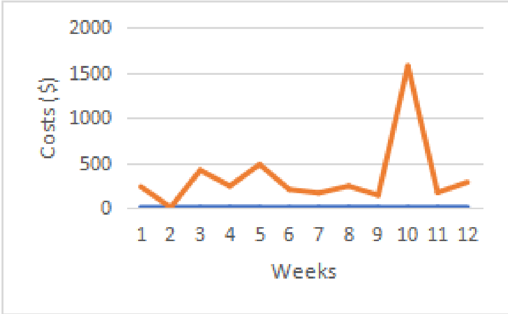

Figure 3: Holding and Shortage costs

3.1 State transitions and on-hand inventory

For a particular state transition, given the beginning and ending inventory, the average on-hand inventory was calculated as presented in Table 4.

Table 4: Average on-hand inventory for product A

State transitions (i, j)	Beginning inventory (B)	Ending inventory (E)	Average on-hand inventory $V = (B + E)/2$
FF	4163	6279	5221
FU	4525	2018	3271.5
UF	10160	4149	7154.5
UU	4687	6306	5496.5

From Equation (1) section 2, the average on-hand inventory was calculated giving;

$V_{FF}(A,1) = 5221$ $V_{FU}(A,1) = 3271.5$ $V_{UF}(A,1) = 7154.5$ $V_{UU}(A,1) = 5496.5$

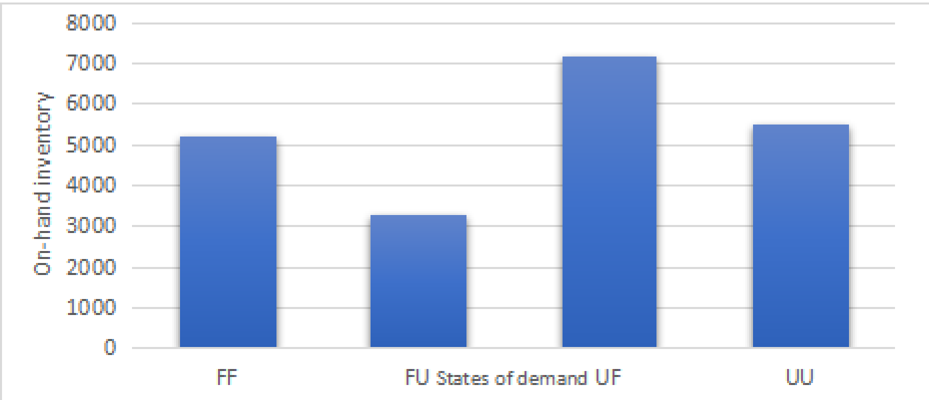

Figure 4: Average on-hand inventory and state transitions

3.2 Demand transition probabilities

Data classification by state-transition was done as illustrated in Table 5 and then used to calculate the demand transition probabilities for the product

Table 5: *Data classification by state-transition for product A*

Month	State transition (i, j)	Number of customers $N_{ij}(A,1)$	Demand $D_{ij}(A,1)$
1	FF	0	0
	FU	0	0
	UF	25	10440
	UU	41	15758
2	FF	0	0
	FU	22	6702
	UF	43	10636
	UU	0	0
3	FF	61	15060
	FU	0	0
	UF	20	6883
	UU	0	0

From Table 5, the Totals for customers and demand as it changes from one state to another are;

Customers:

$N_{FF}(A,1) = 0 + 0 + 61 = 61$ $\quad$ $N_{FU}(A,1) = 0 + 22 + 0 = 22$

$N_{UF}(A,1) = 25 + 43 + 20 = 88$ $\quad$ $N_{UU}(A,1) = 41 + 0 + 0 = 41$

Demand:

$D_{FF}(A,1) = 0 + 0 + 15060 = 15060$ $\quad$ $D_{FU}(A,1) = 0 + 6702 + 0 = 6702$

$$D_{UF}(A,1) = 10440 + 10636 + 6883 = 27959$$

$$D_{UU}(A,1) = 15758 + 0 + 0 = 15758$$

From Equation (3) in section 2, the demand transition probabilities are;

$$Q_{FF}(A,1) = \frac{N_{FF}(A,1)}{N_{FF}(A,1)+N_{FU}(A,1)} = \frac{61}{61+22} = 0.7349$$

$$Q_{FU}(A,1) = \frac{N_{FU}(A,1)}{N_{FF}(A,1)+N_{FU}(A,1)} = \frac{22}{61+22} = 0.2651$$

$$Q_{UF}(A,1) = \frac{N_{UF}(A,1)}{N_{UF}(A,1)+N_{UU}(A,1)} = \frac{88}{88+41} = 0.6822$$

$$Q_{UU}(A,1) = \frac{N_{UU}(A,1)}{N_{UF}(A,1)+N_{UU}(A,1)} = \frac{41}{88+41} = 0.3178$$

Hence the demand transition matrix as from equation (4),

$$Q(A,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \begin{matrix} \boldsymbol{F} & \quad\quad \boldsymbol{U} \end{matrix} \\ \begin{pmatrix} 0.7349 & 0.2651 \\ 0.6822 & 0.3178 \end{pmatrix} \end{matrix}$$

3.3 Demand matrix, inventory matrix and production-inventory cost matrix

The demand matrix, the inventory matrix and the production-inventory cost matrix were developed as follows.

From Equation (5), the demand matrix becomes;

$$D(A,1) = \begin{matrix} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{matrix} \begin{matrix} \begin{matrix} \boldsymbol{F} & \quad\quad \boldsymbol{U} \end{matrix} \\ \begin{pmatrix} 15060 & 6702 \\ 27959 & 15758 \end{pmatrix} \end{matrix}$$

From Equation (6), the Inventory matrix becomes;

$$V(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 5221 & 3271.5 \\ \boldsymbol{U} & 7154.5 & 5496.5 \end{matrix}$$

Production-inventory cost matrix

The production-inventory cost matrix is then computed for the product From Equations (7), (8) and (9).

Unit production cost, $C_p(A) = \$\ 7.2222$

Unit holding cost, $C_h(A) = \$\ 0.1083$

Unit shortage cost, $C_s(A) = \$\ 0.7944$

$$C_{FF}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{FF}(A,1) - V_{FF}(A,1)\right)$$
$$C_{FF}(A,1) = (7.2222 + 0.1083 + 0.7944)(15060 - 5221) = 79940.9$$
$$C_{FU}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{FU}(A,1) - V_{FU}(A,1)\right)$$
$$C_{FU}(A,1) = (7.2222 + 0.1083 + 0.7944)(6702 - 3271.5) = 27872.5$$
$$C_{UF}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{UF}(A,1) - V_{UF}(A,1)\right)$$
$$C_{UF}(A,1) = (7.2222 + 0.1083 + 0.7944)(27959 - 7154.5) = 169034.5$$
$$C_{UU}(A,1) = \left(C_h(A)\right)\left(D_{UU}(A,1) - V_{UU}(A,1)\right)$$
$$C_{UU}(A,1) = (0.1083)(15758 - 5496.5) = 1111.3$$

Hence,

$$C(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(A,1) & C_{FU}(A,1) \\ \boldsymbol{U} & C_{UF}(A,1) & C_{UU}(A,1) \end{matrix}$$

$$C(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 79940.9 & 27872.5 \\ \boldsymbol{U} & 169034.5 & 1111.3 \end{matrix}$$

3.4 Expected demand, inventory, production-inventory costs and manufacturing lot-size

Expected demand

After generating the demand transition matrix and formulating the production-inventory cost matrix, the expected demand, expected inventory and expected production-inventory costs are computed for the product considering both favorable and unfavorable demand as shown below;

Favorable demand (F) was computed from equation (10)

$$E[D_F(A,1)] = Q_{FF}(A,1) * D_{FF}(A,1) + Q_{FU}(A,1) * D_{FU}(A,1)$$
$$E[D_F(A,1)] = (0.7349 * 15{,}060) + (0.2651 * 6{,}702)$$
$$E[D_F(A,1)] = 12{,}844.3\ units$$

Unfavorable demand (U) was computed from equation (11)

$$E[D_U(A,1)] = Q_{UF}(A,1) * D_{UF}(A,1) + Q_{UU}(A,1) * D_{UU}(A,1)$$
$$E[D_U(A,1)] = (0.6822 * 27{,}959) + (0.3178 * 15{,}758)$$
$$E[D_U(A,1)] = 24{,}081.5\ units$$

Expected Inventory

Computation of the expected inventory considering both favorable and unfavorable demand for the product was computed From equation (12) as follows:

Favorable demand (F)

$$E[V_F(A,1)] = Q_{FF}(A,1) * V_{FF}(A,1) + Q_{FU}(A,1) * V_{FU}(A,1)$$

$$E[V_F(A,1)] = (0.7349 * 5221) + (0.2651 * 3271.5)$$

$$E[V_F(A,1)] = 4{,}704.2 \; units$$

Unfavorable demand (U) was computed from equation (13) as follows

$$E[V_U(A,1)] = Q_{UF}(A,1) * V_{UF}(A,1) + Q_{UU}(A,1) * V_{UU}(A,1)$$

$$E[V_U(A,1)] = (0.6822 * 7154.5) + (0.3178 * 5496.5)$$

$$E[V_U(A,1)] = 6{,}627.6 \; units$$

Expected production-Inventory costs

The expected production-Inventory costs are then computed for the product considering both favorable and unfavorable demand results were computed from equations (14) and (15) as follows;

Favorable demand (F)

$$E[C_F(A,1)] = Q_{FF}(A,1) * C_{FF}(A,1) + Q_{FU}(A,1) * C_{FU}(A,1)$$

$$E[C_F(A,1)] = (0.7349 * 79940.9) + (0.2651 * 27872.5)$$

$$E[C_F(A,1)] = \$ \; 66{,}137.6$$

Unfavorable demand (U)

$$E[C_U(A,1)] = Q_{UF}(A,1) * C_{UF}(A,1) + Q_{UU}(A,1) * C_{UU}(A,1)$$

$$E[C_U(A,1)] = (0.6822 * 169034.5) + (0.3178 * 1111.3)$$

$$E[C_U(A,1)] = \$ \; 115{,}668.5$$

Expected manufacturing lot size

Computation of the expected manufacturing lot size considering both favorable and unfavorable demand for the product yields was computed from equations (16) and (17) as follows:

Favorable demand (F)

$$E[M_F(A,1)] = \begin{pmatrix} E[D_F(A,1)] - E[V_F(A,1)] \; if \; E[D_F(A,1)] > E[V_F(A,1)] \\ 0 \qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_F(A,1)] = E[D_F(A,1)] - E[V_F(A,1)]$$

$$E[M_F(A,1)] = 12{,}844.3 - 4{,}704.2 = 8{,}140.1 \; units$$

Unfavorable demand (U)

$$E[M_U(A,1)] = \begin{pmatrix} E[D_U(A,1)] - E[V_U(A,1)] \; if \; E[D_U(A,1)] > E[V_U(A,1)] \\ 0 \qquad\qquad\qquad otherwise \end{pmatrix}$$

$$E[M_U(A,1)] = E[D_U(A,1)] - E[V_U(A,1)]$$

$$E[M_U(A,1)] = 24{,}081.5 - 6{,}627.6 = 17{,}453.9 \; units$$

3.5 Stochastic goal programming model

The stochastic goal programming model for the product was formulated by setting priorities, defining the objective function and formulating the goal constraints as follows:

Priorities set

$P_1: Produce\ a\ batch\ of\ 8{,}140.1\ units\ when\ demand\ is\ initially\ favorable$

$P_2: Produce\ a\ batch\ of\ 17{,}453.9\ units\ when\ demand\ is\ initially\ unfavorable$

$P_3: Total\ production\_inventory\ costs\ must\ not\ exceed\ \$\ 66{,}137.6\ when\ demand\ is\ favorable$

$P_4: Total\ production\_inventory\ costs\ must\ not\ exceed\ \$\ 115{,}668.5\ when\ demand\ is\ unfavorable$

Objective function

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(A,1)d_k^+ + P_K(A,1)d_k^-]$$

Goal constraints

Manufacturing lot size

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1\ (Favorable\ demand)$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$79940.9X_{FF}(A,1) + 27872.5X_{FU}(A,1) - d_3^+ = 66{,}137.6\ (Favorable\ demand)$$

$$169034.5X_{UF}(A,1) + 1111.3X_{UU}(A,1) - d_4^+ = 115{,}668.5\ (Unfavorable\ demand)$$

Non negativity

$$X_{FF}(A,1), X_{FU}(A,1), X_{UF}(A,1), X_{UU}(A,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

3.6 Stochastic goal programming model for manufacturing lot size

The stochastic goal programming model for manufacturing lot size was then developed for the product as below. This determines the quantity of the product to manufacture in the first quarter of the year when demand changes from state *i* to state *j* for $i, j \in \{F, U\}$, establishing the over-achievement or under achievement of the manufacturing lot size priorities desired.

