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AFRICAN JOURNAL OF PURE AND APPLIED MATHEMATICS

Imhotep Mathematical Proceedings
Volume 1, Numéro 1, (2014),
pp. 59 – 63.

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Abstract

Search for experimental designs which aid in research studies involving large number of treatments with minimal experimental units has been desired overtime. This paper constructs some new series of three associate Partially Balanced Incomplete Block (PBIB) designs having $n(n-2)/4$ treatments with three associate classes in two replicates using the concept of triangular association scheme. The design is constructed from an even squared array of n rows and n columns ($n \geq 8$) with its both diagonal entries bearing no treatment entries and that given the location of any treatment in the squared array, the other location of the same treatment in the array is predetermined. The design and association parameters for a general case of an even integer $n \geq 8$ are obtained with an illustrated case for $n = 8$. Efficiencies of the designs within the class of designs are obtained for a general case of even $n \geq 8$ with a listing of efficiencies of designs with blocks sizes in the interval [8,22]. The designs constructed have three associate classes and are irreducible to minimum number of associate classes.

Proceedings of the 2nd Strathmore International Mathematics Conference (SIMC 2013), 12 - 16 August 2013, Strathmore University, Nairobi, Kenya.

http://imhotep-journal.org/index.php/imhotep/
Imhotep Mathematical Proceedings

IMHOTEP - Math. Proc. **1** (2014), 59–63
1608-9324/010059–58, DOI 13.1007/s00009-003-0000

IMHOTEP Mathematical Proceedings

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Abstract. Search for experimental designs which aid in research studies involving large number of treatments with minimal experimental units has been desired overtime. This paper constructs some new series of three associate Partially Balanced Incomplete Block (PBIB) designs having $n(n-2)/4$ treatments with three associate classes in two replicates using the concept of triangular association scheme. The design is constructed from an even squared array of n rows and n columns ($n \geq 8$) with its both diagonal entries bearing no treatment entries and that given the location of any treatment in the squared array, the other location of the same treatment in the array is predetermined. The design and association parameters for a general case of an even integer $n \geq 8$ are obtained with an illustrated case for $n = 8$. Efficiencies of the designs within the class of designs are obtained for a general case of even $n \geq 8$ with a listing of efficiencies of designs with blocks sizes in the interval [8,22]. The designs constructed have three associate classes and are irreducible to minimum number of associate classes.

Keywords. Partially Balanced Incomplete Block (PBIB), Associate class, three associate classes.

Introduction

By changing the arrangement of treatments or omitting certain blocks and or treatments, we obtain designs that may belong to a class of new designs. Using this technique Bose and Nair (1939) introduced some PBIB designs. Atiqullah (1958) established that considering PBIB designs based on triangular association scheme with $v = n((n-2))/2$, $b = (n-1)((n-2))/2$, $r = k = n-2$, $\lambda_1 = 1$ and $\lambda_2 = 2$. Other methods were given by Shrikande (1960, 1965) and Chang et al (1965) based on the existence of certain BIB designs and considering the dual of a BIB design as omitting certain blocks from the BIB design. John (1966) showed that triangular association scheme can be described by representing the treatments by ordered pairs(x,y) with $1 \leq x < y \leq q$ and then generalized (John, 1966) for the case that $v = q(q-1)((q-2))/6$, $(q > 3)$. Arya and Narain (1981) discussed a new association scheme called truncated triangular (TT) with five associate classes when $v = p((p-2))/2$ with p an even positive integer ≥ 8, and used to construct partial diallel crosses. Ching-Shui et al (1984) came up with a general and simple method of construction based on the relation of triangular and 1_2 type of PBIB design

Paper presented at the 2nd Strathmore International Mathematics Conference (SIMC 2013), 12 - 16 August 2013, Strathmore University, Nairobi, Kenya.

to the line graph theory. A construction of triangular designs with nested rows and columns is given by Agrawal and Prasad (1984). Sinha and Sanpei (2004) gave a series of triangular (tenary) designs with nested row and columns. Garg et al (May, 2011) constructed some new series of triangular and four associate PBIB designs with two replications by using dualization technique. Silver et al (April 2012) constructed some new three associate class Pbib Design with two replicates. Recently, Kipkemoi et al (January-June, 2013) constructed a three associate class partially balanced incomplete Block designs in two replicates

Construction of designs

The design is constructed from a squared array of n rows and n columns (n is even positive integer ≥ 8) with both diagonal entries in the array having no treatments allocated to as illustration in the figure below