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(A,1)d_k^+ + P_K(A,1)d_k^-]$$

Subject to:

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9$$

$$79940.9X_{FF}(A,1) + 27872.5X_{FU}(A,1) - d_3^+ = 66{,}137.6$$

$$169034.5X_{UF}(A,1) + 1111.3X_{UU}(A,1) - d_4^+ = 115{,}668.5$$

$$X_{FF}(A,1), X_{FU}(A,1), X_{UF}(A,1), X_{UU}(A,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(A,1)$ – manufacturing lot size of product A when initially favorable demand remains favorable

$X_{FU}(A,1)$ - manufacturing lot size of product A when initially favorable demand becomes unfavorable

$X_{UF}(A,1)$ - manufacturing lot size of product A when initially unfavorable demand becomes favorable

$X_{UU}(A,1)$ - manufacturing lot size of product A when initially unfavorable demand remains unfavorable

4. Results and Discussions

In this study, the stochastic goal programming model for the product was solved using the using the linear programming (linprog) solver in MATLAB™ ([29], [30])., an optimal solution was obtained with the values as shown in Table 6:

Table 6: Optimal solution from MATLAB

Variables	X_{FF} (A,1)	X_{FU} (A,1)	X_{UF} (A,1)	z	X_{UU} (A,1)	d_1^-	d_2^-	d_3^+	d_4^+
values	0	2.3729	0		104.0840	8137.7	17350	0	0

The results highlight the optimal values of the manufacturing lot size of product A in the first quarter of the year as demand changes from one state to another. The results were analyzed and discussed based on the priorities set and the optimal values achieved as seen from Table 6.

The improvement of the solution from the case is establishing the over-achievement and under achievement of the manufacturing lot size priorities desired during production planning. An expansion in this case is incorporating in Markov chains which considers changes form one state to another. As seen from Table 6, for cases where initially demand is favorable and unfavorable, more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand since the model predicts 0 manufacturing lot size of product A in the first quarter of the year.

The model also predicts the manufacturing lot size of product A of 2.3729 units and 104.0840 units when initially favorable demand becomes unfavorable and unfavorable demand remains unfavorable respectively. Meaning these number of products should be produced to meet demand.

Table 7: Expected goal values and actual stochastic solution with over and under achievement

Goals/ priorities	Expected value from Goal	Value of the stochastic solution	Deviation	Over-achievement	Under-achievement
1	8140.1	8140.07	0.03		8137.7
2	17453.9	17454.08	0.18		17350
3	66137.6	66138.66	1.06	0	
4	115668.5	115668.55	0.05	0	

With the set priorities and expected values from each goal, the results from Table 7 show the importance of utilizing the available sources of information when generating a plan.

As observed from Table 7, Priority 1 and 2 can be fully achieved however, an underachievement of 8137.7 units 17350 units respectively is realized in the first quarter when demand is initially favorable (state F) and unfavorable (state U) respectively.

Priority 3 is partially achieved as the actual stochastic solution is slightly higher than the expected goal value targeted production-inventory costs in the first quarter when demand is initially favorable (state F). And priority 4 is fully achieved in the first quarter when demand is initially unfavorable (state U). Both priority 3 and 4 have no over-achievement.

5. Conclusion

A stochastic goal programming model that optimizes the manufacturing lot size under demand uncertainty was presented in this paper. The model determines the quantity of the product (with demand uncertainty) to be produced in the first quarter of the year when demand changes from state *i* to state *j* for $i, j \in \{F, U\}$, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired. The decision of whether or not to produce more units is modelled using Markov chains in conjunction with stochastic goal programming. The model was solved with the help of MATLAB software environment and the results indicate the optimal manufacturing lot sizes as demand changes from one state to another, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired.

Further research is sought to extend the proposed model in order to handle multiple products under demand and price uncertainty. In addition, weighted goal programming can be introduced to improve computational efficiency while handling pre-emptive priorities of the product.

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Appendix 3, Paper 3: "Multi-Objective Optimization of Manufacturing Lot Size under Stochastic Demand"

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Multi-objective optimization of manufacturing lot size under stochastic demand

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Abstract
In many manufacturing problems, multi-objective optimizations are representative models, as objectives are considered a conflict with one another. In real-life applications, optimizing a specific solution concerning one objective may end up in unacceptable results concerning the other objectives. Many Manufacturing companies operate under uncertainties and this affects the system performance. Stochastic product demand is one of the challenges faced by manufacturing companies and often affects the manufacturing system's performance and decision-making. Making the proper decisions regarding manufacturing lot-sizing problems is critical for any manufacturer because it makes the firm compete within the market. In this paper, Markov chains in conjunction with stochastic goal programming were used to develop an optimization model for the manufacturing lot size. The over-achievement or under-achievement of the manufacturing lot size was determined by defining the goal constraints, deviation variables, priorities, and objective function. The different states of demand for the product with stochastic demand were represented by states of a Markov chain. Using the applied mathematics solver in MATLAB TM, the optimization model was then solved, determining the quantity of product to be manufactured in a given quarter of the year as demand changes from one state to another.

Keywords: Optimization, multi-objective, manufacturing lot size, stochastic product demand, stochastic goal programming

1. Introduction

Manufacturing companies experience a rapid and growing change where developments like customer orientation, globalization, and increasing market dynamics have led to a shift in both manufacturing and managerial principles which require more flexibility and effectiveness [1]. Many manufacturing companies operate under uncertainties [2] and this affects the system performance hence the ultimate

decision on utilizing a production system at the initial stages [3]. Manufacturing companies are continuously trying to find efficiency to beat the challenges related to the market dynamics. Stochastic product demand is one of the important factors that affect the manufacturing system's performance. Practically, stochastic product demand is more realistic than other demand types, like constant or functions [4]. Understanding these uncertainties and their impacts (which can make it difficult to predict performance) when assessing the risk associated with a decision, are of major concern [5]. Having more orders, more different products, enlargement of factories, and increased number and size of workshops, have all led to more complications in production planning making the ordinary methods of optimization not able to resolve them [1]. Production planning is the pillar of any manufacturing operation, with the key purpose of determining the number of products to be manufactured considering the level of inventory to be shifted from one period to another to lessen both the overall costs of production and the inventory, meeting the customers' demand [6].
Making the proper decisions about the manufacturing lot size is incredibly important because it directly affects the system performance and productivity [7] and this is often key for any manufacturing firm that wishes to compete within the Market. Lot sizing problems have gotten an immediate effect on the system performance and productivity. Manufacturing Lot sizing is determining the amount of a given product that has to be manufactured in a specified period. Every production plan has got the main problem of determining the manufacturing lot size for every product. To own efficient production planning, lot allocation issues must be solved centered on the demand that has to be achieved and also the availability of inventory stock minimizing production costs by determining the optimal production quantity [8]. The smaller the manufacturing lot size, the less the holding cost but raises the ordering cost whereas the larger the manufacturing lot size, the more the holding cost but reducing the ordering cost. Based on the concepts of lean production, it's preferable to have a smaller lot size because it prevents the buildup of inventory which comes with management and holding costs. The lot size recommended by a mathematical manufacturing lot size model would be the most effective because it accounts for the tradeoff between the costs involved [7]. Optimization is the process of finding (activity of selecting [9]) the simplest possible solution to a given problem by examining several alternatives (assessed after a predefined criterion) [10] and maybe done by adjusting the inputs to or characteristics of a device, mathematical process, or experiment to determine the minimum or maximum output [11]. The optimization problem contains three basic parameters that must be considered, that is, the objective function, a collection of variables, and a collection of constraints [12]. The objective of the optimization model depends on certain characteristics of the system, called variables or unknowns to determine the values of those variables that optimize the objective function, even though these variables are often restricted, or constrained in one way or the other. Brahimi et al. grouped optimization problems into four categories: process planning, layout design, reconfigurability and planning, and, scheduling. In the beginning multi-objective optimization originally developed from areas including economic equilibrium and welfare theories, game theories, and pure mathematics. Consequently, many terms and fundamental ideas stem from these fields [13]. A realistic result to a multi-objective problem is to examine a collection of solutions, each satisfying the objectives at a satisfactory level without being controlled by another solution. Many, or maybe most, real engineering problems do have multiple objectives, that is, minimize cost, maximize performance, maximize reliability, and many others, of which are difficult but realistic problems [14]. The

solution of multi-objective optimization (MOO) problems differs from single-objective optimization problems because there's no global optimal solution in an exceedingly mathematical sense, due to the contradictory nature of the set of objectives involved; that's, a result that minimizes all objectives at the same time doesn't exist [15]. Manufacturing companies must have the flexibility to regulate scalable production capacities and to respond rapidly to market demands making planning and scheduling complex in such a dynamic environment [16]. Markov chain is a powerful mathematical tool that's extensively accustomed to capturing the stochastic process of systems transitioning among different states [17]. Markov chains may be applied in modeling and performance evaluation as manufacturing systems show any unplanned behavior relating to breakdowns, unplanned time to process a component, and many others [18]. To tackle uncertainties in real-world manufacturing systems, goal programming and stochastic analysis must be put into the whole structure. Stochastic Goal Programming is a multi-criteria decision support model that provides "satisficing" solutions to a linear system under an uncertainty case from the normally expected utility viewpoint [19], [20].

Because many real-world optimization problems have got several inaccurate information estimates & goals and conflicting criteria, the stochastic goal programming method suggests an analytical structure aid in modeling and solving such problems. Stochastic goal programming can cope with the inherent uncertainty and has been applied in several fields including Portfolio selection, project selection, resource allocation, Healthcare management, transportation, marketing [21], cash management [22], wealth management [23], economic development, energy consumption, workforce allocation, and greenhouse gas emissions [24], forest planning [25]. Little applications of stochastic goal programming in production planning in manufacturing systems are observed hence the necessity for multi-objective optimization of the manufacturing lot size under stochastic demand. This may be considered as a suggestion for production planners and practitioners accustomed to solving specific decision-making problems (optimal manufacturing lot size). Manufacturing companies will minimize overproduction when demand is low or underproduction when demand is high. As a result of fluctuations and uncertainties in demand, manufacturing companies are always challenged with determining optimal manufacturing lot sizes in production planning systems. Manufacturing companies are continuously searching for efficiency to beat the challenges related to the market dynamics. It's therefore important that these uncertain parameters be considered within the production planning process when developing a strong production plan because when neglected, production efficiency and system performance are affected [26]. Centering on external demands, manufacturing industries form their production plans having the principal goal of establishing the number (lot size) of products that can be produced for each period but meeting the demand and minimizing total costs [27]. In production planning, making the proper decisions about the lot size is extremely important because it directly affects the system performance and productivity [7] and this is often key for any manufacturing firm that desires to compete on market. As this is often complex moreover as important, it's been highly studied although, there's still a niche about showing the contributions to clarify the suitability of these methods used concerning each quite underlying manufacturing environment (regarding variations in demand and peaks of seasonality) [28]. Therefore this study aimed toward the development of a multi-objective optimization model for the manufacturing lot size under stochastic demand, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired.

Mathematical model formulation

A case of a manufacturing company whose products have got stochastic demand was considered. The demand for these products during each period over a finite fixed planning horizon was described as either favorable or unfavorable. The Markov chain approach ([29], [30], [31], [32], [1]) together with stochastic goal programming ([20], [33], [21], [34], [25], [22]) was adopted and also the states of a Markov chain represent possible states of demand for the finished products with the notations shown in Table 1.

Table 4: Key notations used in the Markov model

i, j	Set of states of demand	M	Manufacturing lot-size
F	Favorable demand	$X_{ij}(p,q)$	Quantity of product p to be manufactured in quarter q
U	Unfavorable demand	N	Customer matrix
Q	Demand transition matrix	C_p	Unit production cost
p	Product	C_h	Unit holding cost
q	A quarter of the year	C_s	Unit shortage cost
FF, FU, UF, UU	State transitions	D	Demand matrix
Z	Value of the objective function	V	Inventory matrix
P_k	Preemptive priority of the kth goal	C	Production-Inventory cost matrix
d_k^+	Over achievement of the kth goal	B	Beginning Inventory
d_k^-	Under achievement of the kth goal	E	Ending Inventory

Average on-hand inventory, $V = (B+E)/2$ (1)

Consider the customer matrix:

$$N(p,q) = \begin{bmatrix} N_{FF}(p,q) & N_{FU}(p,q) \\ N_{UF}(p,q) & N_{UU}(p,q) \end{bmatrix} \quad (2)$$

2.1 Demand transition probability

As demand changes from state *i* to ievementstate *j* for $i, j \in \{F, U\}$, the associated demand transition probabilities are calculated as:

$$Q_{ij}(p,q) = \frac{N_{ij}(p,q)}{N_{if}(p,q)+N_{iu}(p,q)} \quad (3)$$

This yields the demand transition matrix:

$$Q(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & Q_{FF}(p,q) & Q_{FU}(p,q) \\ \boldsymbol{U} & Q_{UF}(p,q) & Q_{UU}(p,q) \end{matrix} \quad (4)$$

Then the demand matrix, the inventory matrix and the production-inventory cost matrix.

Demand matrix;

$$D(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & D_{FF}(A,1) & D_{FU}(A,1) \\ \boldsymbol{U} & D_{UF}(A,1) & D_{UU}(A,1) \end{matrix} \quad (5)$$

Inventory matrix;

$$V(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(A,1) & V_{FU}(A,1) \\ \boldsymbol{U} & V_{UF}(A,1) & V_{UU}(A,1) \end{matrix} \quad (6)$$

Production-inventory cost matrix;
When demand outweighs the amount produced then,

$$C(p,q) = \begin{bmatrix} C_p \\ + \\ C_h \\ + \\ C_s \end{bmatrix} [D(p,q) - V(p,q)] \quad (7)$$

Similarly, when the demand is less than the amount produced then,

$$C(p,q) = C_h[V(p,q) - D(p,q)] \quad (8)$$

Hence, as demand changes from state *i* to state *j* ($i, j \in \{F, U\}$)

$$C(p,q) = \begin{array}{c} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{array} \begin{array}{c} \begin{array}{cc} \boldsymbol{F} & \boldsymbol{U} \end{array} \\ \begin{pmatrix} C_{FF}(p,q) & C_{FU}(p,q) \\ C_{UF}(p,q) & C_{UU}(p,q) \end{pmatrix} \end{array} \quad (9)$$

where $C(p,q) =$ production-inventory cost matrix.

2.2 Expected demand, inventory, production-inventory costs, and manufacturing lot-size

Expected demand

Favorable Demand $E[D_F(p,q)] = Q_{FF}(p,q)D_{FF}(p,q) + Q_{FU}(p,q)D_{FU}(p,q)$ (10)

Unfavorable Demand $E[D_U(p,q)] = Q_{UF}(p,q)D_{UF}(p,q) + Q_{UU}(p,q)D_{UU}(p,q)$ (11)

Expected inventory

Favorable Demand $E[V_F(p,q)] = Q_{FF}(p,q)V_{FF}(p,q) + Q_{FU}(p,q)V_{FU}(p,q)$ (12)

Unfavorable Demand $E[V_U(p,q)] = Q_{FU}(p,q)V_{UF}(p,q) + Q_{UU}(p,q)V_{UU}(p,q)$ (13)

Expected production-inventory costs

Favorable Demand $E[C_F(p,q)] = Q_{FF}(p,q)C_{FF}(p,q) + Q_{FU}(p,q)C_{FU}(p,q)$ (14)

Unfavorable Demand $E[C_U(p,q)] = Q_{UF}(p,q)C_{UF}(p,q) + Q_{UU}(p,q)C_{UU}(p,q)$ (15)

Expected manufacturing lot-size

Favorable demand

$$E[M_F(p,q)] = \begin{cases} E[D_F(p,q)] - E[V_F(p,q)] & \text{if } E[D_F(p,q)] > E[V_F(p,q)] \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

Unfavorable demand

$$E[M_U(p,q)] = \begin{cases} E[D_U(p,q)] - E[V_U(p,q)] & \text{if } E[D_U(p,q)] > E[V_U(p,q)] \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

2.3 Stochastic goal programming formulation

The stochastic goal programming model was formulated by setting priorities, defining the objective function, and formulating the goal constraints as follows:

Set priorities

P_1: Produce a batch of $E[M_F(p,q)]$ units when demand is favorable

P_2: Produce a batch of $E[M_U(p,q)]$ units when demand is unfavorable

P_3: Total production-inventory cost must not exceed $E[C_F(p,q)]$ when demand is favorable

P_4: Total production-inventory cost must not exceed $E[C_U(p,q)]$ when demand is unfavorable

Objective function

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \quad (18)$$

Goal constraints

P1: Manufacturing lot-size $E[M_F(p,q)]$ - favorable demand

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \quad (18.1)$$

P2: Manufacturing lot-size $E[M_U(p,q)]$ - unfavorable demand

$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \quad (18.2)$$

P3: Total production-inventory cost – favorable demand

$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad (18.3)$$

P4: Total production-inventory cost – unfavorable demand

$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad (18.4)$$

1.4 Stochastic goal programming model for manufacturing lot-size

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \quad (19)$$

Subject to:

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \quad (19.1)$$

$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \quad (19.2)$$

$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad (19.3)$$

$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad (19.4)$$

$$X_{FF}(p,q), X_{FU}(p,q), X_{UF}(p,q), X_{UU}(p,q), d_1^-, d_1^+, d_2^-, d_2^+, d_3^+, d_4^+ \geq 0 \quad (19.5)$$

2. Case study

In this section, a real case application from Movit Products Uganda limited was used to demonstrate the applicability of the proposed mathematical models. The manufacturing industry manufactures, distributes, and sells skin care, hair & nail care products. The numerical illustration contains real data for the first quarter of the year, which was collected and then reduced to usable dimensions as shown in Table 2. Data classification by state of demand was made, analyzed, and used in the proposed mathematical model.

Considering a product D, for a given week, demand is favorable (state F) if $N_{ij} > 26$ otherwise demand is unfavorable (state U) if $N_{ij} \leq 26$ as shown in Table 2.

Table 5: Data classification by state of demand for product D

Month	Week	Customers (N)	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	State of demand (*i*)
1	1	15	308	5263	U
	2	29	2891	7337	F
	3	24	1757	7081	U
	4	38	6619	5654	F
2	1	8	231	3525	U
	2	17	2046	6243	U
	3	15	1617	5922	U
	4	45	4443	5951	F
3	1	14	559	3765	U
	2	37	3686	4738	F
	3	28	1537	4980	F
	4	44	5626	5746	F

Tables 3a, 3b, and 3c show the overstocking or understocking of product D with the corresponding holding or shortage costs in the first quarter of the year.

Table 6a: Overstocking and understocking with holding and shortage costs for month 1

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	308	5263	4955	3121.65
2	2891	7337	4446	2800.98
3	1757	7081	5324	3354.12
4	6619	5654	-965	3343.725

Table 3b: Overstocking and understocking with holding and shortage costs for month 2

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	231	3525	3294	2075.22
2	2046	6243	4197	2644.11
3	1617	5922	4305	2712.15
4	4443	5951	1508	950.04

Table 3c: Overstocking and understocking with holding and shortage costs for month 3

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	559	3765	3206	2019.78
2	3686	4738	1052	662.76
3	1537	4980	3443	2169.09
4	5626	5746	120	75.6

Figure 5: Overstocking and understocking of product D

Figure 6: Holding and Shortage costs

3.1 State transitions and on-hand inventory

For a particular state transition, given the beginning and ending inventory, the average on-hand inventory was calculated as presented in Table 4.

Table 4: Average on-hand inventory for product D

State transitions (i, j)	Beginning inventory (B)	Ending inventory (E)	Average on-hand inventory $V = (B + E)/2$
FF	4980	5746	5363
FU	7081	3765	5423
UF	7337	4738	6037.5
UU	6243	5922	6082.5

From Equation (1) section 2, the average on-hand inventory was calculated giving;

$V_{FF}(D,1) = 5363 \qquad V_{FU}(D,1) = 5423 \qquad V_{UF}(D,1) = 6037.5 \qquad V_{UU}(D,1) = 6082.5$

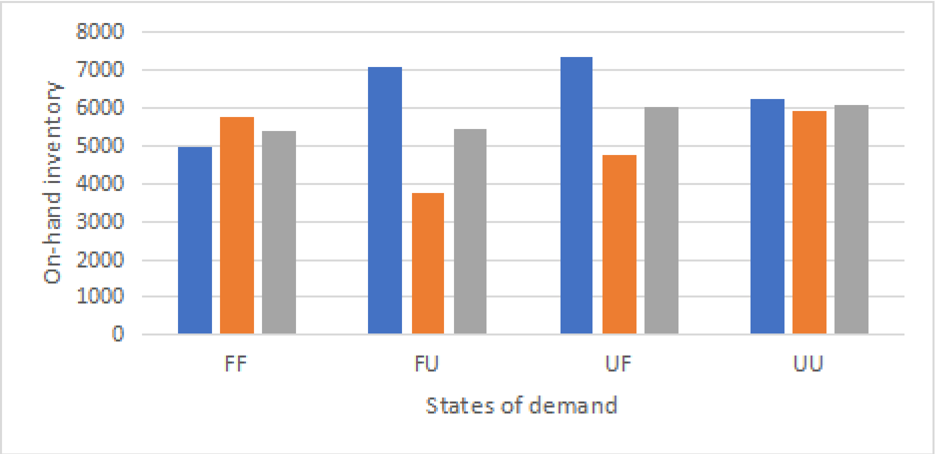

Figure 7: Average on-hand inventory and state transitions

3.2 Demand transition probabilities

Data classification by state transition was done as illustrated in Table 5 and then used to calculate the demand transition probabilities for the product

Table 5: *Data classification by state-transition for product D*

Month	State transition (i,j)	Number of customers $N_{ij}(A,1)$	Demand $D_{ij}(A,1)$
1	FF	0	0
	FU	53	4648
	UF	106	11575
	UU	0	0
2	FF	0	0
	FU	0	0
	UF	60	6060
	UU	57	5940
3	FF	137	12386
	FU	0	0
	UF	51	4245
	UU	0	0

From table 5, the Totals for customers and demand as it changes from one state to another are;

Customers: $N_{FF}(D,1) = 137$ $\quad N_{FU}(D,1) = 53$

$N_{UF}(D,1) = 106 + 60 + 51 = 217$ $\quad N_{UU}(D,1) = 57$

Demand: $D_{FF}(D,1) = 12386$ $\quad D_{FU}(D,1) = 4648$

$D_{UF}(D,1) = 11575 + 6060 + 4245 = 21880$ $\quad D_{UU}(D,1) = 5940$

From Equation (3) in section 2, the demand transition probabilities are;

$$Q_{FF}(D,1) = \frac{N_{FF}(D,1)}{N_{FF}(D,1)+N_{FU}(D,1)} = \frac{137}{137+53} = 0.7211$$

$$Q_{FU}(D,1) = \frac{N_{FU}(D,1)}{N_{FF}(D,1)+N_{FU}(C,1)} = \frac{53}{137+53} = 0.2789$$

$$Q_{UF}(D,1) = \frac{N_{UF}(D,1)}{N_{UF}(D,1)+N_{UU}(D,1)} = \frac{217}{217+57} = 0.7920$$

$$Q_{UU}(D,1) = \frac{N_{UU}(D,1)}{N_{UF}(D,1)+N_{UU}(D,1)} = \frac{57}{217+57} = 0.2080$$

Hence the demand transition matrix as from equation (4),

$$Q(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 0.7211 & 0.2789 \\ \boldsymbol{U} & 0.7920 & 0.2080 \end{matrix}$$

3.3 Demand matrix, inventory matrix, and production-inventory cost matrix

The demand matrix, the inventory matrix, and the production-inventory cost matrix were developed as follows.