*	n_{12}	n_{13}	n_{14}	n_{15}	n_{16}	n_{17}	*
n_{21}	*	n_{23}	n_{24}	n_{25}	n_{26}	*	n_{28}
n_{31}	n_{32}	*	n_{34}	n_{35}	*	n_{37}	n_{38}
n_{41}	n_{42}	n_{45}	*	*	n_{46}	n_{47}	n_{48}
n_{51}	n_{52}	n_{53}	*	*	n_{56}	n_{57}	n_{58}
n_{61}	n_{62}	*	n_{64}	n_{65}	*	n_{67}	n_{68}
n_{71}	*	n_{73}	n_{74}	n_{75}	n_{76}	*	n_{78}
*	n_{82}	n_{83}	n_{84}	n_{85}	n_{86}	n_{87}	*

i and j are integers $(1 \leq i,\, j \geq n)$ such that;
Each row and column of the square array has $n-2$ treatment entries.
The treatment entries are allocated in the array by following two subsequent steps

1. The initial set of v treatment entries are first filled on one triangle enclosed by the two diagonal and a side of the square array.
2. The second set of v treatment entries are replicated in each of the remaining triangles by simply reflecting the initial set of v treatment entries subsequently with the diagonal blank entries as mirrors in such a way that given any two entries n_{ij} and n_{ij} are allocated to treatment x if and only if the subscripts $i+\acute{i}=j+\acute{j}$ or $i+\acute{i}+j+\acute{j}=2(n+1)$.

Taking each row and column to constitute a block we obtain $n/2$ distinct blocks and thus a design with parameters
$v=(n(n-2))/4$ $b=n/2$ $k=n-2$ $r=2$ $\lambda_1=2$ $\lambda_2=1$ $\lambda_3=0$

Association scheme

Two treatments are said to be:

- First associates if they both occur in the same row and column.
- Second associates if they both occur in the same row or the same column but not both.
- Third associates if they neither occur in the same row nor in the same column. Giving rise to the following association parameters.

  $n_1=1$ $n_2=2(n-4)$ $n_2=\frac{n(n-10)+24}{4}$

$$P_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2(n-4) & 0 \\ 0 & 0 & 0 & \frac{n(n-10)+24}{4} \end{pmatrix} P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2(n-4) & 0 \\ 0 & 0 & 0 & \frac{n(n-10)+24}{4} \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & n-4 & n-6 \\ 0 & 0 & n-6 & \frac{n(n-14)+48}{4} \end{pmatrix} P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 8 & 2(n-8) \\ 1 & 1 & 2(n-8) & \frac{n(n-18)+80}{4} \end{pmatrix}$$

Illustration: taking $n = 8$ we obtain three associate class PBIB design with the parameters

$v = 12$ $b = 4$ $k = 6$ $r = 2$

$\lambda_1 = 2$ $\lambda_2 = 1$ $\lambda_3 = 0$

$n_1 = 1$ $n_2 = 8$ $n_3 = 2$

Whose blocks are:

1. (1, 2, 3, 4, 5, 6)
2. (1, 6, 7, 8, 9, 10)
3. (2, 5, 7, 10, 11, 12)
4. (3, 4, 8, 9, 11, 12)

With the parameters of the second kind given by:

$$P_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} P_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 8 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

The efficiency factors for this class of designs are given by

$E_1 = \frac{n-4}{n-2}$

$E_2 = -\frac{n(n-4)^2}{7n^2-6n-16-n^3}$

$E_3 = -\frac{n(n-4)^2}{6n^2+2n-32-n^3}$

and the overall efficiency factor of the design is

$$E = \frac{1}{n(n-2)-4}\left(\frac{4(n-4)}{n-2} - \frac{8n(n-4)^3}{7n^2-6n-16-n^3} + \frac{n(n-6)(n-4)^3}{6n^2+2n-32-n^3}\right)$$

Efficiencies of three associate class PBIB designs having $(n(n-2))/4$ treatments with two replicates for $6 \leq k \leq 22$ are give in the table below

n	v	b	k	r	λ_1	λ_2	λ_3	E	E_1	E_2	E_3
8	12	4	6	2	2	1	0	0.9495	0.6667	1	0.8889
10	20	5	8	2	2	1	0	0.9201	0.7500	0.9574	0.8738
12	30	6	10	2	2	1	0	0.9164	0.8000	0.9505	0.8807
14	42	7	12	2	2	1	0	0.9187	0.8333	0.9511	0.8906
16	56	8	14	2	2	1	0	0.9226	0.8571	0.9536	0.9000
18	72	9	16	2	2	1	0	0.9269	0.8750	0.9566	0.9083
20	90	10	18	2	2	1	0	0.9311	0.8889	0.9595	09156
22	110	11	20	2	2	1	0	0.9350	0.9000	0.9622	0.9219
24	132	12	22	2	2	1	0	0.9386	0.9091	0.9646	0.9274

Conclusion

In this paper we have constructed some new series of three associate class PBIB designs having $(n(n-2))/4$ treatments with two replications. The restriction of the number of replications to two helps to minimize cost. The average efficiency factors of these designs along with the three efficiencies factors E_1, E_2 and E_3 are quite high for practical purposes.

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