From Equation (5), the demand matrix becomes;

$$D(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 12386 & 4648 \\ \boldsymbol{U} & 21880 & 5940 \end{matrix}$$

From Equation (6), the Inventory matrix becomes;

$$V(D,1) = \begin{matrix} & F & U \\ F & 5363 & 5423 \\ U & 6037.5 & 6082.5 \end{matrix}$$

Production-inventory cost matrix

The production-inventory cost matrix is then computed for the product From Equations (7), (8), and (9).

Unit production cost, $C_p(D) = \$\ 31.5$

Unit holding cost, $C_h(D) = \$\ 0.63$

Unit shortage cost, $C_s(D) = \$\ 3.465$

$C_{FF}(D,1) = \left(C_p(D) + C_h(D) + C_s(D)\right)\left(D_{FF}(D,1) - V_{FF}(D,1)\right)$
$C_{FF}(D,1) = (31.5 + 0.63 + 3.465)(12386 - 5363) = 249983.685$
$C_{FU}(D,1) = C_h(D)\left(V_{FU}(D,1) - D_{FU}(D,1)\right)$
$C_{FU}(D,1) = 0.63(5423 - 4648) = 488.25$
$C_{UF}(D,1) = \left(C_p(D) + C_h(D) + C_s(D)\right)\left(D_{UF}(D,1) - V_{UF}(D,1)\right)$
$C_{UF}(D,1) = (31.5 + 0.63 + 3.465)(21880 - 6037.5) = 563913.7875$
$C_{UU}(D,1) = C_h(D)\left(V_{UU}(D,1) - D_{UU}(D,1)\right)$
$C_{UU}(D,1) = (0.63)(6082.5 - 5940) = 89.775$
Hence,

$$C(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(D,1) & C_{FU}(D,1) \\ \boldsymbol{U} & C_{UF}(D,1) & C_{UU}(D,1) \end{matrix}$$

$$C(D,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & 249983.685 & 488.25 \\ \boldsymbol{U} & 563913.7875 & 89.775 \end{matrix}$$

3.4 Expected demand, inventory, production-inventory costs, and manufacturing lot-size

Expected demand

After generating the demand transition matrix and formulating the production-inventory cost matrix, the expected demand expected inventory, and expected production-inventory costs are computed for the product considering both favorable and unfavorable demand as shown below;

Favorable demand (F) was computed from equation (10)

$$E[D_F(D,1)] = Q_{FF}(D,1) * D_{FF}(D,1) + Q_{FU}(D,1) * D_{FU}(D,1)$$
$$E[D_F(D,1)] = (0.7211 * 12386) + (0.2789 * 4648)$$
$$E[D_F(D,1)] = 11227.8718\ units$$

Unfavorable demand (U) was computed from equation (11)

$$E[D_U(D,1)] = Q_{UF}(D,1) * D_{UF}(D,1) + Q_{UU}(D,1) * D_{UU}(D,1)$$
$$E[D_U(D,1)] = (0.7920 * 21880) + (0.2080 * 5940)$$
$$E[D_U(D,1)] = 18564.48\ units$$

Computation of the expected inventory considering both favorable and unfavorable demand for the product was computed from equation (12) as follows:

Favorable demand (F)

$$E[V_F(D,1)] = Q_{FF}(D,1) * V_{FF}(D,1) + Q_{FU}(D,1) * V_{FU}(D,1)$$
$$E[V_F(D,1)] = (0.7211 * 5363) + (0.2789 * 5423)$$
$$E[V_F(D,1)] = 5379.734\ units$$

Unfavorable demand (U) was computed from equation (13) as follows

$$E[V_U(D,1)] = Q_{UF}(D,1) * V_{UF}(D,1) + Q_{UU}(D,1) * V_{UU}(D,1)$$
$$E[V_U(D,1)] = (0.7920 * 6037.5) + (0.2080 * 6082.5)$$
$$E[V_U(D,1)] = 6046.86\ units$$

Expected production-Inventory costs

The expected production-Inventory costs are then computed for the product considering both favorable and unfavorable demand results were computed from equations (14) and (15) as follows;

Favorable demand (F)

$$E[C_F(D,1)] = Q_{FF}(D,1) * C_{FF}(D,1) + Q_{FU}(D,1) * C_{FU}(D,1)$$
$$E[C_F(D,1)] = (0.7211 * 249983.685) + (0.2789 * 488.25)$$
$$E[C_F(D,1)] = \$\ 180399.4082$$

Unfavorable demand (U)

$$E[C_U(D,1)] = Q_{UF}(D,1) * C_{UF}(D,1) + Q_{UU}(D,1) * C_{UU}(D,1)$$
$$E[C_U(D,1)] = (0.7920 * 563913.7875) + (0.2080 * 89.775)$$
$$E[C_U(D,1)] = \$\ 446638.3929$$

Expected manufacturing lot size

Computation of the expected manufacturing lot size considering both favorable and unfavorable demand for the product yields was computed from equations (16) and (17) as follows:

Favorable demand (F)

$$E[M_F(D,1)] = \begin{pmatrix} E[D_F(D,1)] - E[V_F(D,1)]\ if\ E[D_F(D,1)] > E[V_F(D,1)] \\ 0 \qquad\qquad otherwise \end{pmatrix}$$
$$E[M_F(D,1)] = E[D_F(D,1)] - E[V_F(D,1)]$$
$$E[M_F(D,1)] = 11227.8718 - 5379.734 = 5848.1378\ units$$

Unfavorable demand (U)

$$E[M_U(D,1)] = \begin{pmatrix} E[D_U(D,1)] - E[V_U(D,1)]\ if\ E[D_U(D,1)] > E[V_U(D,1)] \\ 0 \qquad\qquad otherwise \end{pmatrix}$$
$$E[M_U(D,1)] = E[D_U(D,1)] - E[V_U(D,1)]$$
$$E[M_U(D,1)] = 18564.48 - 6046.86 = 12517.62\ units$$

3.5 Stochastic goal programming model

The stochastic goal programming model for the product was formulated by setting priorities, defining the objective function, and formulating the goal constraints as follows:

Priorities set

P_1: *Produce a batch of* 5848.1378 *units when demand is initially favorable*

P_2: *Produce a batch of* 12517.62 *units when demand is initially unfavorable*

P_3: *Total production_inventory costs must not exceed* \$ 180399.4082 *when demand is favorable*

P_4: *Total production_inventory costs must not exceed* \$ 446638.3929 *when demand is unfavorable*

Objective function

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(D,1)d_k^+ + P_K(D,1)d_k^-]$$

Goal constraints

Manufacturing lot size

$$X_{FF}(D,1) + X_{FU}(D,1) + d_1^- = 5848.1378\ (Favorable\ demand)$$
$$X_{UF}(D,1) + X_{UU}(D,1) + d_2^- = 12517.62\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$249983.685X_{FF}(D,1) + 488.25X_{FU}(D,1) - d_3^+ = 180399.4082\ (Favorable\ demand)$$

$$563913.7875X_{UF}(D,1) + 89.775X_{UU}(D,1) - d_4^+ = 446638.3929\ (Unfavorable\ demand)$$

Non-negativity

$$X_{FF}(D,1), X_{FU}(D,1), X_{UF}(D,1), X_{UU}(D,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

3.6 Stochastic goal programming model for manufacturing lot size

The stochastic goal programming model for manufacturing lot size was then developed for the product as below. This determines the quantity of the product to manufacture in the first quarter of the year when demand changes from state *i* to state *j* for $i, j \in \{F, U\}$, establishing the over-achievement or under achievement of the manufacturing lot size priorities desired.

$$Minimize\ Z = \sum_{k=1}^{4} [P_K(D,1)d_k^+ + P_K(D,1)d_k^-]$$

Subject to:

$$X_{FF}(D,1) + X_{FU}(D,1) + d_1^- = 5848.1378$$
$$X_{UF}(D,1) + X_{UU}(D,1) + d_2^- = 12517.62$$
$$249983.685X_{FF}(D,1) + 488.25X_{FU}(D,1) - d_3^+ = 180399.4082$$
$$563913.7875X_{UF}(D,1) + 89.775X_{UU}(D,1) - d_4^+ = 446638.3929$$
$$X_{FF}(D,1), X_{FU}(D,1), X_{UF}(D,1), X_{UU}(D,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(D,1)$ – manufacturing lot size of product D when initially favorable demand remains favorable

$X_{FU}(D,1)$ - manufacturing lot size of product D when initially favorable demand becomes unfavorable

$X_{UF}(D,1)$ - manufacturing lot size of product D when initially unfavorable demand becomes favorable

$X_{UU}(D,1)$ - manufacturing lot size of product D when initially unfavorable demand remains unfavorable

3. Results and Discussions

In this study, the stochastic goal programming model for the product was solved using MATLAB. The values were inserted in MATLAB TM ([35], [36]) and using the linprog solver, an optimal solution was obtained with the values as shown in Table 6:

Table 6: Optimal solution from MATLAB

Variables	X_{FF} (D,1)	X_{FU} (D,1)	X_{UF} (D,1)	X_{UU} (D,1)	d_1^-	d_2^-	d_3^+	d_4^+
values	0	369.4816	0	12518	5478.7	0	0	677130

The results highlight the optimal values of the manufacturing lot size of product A in the first quarter of the year as demand changes from one state to another. The results were analyzed and discussed based on the priorities set and the optimal values achieved as seen from table 6.

The improvement of the solution from the case is establishing the over-achievement and under achievement of the manufacturing lot size priorities desired during production planning. An expansion, in this case, is incorporated in Markov chains which considers changes from one state to another. As seen from table 6, for cases where initially demand is favorable and unfavorable, more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand since the model predicts 0 manufacturing lot size of product A in the first quarter of the year.

The model also predicts the manufacturing lot size of product A of 2.3729units and 104.0840 units when initially favorable demand becomes unfavorable and unfavorable demand remains unfavorable respectively. Meaning these number of products should be produced to meet demand.

Table 7: Expected goal values and actual stochastic solution with over and under achievement

Goals/ priorities	Expected value from Goal	Value of the stochastic solution	Deviation	Over-achievement	Under-achievement
1	5848.1378	5848.1816	0.0438		5478.7
2	12517.62	12518	0.38		0
3	180399.4082	180399.3912	0.017	0	
4	446638.3929	446673.45	35.0571	677130	

With the set priorities and expected values from each goal, the results from table 7 show the importance of utilizing the available sources of information when generating a plan.

As observed from table 7, Priorities 1, 2, and 3 can be fully achieved however, an underachievement of 5478.7 units is realized in the first quarter when demand is initially favorable (state F).

Priority 4 is partially achieved as the actual stochastic solution is slightly higher than the expected goal value targeted production-inventory costs in the first quarter when demand is initially unfavorable (state U) and an over-achievement of 677130 units is realized.

4. Conclusion

A stochastic goal programming model that optimizes the manufacturing lot size under demand uncertainty was presented in this paper. The model determines the quantity of the product (with demand uncertainty to be produced in the first quarter of the year when demand changes from state *i* to state *j* for $i, j \in \{F, U\}$, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired. The decision of whether or not to produce more units is modeled using Markov chains in conjunction with stochastic goal programming. The model was solved with the help of MATLAB software environment and the results indicate the optimal manufacturing lot sizes as demand changes from one state to another, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired.

Further research is sought to extend the proposed model to handle multiple products under demand and price uncertainty. In addition, weighted goal programming can be introduced to improve computational efficiency while handling pre-emptive priorities of the product.

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Appendix 4, Paper 4: "A Review on Stochastic Goal Programming Approach in Production Planning in Manufacturing"

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A review on stochastic goal programming approach in production planning in manufacturing

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Abstract
Manufacturing system performance is very important for any manufacturing company and one of the key tools that aid in maintaining and improving its performance is production planning.
Manufacturing systems function in an uncertain environment and this affects its system performance in one way or the other. Production planning tactics that don't put into consideration the uncertainties will produce substandard planning decisions linked to the ones that clearly consider uncertainty. Stochastic goal programming is applied in decision-making circumstances with uncertainty by means of stochastic calculus and therefore the decision-maker isn't in the position to evaluate exactly the several factors but gives certain information regarding the likelihood of the existence of the decision-making factor values. During this paper, existing literature about the application of stochastic goal programming in production planning in manufacturing has been reviewed to provide the reader, optimization practitioners, and researchers with the important matters that arise once dealing with uncertainty modeling in manufacturing systems using stochastic goal programming.

Keywords
Production planning, uncertainty, manufacturing, Goal programming, Stochastic programming

1. Introduction

Manufacturing is a vital aspect of the global economy and prosperity. There's always an issue of the supply chain within the manufacturing domain, and production planning is one of its stages. Many industries have manufacturing systems characterized as large and sophisticated and operate in an uncertain environment. There are several different methods by which the complexity within the manufacturing system can be reduced and one of them is by modeling uncertainties within the production planning problem [1], [2]. With manufacturing systems functioning in uncertain environments, production planning is a significant element in refining its performance [3]. The production planning process becomes more difficult and complicated concerning product demand uncertainty [4]. Making the production planning process well-organized and enhanced (having minimum expenses and meeting the market demand) is the desire of any manufacturing company.

Optimizing the production planning process is very essential in the production of high-quality products (while maximizing profit), especially given the competitive nature of the market globally.

Examining the decision-making process (to get an optimal alternative possible solution to a specific problem) within the production planning process, the application of mathematical modeling is essential.

One of the objectives in production planning is maximizing profit with the production of products at minimum cost [5]. The process of defining the modest way that proficiently utilizes resources necessary for production (manpower, materials, and equipment) is the production planning process. The usual decisions in Production planning are affected by compromises between productivity efficiency and financial objectives.

Financial targets are precisely linked to cumulative profits or decreasing costs, (including production costs, labor costs, material costs, and inventory costs).

Concerning production efficiency, production planning must reveal the capability to supply products & the distinct effects of extra concerns all through the operation (inventory levels, overtime, and backorders) [1]. Ali Cheraghalikhani, Farid Khoshalhan, and Hadi Mokhtari in their paper characterized the production planning models as shown in Table 1 [6].

Table 1: Model characterization by type of data and number of objective function

Type of data	Model	Objective function
Deterministic		Single
		Multiple
Uncertain	Fuzzy	Single
		Multiple
	Stochastic	Single
		Multiple

Mula et al.'s research demonstrates the factors of uncertainty within the production system (as demand, environment, system resource, lead time, and yield) and further amalgamate common methods to uncertainty (including stochastic model, dynamic programming, fuzzy theory, and simulation-based approaches) [2]. Many times uncertainty in production planning is perceived as demand uncertainty (fluctuations) within the production process (production times or material loss). As observed in Bakir & Byrne's research, the variation in the solutions given by the deterministic model & stochastic model is analyzed and the uncertainty factor is market demand (the analytical results showing that the difference relies on the variance of uncertain demand) [1].

Considering the summarized fundamental concerns in table 2 [6], the purpose of the production planning models is to determine an optimum rate of production and labor force, minimizing the costs associated with satisfying the known demand

Table 2: Fundamental concerns in production planning

Fundamental concerns	Definition
Market demand	Demand per period satisfied by product, inventory or backorder
Inventory	Products held in stock per period
Backorder	Part of demand not satisfied per period
Production capacity	Maximum amount of products that can be produced per

	period by system
Warehouse space	Capacity of the warehouse for the holding inventory
Costs of production	Regular time & overtime production and costs of inventory carrying & backorders
Subcontracting	Hiring capacity of other firms temporarily to make component parts
Labor level	Number of workers per period (regular & overtime workers)
Hiring and Layoff cost	Additional workers recruited to handle extra production loading and redundant workers laid-off to reduce overheads.
Product Price	Selling price of products

Given today's industrial competitiveness, and the uncertain environment these manufacturing companies operate in, it's crucial for the decision makers to maintain optimal strategies or solutions to such problems hence stochastic goal programming method because it proposes a logical structural guide in modeling and resolving such problems.

2. Goal Programming

Goal programming (GP), is one of the well-known multi-objective optimization models and has recurrently been cultivated by both theoretical advances and new applications through categorical success. Abraham Charnes and William W Cooper introduced the first formulation of goal programming spreading its attractiveness to current periods [7], [8], [9], [10], [11]. Having been introduced in the early 1960s, and subsequently, significant additions and several applications have been recommended, one of them being the stochastic goal programming model. This is where the result of the best negotiation reduces total deviances between the achievement $f_i(x)$ and aspiration levels g_i [12]. Goal programming, generally a linear programming tool (useful tool to balance conflicting aspects of the competing criteria), attempts to achieve predefined targets for a set of goals (satisficing philosophy) other than an optimal result subject to stringent constraints (optimizing philosophy).

Stochastic programming must be applied to proficiently assimilate information concerning an aspect of uncertainty. Goals are governed by the decision-makers' perspective (and vary with time due to related factors) [13] , follows a sustaining logic conveyed by means of targets and he appreciates the notion of setting targets & thus being directly involved in the development of other solutions [14]. Goal programming combines several objectives to get the result that minimizes in totality the deviations between achievement & the aspiration levels of the goals. It is essential to specify for each goal g_i, the aspiration level or target $G_i \in R$, with i = 1; 2; : : : ; q introducing positive & negative deviation auxiliary variables to associate goal achievement and targets [15].

goal programming purposes to reconcile the achievement of a set of goals other than optimizing each goal done by instituting an achievement objective function. In terms of fundamental distance metric, the goal programming types are lexicographic, weighted [15], [16], & Chebyshev (min-max) goal programming [17] and in terms of the mathematical nature of the decision variables or goals used are fuzzy, integer, binary, and fractional goal programming [10]. In weighted goal programming, each objective is multiplied by a weight assigned to it, and the overall objective function

(archimedian sum of all these), is minimized. In lexicographic goal programming, the objectives are assigned priorities, then ranked by priority from highest to lowest, then the first objective is minimized by itself, and a constraint is set after the optimization to prevent the next optimization from obtaining a worse result, and lastly, this procedure is repeated for all of the objectives. In min-max goal programming, the maximum difference between any goal and its objective is minimized [17]. similar to that of a linear programming model, problem is modelled into a goal programming model in the same way, but, the goal programming model has several & frequently contradicting incommensurable goals, in a specific priority hierarchy (established by ranking or weighing various goals in accordance with their importance) [18]. GP defines the resources needed to attain a preferred set of objectives, determines the point of achievement of the goals per the available resources, and delivers the best sufficient result with a changing resources & priorities of
the goals. For every objective, a goal is set and the deviancy concerning every objective & its goal are minimized. The general formulation of goal programming consists in transforming multi-objective programming as [19]:

$$Optimize\ f_i(x) \tag{1}$$

Subject to P1

$$x \in A \tag{2}$$

In the following form:

$$\min \sum_{i=1}^{n} w_i(\delta_i^+ + \delta_i^+) \tag{3}$$

Subject to

$$f_i(x) + \delta_i^- - \delta_i^+ = \hat{f}_i \quad i = 1,...,n \tag{4}$$

$$x \in A \tag{5}$$

$$\delta_i^- \text{ and } \delta_i^+ \geq 0 \quad i = 1,...,n$$

Where $f_i(x)$ is the goal function i; $\hat{f}_i$ is the target level of objective i; δ_i^- and δ_i^+ are the negative and positive deviations respectively associated with the objective i from its target; w_i is the weight assigned to the objective i, and A is the set of feasible solutions or system constraints.

Pre-emptive and Non Pre-emptive goal programming are the basic types of goal programming formulations, with Non Pre-emptive [20] having the weighted sum of all the undesirable deviations is minimized (no goal is said to dominate any other goal).

$$Max\ Profit\ Z_1 = 2x_1 + 3x_2 \tag{6}$$

$$Min\ Cost\ Z_2 = x_1 + 5x_2 \tag{7}$$

Subject to:

$$x_1 + x_2 \leq 10 \tag{8}$$

$$x_1 - x_2 \leq 4$$

$$x_1, x_2 \geq 0 \tag{9}$$

Supposing that the decision maker needs to have at least 40,000 profit and the cost should not exceed the limit of 20,000, the above problem can be converted into a goal programming problem as follows ($GP1$):

$$Min\ d_1^- + d_2^+ \tag{10}$$

Subject to:

$$2x_1 + 3x_2 + d_1^- = 40{,}000 \quad (11)$$
$$x_1 + 5x_2 + d_2^+ = 20{,}000 \quad (12)$$
$$x_1 + x_2 \leq 10 \quad (13)$$
$$x_1 - x_2 \leq 4 \quad (14)$$
$$x_1, x_2 \geq 0 \quad (15)$$
$$d_1^-, d_2^+ \geq 0 \quad (16)$$

Rajendran demonstrates Pre-emptive Goal Programming as below;

Assuming in the problem above, having known the fact that the multi-objective situation limit to have any such result that satisfies both goals concurrently, the decision makers states the priorities for both the goals. Assuming in problem $GP1$ the first goal has the higher priority, say $P1$, and the second goal has a lower priority, say $P2$, that is $P1 > P2$. In this condition, the problem $GP1$ is written as follows ($GP2$):

$$Min\{P_1 d_1^-, P_2 d_2^+\} \quad (17)$$

Subject to:

$$2x_1 + 3x_2 + d_1^- = 40{,}000 \quad (18)$$
$$x_1 + 5x_2 + d_2^+ = 20{,}000 \quad (19)$$
$$x_1 + x_2 \leq 10 \quad (20)$$
$$x_1 - x_2 \leq 4 \quad (21)$$
$$x_1, x_2 \geq 0 \quad (22)$$
$$d_1^-, d_2^+ \geq 0 \quad (23)$$
$$P_1 > P_2 \quad (24)$$

3. Stochastic programming (SP)

SP is a method aimed at modeling optimization problems involving uncertainty [21] ("find an optimal decision in problems involving uncertain data" [22]). Stochastic programming ([22]) conveys a useful tool within which a huge range of sources of uncertainty is integrated into the development of the production plans [14]. Some of the applications of stochastic programming include, production planning, manufacturing design, financial planning and control [22], portfolio management [23]. As deterministic optimization problems are expressed with known parameters, real world problems comprise unknown parameters at the time a decision is made.

Stochastic programming may be applied in a very setting during which a one-off decisions must be made.

The top broadly applied & studied stochastic programming models are two-stage (linear) programs where the decision maker acts within the first stage, after which a random event occurs affecting the result of the first-stage decision [21]. The basic stochastic programming problem is:

$$minimize\ F_O(X) = Ef_o(x, w) \quad (25)$$

Subject to: $F_i(X) = Ef_i(x, w) \leq 0, i = 1, \ldots\ldots\ldots, m$ (26)

Where the variable is x, problem data are f_i, distribution of w

If $f_i(x, w)$ are convex in x for each w, F_i are convex hence stochastic programming problem is convex

4. Stochastic Goal Programming (SGP)

Stochastic Goal Programming is a "multi-criteria decision support" model that has "satisficing" results to a linear structure under an uncertainty situation from the usually predictable utility perspective [24], [25]. Because many real-world optimization problems have got numerous erroneous information estimates & goals and conflicting criteria [26], the stochastic goal programming technique proposes a logical physical aid in modeling and resolving these problems.

SGP addresses intrinsic uncertainty & is being applied in numerous areas like economic development, portfolio selection, project selection, resource allocation, healthcare management, transportation, marketing [12], cash management [15], wealth management [27], energy consumption, workforce allocation, greenhouse gas emissions [11], forest planning [14]. Contini, introduced the first formulation of SGP in 1968, considering goals as random variables having statistical distribution & and suggested a model taking into account that the maximization of the probability that the decision belongs to a region surrounding the random goal. This model induces a solution that is near the random goal as much as possible [28]. The conventional formulation of the SGP model is as follows:

$$\max f(x) \quad (27)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}i \; (for\; i = 1, 2, \ldots.., p) \quad (28)$$

$$\underline{x} \geq 0$$

Where $\underline{x}$ represents an "n-dimensional random vector" of the decision variables, a_{ij} symbolizes a mxn matrix A of "deterministic coefficients" and $\tilde{b}i$ represents an "m-dimensional vector" $\underline{b}$ "(stochastic) resource limitations".

Applications of SGP

Numerous studies have revealed the usefulness of stochastic goal programming formulations being supportive in quiet a number of different areas (marketing, transportation [29], portfolio selection [10], health care management [30], [31], [32], resource allocation, project selection,) [12], "cash management" [15], "wealth management" [27], pharmaceutical [20], "economic development, energy consumption, workforce allocation, & greenhouse emission emissions" [11], "forest planning" [14], Coal-fired power stations, Water management [33], data communication networks, Market share scheme, Investments, Blends, Hot Desking, Advertising [34], [25].

Table 3: Applications of stochastic goal programming

Application	Author (s)	Uncertainty	Conclusion
Textile industry	Wang et al., 2021	Demand	Developed "a stochastic multi-objective mixed-integer programming model for global sustainable multi-product production planning".
Water use planning	Bravo & Gonzalez, 2009	Demand	SGP especially designed for water use planning was developed.

Portfolio management	Ji et al., 2005	Asset returns	Presented “a stochastic linear goal programming model for multistage portfolio management emphasizing the investor’s goal & risk preference”.
Sustainable development	Jayaraman et al., 2017	Electricity demand	presented “a scenario-based stochastic goal programming model with satisfaction function for optimal employee allocation across various economic sectors”.
Cash management	Salas-Molina et al., 2020	Cash flows	Developed “a generalized stochastic goal programming model to derive stable policies within cash management systems with multiple bank accounts using cash flow forecasts as a key input”
Groundwater remediation management	Li et al., 2014	Human health-risk	“Stochastic analysis & goal programming were introduced into the framework to handle uncertainties in real-world groundwater remediation systems”
Pharmaceutical	Rajendran et al., 2019	No. “of customers lost due to side effects”	Developed “a multiple criteria stochastic mixed integer programming model, which serves as a decision support system to pharmaceutical companies”.
Forest planning	Eyvindson & Kangas, 2014	Forest inventory	Developed “three stochastic goal programming formulations & highlighted the usefulness of the approach on a small forest holding”.

Table 37: (Continued)

Application	Author (s)	Uncertainty	Conclusion
Industrial production (textile blending)	Ballestero, 2005	Blends	“Choice of fibers to make blends in yam production was developed from empirical information & numerically solved”

Wealth management	Kim et al., 2020	Assets	Proposed “a GBI framework that finds the optimal financial plan for an individual to achieve multiple consumption goals with various priority levels”, (automated financial advising services)
Health care system (blood collection and distribution)	Attari & Jami, 2018	Demand	Developed “a novel hybrid approach based on stochastic programming, MCGP and robust optimization”.
Transportation	Yang, 2007	“direct cost, transportation time, supply abilities, demands”	Three “models were constructed for stochastic solid transportation problem with different modeling Ideas (expected value, chance-constrained & dependent-chance goal programming)”.

5. Conclusion

Having scrutinized through a number of papers published in field of stochastic goal programming both theoretical and applied contributions, the following conclusions are drawn:

- Throughout numerous decades, the goal programming model has demonstrated to be powerful instrument and remains to be “an attractive and flexible” model dealing with “decision-making situations” where numerous “conflicting” and “incommensurable” objectives are to be optimized concurrently.
- Given its “simplicity and satisficing philosophy”, the goal programming model is suitable for aiding the decision marker to advance towards the best recommendations and help in getting to know more about the “decision-making context”.
- SGP is one of greatest widespread applied tools in the “multi-criteria decision aid paradigm” and its acceptance is based on to its being easily “understood and applied”.
- The information in this paper can be used as a guide for “academicians and optimization practitioners” to resolve “specific decision-making” situations.

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MODELING AND OPTIMIZATION OF MANUFACTURING LOT SIZE IN AGGREGATE PRODUCTION PLANNING UNDER DEMAND UNCERTAINTY: A CASE STUDY OF MOVIT PRODUCTS (U) LIMITED

BY

MAUREEN NALUBOWA SSEMPIJJA

2022

Appendix 6: Similarity Index Report

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Appendix 7: Data Collected Tools Used

Below are the 'sample' data collection tools that were used in the collection of data during this research study.

DATA GATHERING SHEETS

Introduction

The main aim of this survey is to collect data that will be used in the development of an optimization model that predicts optimal manufacturing lot size in production planning (PP) under demand uncertainty in Uganda (case study: MMOVIT PRODUCTS (U) LTD).

It is for academic purposes only and I'm requesting you to help in providing the necessary information as it will be treated with outmost confidentiality. Thanks so much.

BACKGROUND QUESTIONS

1. List all the items produced by Movit products (U) ltd in the plastic department

1.		11.	
2.		12.	
3.		13.	
4.		14.	
5.		15.	
6.		16.	
7.		17.	
8.		18.	
9.		19.	
10.		20.	

2. List the five (5) most demanded products from the list above starting with the most demanded to the least demanded
   1) ______________________________
   2) ______________________________
   3) ______________________________
   4) ______________________________
   5) ______________________________

3. What characterizes the Production Planning system at Movit products(U) ltd (Tick the most appropriate answer, please)
   a) Make – on – order
   b) Make – to – stock

4. What is the nature of the Production Planning system at Movit products(U) ltd (Tick the most appropriate answer, please)
    a) Job Method
    b) Flow Method
    c) Mass Production Method
    d) Batch Method
    e) Process Method

5. What are the distinct features of the production planning system with respect to the manufacturing lot sizes at Movit products(U) Ltd? (Tick the appropriate answer, please)
    a) Routing
    b) Scheduling
    c) Dispatching and inspection
    d) Co-ordination and the control of materials
    e) Methods
    f) Machines
    g) Tooling
    h) Operating times.

DATA GATHERING WORKSHEET

Factory: **Movit products(U) Limited** Product: ______________________
Week: ________ Month: ________________ Year: ________ Quarter: _________

Day	Time Duration	Observed Customer orders	Quantity Demanded	On-hand inventory	State	
					Favorable (F)	Unfavorable (U)
Mon	8:00am -9:00am					
	9:00am -10:00am					
	10:00am -11:00am					
	11:00am -12:00pm					
	12:00pm -1:00pm					
	1:00pm -2:00pm					
	2:00pm -3:00pm					
	3:00pm -4:00pm					
	4:00pm -5:00pm					
Tue	8:00am -9:00am					
	9:00am -10:00am					
	10:00am -11:00am					
	11:00am -12:00pm					
	12:00pm -1:00pm					
	1:00pm -2:00pm					
	2:00pm -3:00pm					
	3:00pm -4:00pm					
	4:00pm -5:00pm					
Wed	8:00am -9:00am					
	9:00am -10:00am					
	10:00am -11:00am					
	11:00am -12:00pm					
	12:00pm -1:00pm					
	1:00pm -2:00pm					
	2:00pm -3:00pm					
	3:00pm -4:00pm					
	4:00pm -5:00pm					
Thur	8:00am -9:00am					
	9:00am -10:00am					
	10:00am -11:00am					
	11:00am -12:00pm					
	12:00pm -1:00pm					
	1:00pm -2:00pm					
	2:00pm -3:00pm					
	3:00pm -4:00pm					
	4:00pm -5:00pm					
Fri	8:00am -9:00am					
	9:00am -10:00am					
	10:00am -11:00am					
	11:00am -12:00pm					
	12:00pm -1:00pm					

	1:00pm -2:00pm					
	2:00pm -3:00pm					
	3:00pm -4:00pm					
	4:00pm -5:00pm					

CUSTOMER DATA FOR PRODUCT A IN THE FIRST QUARTER OF THE YEAR

1. What is the average number of customers for product 1 in the first week of the first month? ……………………………………………………….

2. What is the average number of customers for product 1 in the second week of the first month? ……………………………………………………….

3. What is the average number of customers for product 1 in the third week of the first month? ……………………………………………………….

4. What is the average number of customers for product 1 in the fourth week of the first month? ……………………………………………………….

5. What is the average number of customers for product 1 in the first week of the second month? ……………………………………………………….

6. What is the average number of customers for product 1 in the second week of the second month? ……………………………………………………….

7. What is the average number of customers for product 1 in the third week of the second month? ……………………………………………………….

8. What is the average number of customers for product 1 in the fourth week of the second month? ……………………………………………………….

9. What is the average number of customers for product 1 in the first week of the third month? ……………………………………………………….

10. What is the average number of customers for product 1 in the second week of the third month? ……………………………………………………….

11. What is the average number of customers for product 1 in the third week of the third month? ……………………………………………………….

12. What is the average number of customers for product 1 in the fourth week of the third month? ……………………………………………………….

DEMAND DATA FOR PRODUCT A IN THE FIRST QUARTER OF THE YEAR

1. What is the average number of product 1 demanded in the first week of the first month?
   ……………………………………………………….
2. What is the average number of product 1 demanded in the second week of the first month?
   ……………………………………………………….
3. What is the average number of product 1 demanded in the third week of the first month?
   ……………………………………………………….
4. What is the average number of product 1 demanded in the fourth week of the first month?
   ……………………………………………………….
5. What is the average number of product 1 demanded in the first week of the second month?
   ……………………………………………………….
6. What is the average number of product 1 demanded in the second week of the second month?
   ……………………………………………………….
7. What is the average number of product 1 demanded in the third week of the second month?
   ……………………………………………………….
8. What is the average number of product 1 demanded in the fourth week of the second month?
   ……………………………………………………….
9. What is the average number of product 1 demanded in the first week of the third month?
   ……………………………………………………….
10. What is the average number of product 1 demanded in the second week of the third month?
    ……………………………………………………….
11. What is the average number of product 1 demanded in the third week of the third month?
    ……………………………………………………….
12. What is the average number of product 1 demanded in the fourth week of the third month?
    ……………………………………………………….

ONHAND INVENTORY DATA FOR PRODUCT A IN THE FIRST QUARTER OF THE YEAR

1. What is the average number of product 1 in inventory in the first week of the first month?
 ……………………………………………………….
2. What is the average number of product 1 in inventory in the second week of the first month?
 ……………………………………………………….
3. What is the average number of product 1 in inventory in the third week of the first month?
 ……………………………………………………….
4. What is the average number of product 1 in inventory in the fourth week of the first month?
 ……………………………………………………….
5. What is the average number of product 1 in inventory in the first week of the second month?
 ……………………………………………………….
6. What is the average number of product 1 in inventory in the second week of the second month?
 ……………………………………………………….
7. What is the average number of product 1 in inventory in the third week of the second month?
 ……………………………………………………….
8. What is the average number of product 1 in inventory in the fourth week of the second month?
 ……………………………………………………….
9. What is the average number of product 1 in inventory in the first week of the third month?
 ……………………………………………………….
10. What is the average number of product 1 in inventory in the second week of the third month?
 ……………………………………………………….
11. What is the average number of product 1 in inventory in the third week of the third month?
 ……………………………………………………….
12. What is the average number of product 1 in inventory in the fourth week of the third month?
 ……………………………………………………….

Appendix 8: Data collected for fast moving products at MOVIT products (U) Ltd

Favorable when N > 12, else Unfavorable						
				Unit production cost KES	Unit holding cost KES	Unit shortage cost KES
PRODUCT A, Selling price per carton = 1111 KES				722.22	10.83	79.44
MONTH	**DATE**		**WEEK**	**CUSTOMERS (N)**	**DEMAND (D) (x10^3)**	**ON HAND INVENTORY (V) (x10^3)**
1	**1/3/21 - 6/3/21**	**6 DAYS**	**1**	9	3937	6076
	8/3/21 - 13/3/21	**6 DAYS**	**2**	12	4668	4687
	15/3/21 - 20/3/21	**6 DAYS**	**3**	8	2485	6306
	22/3/21 - 27/3/21	**6 DAYS**	**4**	17	7955	10160
2	**1/4/21 - 7/4/21**	**6 DAYS**	**1**	1	110	4525
	8/4/21 - 14/4/21	**6 DAYS**	**2**	15	3832	5681
	15/4/21 - 21/4/21	**6 DAYS**	**3**	7	2870	4363
	22/4/21 - 28/4/21	**6 DAYS**	**4**	20	3824	6028
3	**1/5/21 - 7/5/21**	**6 DAYS**	**1**	4	758	2018
	8/5/21 - 14/5/21	**6 DAYS**	**2**	16	6125	4149
	15/5/21 - 21/5/21	**6 DAYS**	**3**	14	2625	4163
	22/5/21 - 28/5/21	**6 DAYS**	**4**	17	3685	6279

Favorable when N > 25, else Unfavorable						
				Unit production cost KES	Unit holding cost KES	Unit shortage cost KES
PRODUCT B, Selling price per carton = 3667 KES				2566.67	51.33	282.33
MONTH	**DATE**		**WEEK**	**CUSTOMERS (N)**	**DEMAND (D) (x10^3)**	**ON HAND INVENTORY (V) (x10^3)**
1	**1/3/21 - 6/3/21**	**6 DAYS**	**1**	16	2309	2365
	8/3/21 - 13/3/21	**6 DAYS**	**2**	34	3224	4459
	15/3/21 - 20/3/21	**6 DAYS**	**3**	25	2759	3255
	22/3/21 - 27/3/21	**6 DAYS**	**4**	42	6113	5923
2	**1/4/21 - 7/4/21**	**6 DAYS**	**1**	7	414	2095
	8/4/21 - 14/4/21	**6 DAYS**	**2**	22	2422	2564
	15/4/21 - 21/4/21	**6 DAYS**	**3**	16	1269	2994
	22/4/21 - 28/4/21	**6 DAYS**	**4**	36	2981	3372
3	**1/5/21 - 7/5/21**	**6 DAYS**	**1**	12	289	2502
	8/5/21 - 14/5/21	**6 DAYS**	**2**	24	1825	1827
	15/5/21 - 21/5/21	**6 DAYS**	**3**	30	806	1636
	22/5/21 - 28/5/21	**6 DAYS**	**4**	33	2426	2992

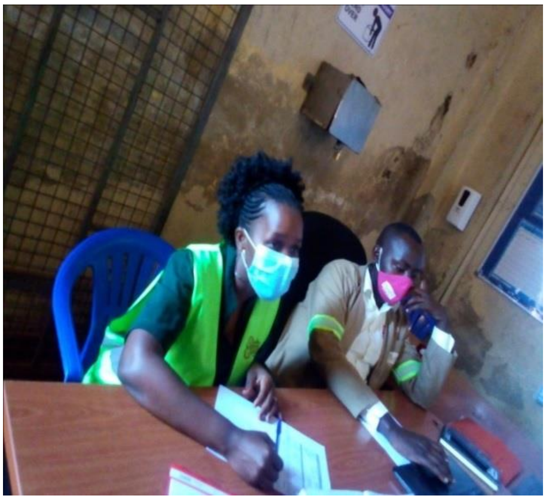
Favorable when N > 27, else Unfavorable						
				Unit production cost KES	Unit holding cost KES	Unit shortage cost KES
PRODUCT C, Selling price per carton = 1194 KES				836.1111111	8.3611111	91.9722222
MONTH	**DATE**		**WEEK**	**CUSTOMERS (N)**	**DEMAND (D) (x10^3)**	**ON HAND INVENTORY (V) (x10^3)**
1	**1/3/21 - 6/3/21**	**6 DAYS**	**1**	16	1746	2581
	8/3/21 - 13/3/21	**6 DAYS**	**2**	39	4929	4656
	15/3/21 - 20/3/21	**6 DAYS**	**3**	19	3347	4538
	22/3/21 - 27/3/21	**6 DAYS**	**4**	37	5020	5514
2	**1/4/21 - 7/4/21**	**6 DAYS**	**1**	7	875	2050
	8/4/21 - 14/4/21	**6 DAYS**	**2**	23	4757	3690
	15/4/21 - 21/4/21	**6 DAYS**	**3**	19	3068	2687

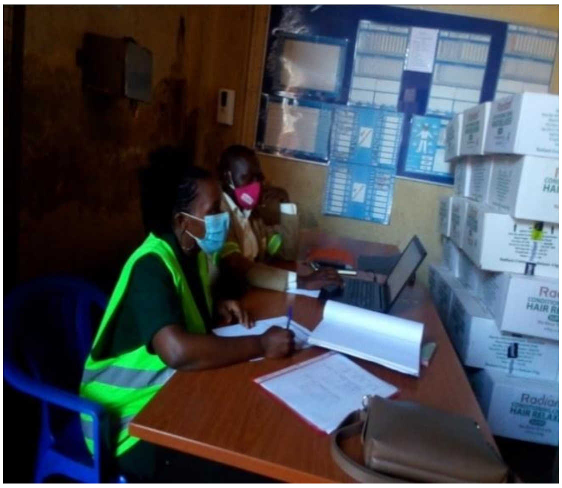
	22/4/21 - 28/4/21	**6 DAYS**	**4**	33	3005	4189
3	**1/5/21 - 7/5/21**	**6 DAYS**	**1**	16	1745	3309
	8/5/21 - 14/5/21	**6 DAYS**	**2**	38	3263	3259
	15/5/21 - 21/5/21	**6 DAYS**	**3**	39	3093	4115
	22/5/21 - 28/5/21	**6 DAYS**	**4**	33	4146	5177

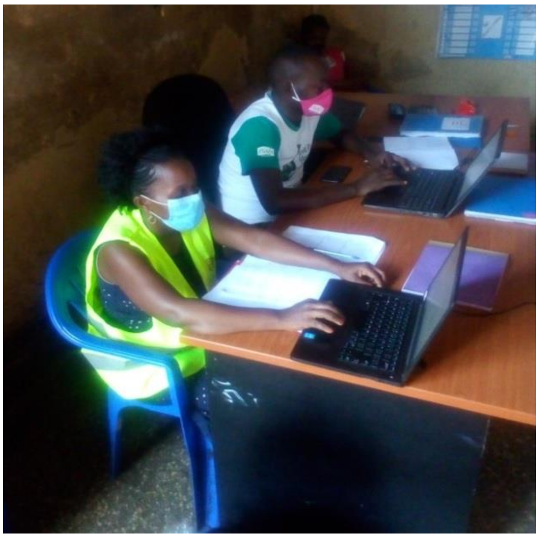
						Favorable when N > 26, else Unfavorable
				Unit production cost KES	Unit holding cost KES	Unit shortage cost KES
PRODUCT D, Selling price per carton = 4500 KES				3150	63	346.5
MONTH	**DATE**		**WEEK**	**CUSTOMERS (N)**	**DEMAND (D) ($x10^3$)**	**ON HAND INVENTORY (V) ($x10^3$)**
1	**1/3/21 - 6/3/21**	**6 DAYS**	**1**	15	308	5263
	8/3/21 - 13/3/21	**6 DAYS**	**2**	29	2891	7337
	15/3/21 - 20/3/21	**6 DAYS**	**3**	24	1757	7081
	22/3/21 - 27/3/21	**6 DAYS**	**4**	38	6619	5654
2	**1/4/21 - 7/4/21**	**6 DAYS**	**1**	8	231	3525
	8/4/21 - 14/4/21	**6 DAYS**	**2**	17	2046	6243
	15/4/21 - 21/4/21	**6 DAYS**	**3**	15	1617	5922
	22/4/21 - 28/4/21	**6 DAYS**	**4**	45	4443	5951
3	**1/5/21 - 7/5/21**	**6 DAYS**	**1**	14	559	3765
	8/5/21 - 14/5/21	**6 DAYS**	**2**	37	3686	4738
	15/5/21 - 21/5/21	**6 DAYS**	**3**	28	1537	4980
	22/5/21 - 28/5/21	**6 DAYS**	**4**	44	5626	5746

						Favorable when N > 34, else Unfavorable
				Unit production cost KES	Unit holding cost KES	Unit shortage cost KES
PRODUCT E, Selling price per carton = 3750				2625	78.75	288.75
MONTH	**DATE**		**WEEK**	**CUSTOMERS (N)**	**DEMAND (D) ($x10^3$)**	**ON HAND INVENTORY (V) ($x10^3$)**
1	**1/3/21 - 6/3/21**	**6 DAYS**	**1**	20	904	2333
	8/3/21 - 13/3/21	**6 DAYS**	**2**	50	2220	4800
	15/3/21 - 20/3/21	**6 DAYS**	**3**	28	1200	5341
	22/3/21 - 27/3/21	**6 DAYS**	**4**	58	3827	6400
2	**1/4/21 - 7/4/21**	**6 DAYS**	**1**	12	335	2802
	8/4/21 - 14/4/21	**6 DAYS**	**2**	31	1672	5037
	15/4/21 - 21/4/21	**6 DAYS**	**3**	24	1893	6102
	22/4/21 - 28/4/21	**6 DAYS**	**4**	37	1480	5750
3	**1/5/21 - 7/5/21**	**6 DAYS**	**1**	17	608	4906
	8/5/21 - 14/5/21	**6 DAYS**	**2**	39	1528	5433
	15/5/21 - 21/5/21	**6 DAYS**	**3**	41	1570	5576
	22/5/21 - 28/5/21	**6 DAYS**	**4**	47	2224	5614

Appendix 9: Photos at MOVIT products (U) Ltd during data collection





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Appendix 10: MATLAB Solutions

PRODUCT A

Objective function

$$\sum_{k=1}^{4}[P_k(A,1)d_k^+ + P_k(A,1)d_k^-]$$

Let $P_k(A,1) = P_k$

$$Z = \sum_{k=1}^{4}[P_k d_k^+ + P_k d_k^-]$$

Focus objective function needed to satisfy the given goals;

$Z = P_1d_1^- + P_2d_2^- + P_3d_3^+ + P_4d_4^+$

Constraints

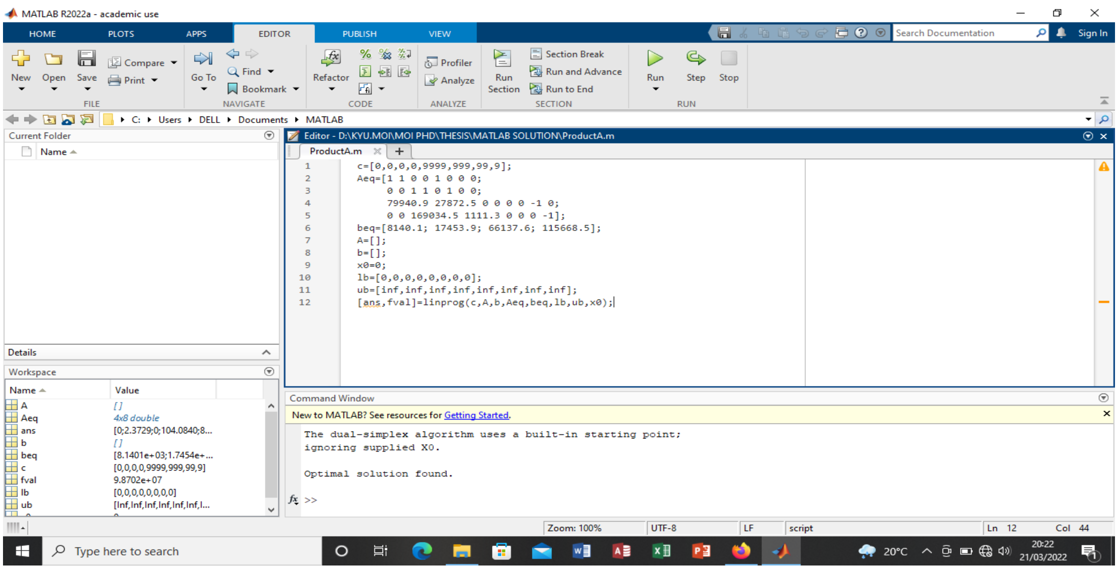
$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8,140.1$

$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17,453.9$

$79940.9X_{FF}(A,1) + 27872.5X_{FU}(A,1) - d_3^+ = 66,137.6$

$169034.5X_{FU}(A,1) + 1111.3X_{UU}(A,1) - d_4^+ = 115,668.5$

The values were then inserted in the editor window of MATLAB with Aeq being the values of the LHS of the constraints and Beq being the values of RHS of the constraints. The Big M method was also used to determine the coefficients of the priorities. The lower bounds (lb) were set to zero and the upper bounds (ub) set to positive infinity. (Used the linprog solver in MATLAB)



PRODUCT B

Objective function

$$\sum_{k=1}^{4} [P_k(B,1)\, d_k^{+} + P_k(B,1)\, d_k^{-}]$$

Let $P_k(B,1) = P_k$

$$Z = \sum_{k=1}^{4} [P_k d_k^{+} + P_k d_k^{-}]$$

Focus objective function needed to satisfy the given goals;

$Z = P_1 d_1^{-} + P_2 d_2^{-} + P_3 d_3^{+} + P_4 d_4^{+}$

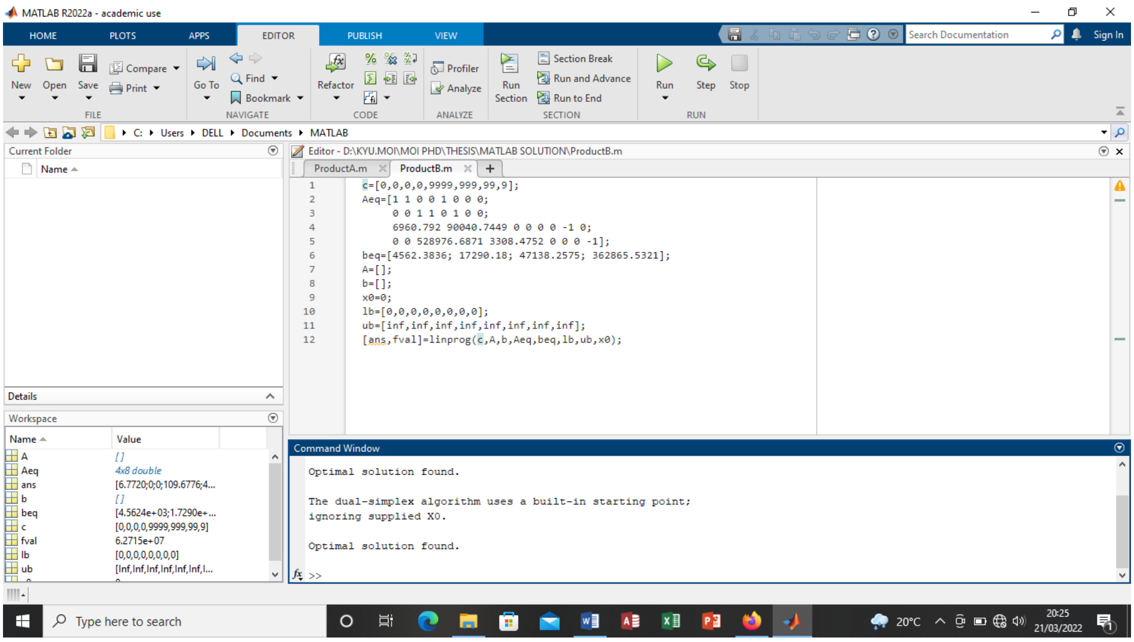
Constraints

$X_{FF}(B,1) + X_{FU}(B,1) + d_1^{-} = 4{,}562.3836$

$X_{UF}(B,1) + X_{UU}(B,1) + d_2^{-} = 17{,}290.18$

$6{,}960.792 X_{FF}(B,1) + 90{,}040.7449 X_{FU}(B,1) - d_3^{+} = 47{,}138.2572$

$528{,}976.6871 X_{FU}(B,1) + 3{,}308.4752 X_{UU}(B,1) - d_4^{+} = 362{,}865.5321$



PRODUCT C

Objective function

$$\sum_{k=1}^{4} [P_k(C,1)\,d_k^+ + P_k(C,1)\,d_k^-]$$

Let $P_k(C,1) = P_k$

$$Z = \sum_{k=1}^{4} [P_k d_k^+ + P_k d_k^-]$$

Focus objective function needed to satisfy the given goals;

$Z = P_1d_1^- + P_2d_2^- + P_3d_3^+ + P_4d_4^+$

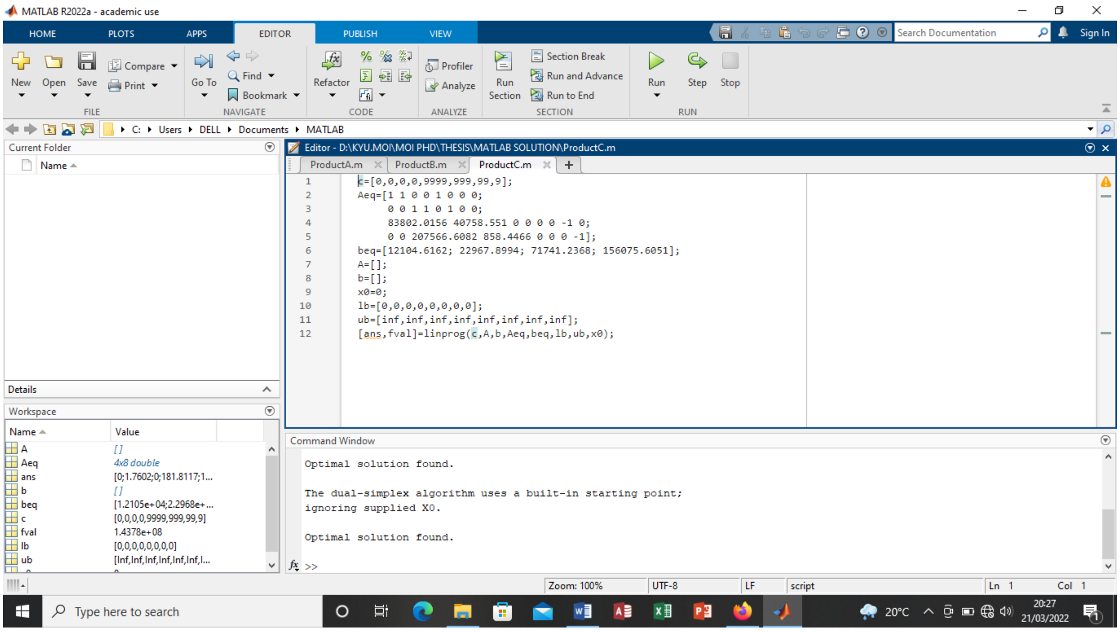
Constraints

$X_{FF}(C,1) + X_{FU}(C,1) + d_1^- = 12{,}104.6162$

$X_{UF}(C,1) + X_{UU}(C,1) + d_2^- = 22{,}967.8994$

$83802.0156X_{FF}(C,1) + 40758.551X_{FU}(C,1) - d_3^+ = 71{,}741.2368$

$207566.6082X_{FU}(C,1) + 858.4466X_{UU}(C,1) - d_4^+ = 15{,}6075.6051$



PRODUCT D

Objective function

$$\sum_{k=1}^{4} \quad [P_k(D,1)\,d_k^+ + P_k(D,1)\,d_k^-]$$

Let $P_k(D,1) = P_k$

$$Z = \sum_{k=1}^{4} \quad [P_k d_k^+ + P_k d_k^-]$$

Focus objective function needed to satisfy the given goals;

$Z = P_1d_1^- + P_2d_2^- + P_3d_3^+ + P_4d_4^+$

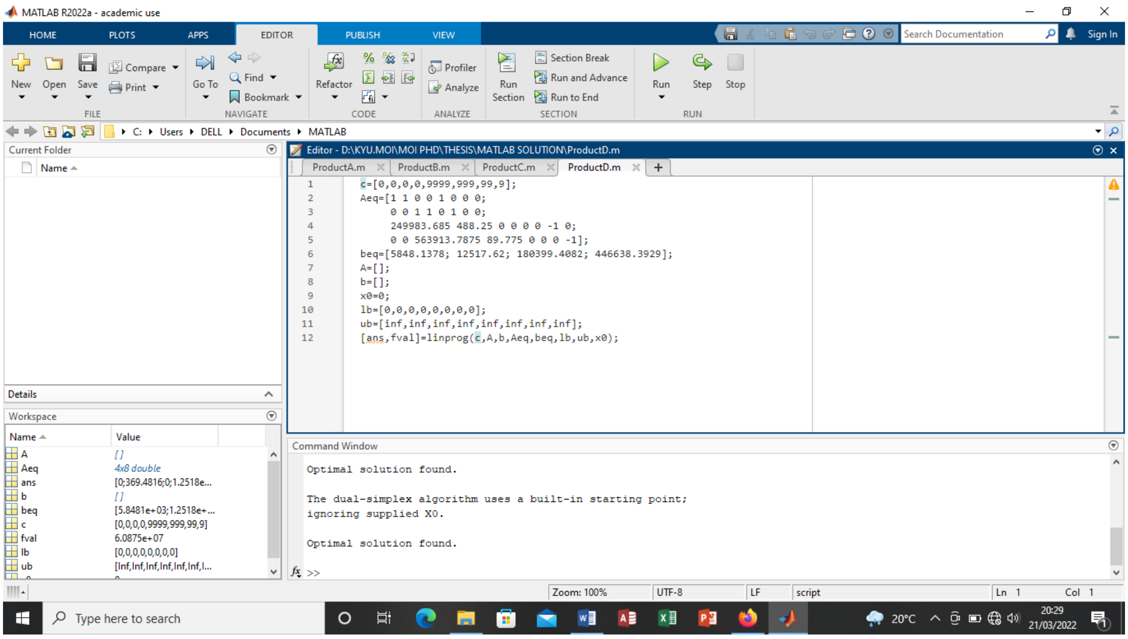
Constraints

$X_{FF}(D,1) + X_{FU}(D,1) + d_1^- = 5848.1378$

$X_{UF}(D,1) + X_{UU}(D,1) + d_2^- = 12517.62$

$249983.685X_{FF}(D,1) + 488.25X_{FU}(D,1) - d_3^+ = 180399.4082$

$563913.7875X_{FU}(D,1) + 89.775X_{UU}(D,1) - d_4^+ = 446638.3929$



PRODUCT E

Objective function

$$\sum_{k=1}^{4}[P_k(E,1)d_k^+ + P_k(E,1)d_k^-]$$

Let $P_k(E,1) = P_k$

$$Z = \sum_{k=1}^{4}[P_k d_k^+ + P_k d_k^-]$$

Focus objective function needed to satisfy the given goals;

$Z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^+ + P_4 d_4^+$

Constraints

$X_{FF}(E,1) + X_{FU}(E,1) + d_1^- = 69.6644$

$X_{UF}(E,1) + X_{UU}(E,1) + d_2^- = 6286.9678$

$38812.725X_{FF}(E,1) + 2026.6313X_{FU}(E,1) - d_3^+ = 27147.8547$

$255664.2375X_{FU}(E,1) + 2.1263X_{UU}(E,1) - d_4^+ = 188118.3077$