LEARNER COMPREHENSION AND PERFORMANCE ON THRESHOLD TOPICS IN MATHEMATICS

BY

GILBERT KIPKOECH TONUI

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DECLARATION

Declaration by the Candidate

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Signature: ……………………………. Date: ………………………..

Gilbert Kipkoech Tonui

Reg. no. EDU/D. Phil. CM/1006/16

Declaration by the Supervisors

This research thesis has been submitted with our approval as university supervisors.

Signature: ……………………………. Date: ………………………..

Prof. Laban Ayiro
Department of Management and Policy Studies,
School of Education,
Moi University

Signature: ……………………………. Date: ………………………..

Prof. Khaemba Ongeti
Department of CIEM,
School of Education,
Moi University

DEDICATION

This dedication goes to my beloved family for their unconditional love, continuous support, patience, and prayers for me to become who I am today.

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ABSTRACT

Mathematics is the cornerstone of development of any contemporary society as attaining self-reliance requires creative and problem solving individuals who can identify opportunities in their environment. Hence the concern for continued poor performance by learners leads to increasing research to identify possible factors contributing to the decline in learners' performance. However, a study on learner's threshold concepts in Mathematics would provide a useful framework for improving teaching and learning in secondary school education and therefore, the study aimed to establish those threshold concepts learners' faces. The objectives of the study included to: discuss the influence of teaching strategy on learners' performance in quadratic equations and functions with one known; describe learners' score performance in solving quadratic equations and functions with one known; analyze learner's threshold concepts in solving quadratic equations and functions with one known that may attribute to gender. to determine gender difference if any, that may exist in the cognitive level and school type performance of quadratic equations and functions with one known and determine relationships if any between gender, teaching strategy and school type on one hand and performance in quadratic equations and functions with one known on the other hand. Piaget's cognitive development and Vygotzy theories of learning guided the study. A descriptive survey research design and a mixed method research paradigm was employed. Learner's diagnostic test instrument containing quadratic equations and functions was designed based on Bloom's cognitive domains of learning was administered. Quantitative data collected was analyzed using the statistical package for social sciences (SPSS and SPSS, macro processing for interaction effects) and both descriptive and inferential statistics was used in making interpretations based on the objectives of the study. Computer Aided Qualitative Data Analysis (CAQDA), Nvivo pro 11 software was used to analyze qualitative data. The study found that teachers used problem solving, use of examples and lecture methods as the teaching strategies. Learners used factorization, completing square and quadratic formula methods to solve quadratic equations. Also teaching strategy was a significant determinant of learners' performance than school type. More detailed exploration of the students' difficulties in solving quadratic equations and functions with one known is a crucial prerequisite for any further attempt to improve the quality of Mathematics education and the levels of performance. Considering these issues, teachers should ask learners to explain a threshold concept, to represent it in new ways, to apply it to new situations, to connect it to their lives. The emphasis is equally strong that they should not simply recall the concept in the form in which it was presented. Teachers should be cautious when making assumptions about what learners' uncertainties might be.

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CHAPTER ONE

INTRODUCTION

1.1 Introduction

Secondary school Mathematics has got so many topics for instance, integers, algebra, commercial arithmetic, matrices, probability and statistics, among others. Threshold concept in Mathematics topics represents knowledge that is difficult, counter-intuitive or alien at the face value of the learner. The study narrowed down its focus to threshold concepts in quadratic equations and functions. Consequently, a threshold concepts in quadratic equations and functions can be considered akin to a portal, opening up a new and previously inaccessible way of thinking about the topic. It represents a transformed way of understanding or interpreting or viewing quadratics without which the learner cannot progress. As a consequence of comprehending, a threshold concept there may thus be a transformed internal view of the subject matter, subject land scape or even world view. Therefore, threshold concepts may potentially cause a significant shift in the learner's perception of or part thereof and it even transforms a learner's personal identity (Meyer & Land. 2003, as cited in Breen & O'Shea, 2016).

Quadratic equations and functions are a topic in mathematics which involves second degree polynomial functions of the form$y = ax^2 + bx + c$, where y is defined as the quadratic function of the form $ax^2 + bx + c$. In secondary school mathematics, quadratic equations and functions are taught in which a, b, and c are integers and solve quadratic equations whose expressions are set equals to a constant zero. But in order for learners to solve these equations especially of the higher order, they ought to have developed threshold concepts and apply it to new situations which require them to transfer the concepts to the new situations (Gillies, Nichols, Burgh & Haynes, 2014).

Secondary school learners in Kericho County take a series of topics in the 8.4.4 Mathematics K.N.E.C syllabus and quadratic expressions and equations is taken in form two as 15th topic out of the 20 topics and estimated to be completed in 12 lessons while quadratic functions is taught as the first topic in form three out of the 15 topics and supposed to be covered in 22 lessons. Quadratic functions are second-degree functions of the form $ax^2 + bx + c$ in which $a, b \; and \; c$ are constants and $a \neq 0$. Any quadratic function can be represented by an algebraic expression or graph. If f denotes a quadratic function, with x being the independent variable, the function can be written in the form $f(x) = ax^2 + bx + c$. In this case, the function f *is* defined as the function given by the expression $ax^2 + bx + c$, which maps each value d of x in the domain to a value $f(d)$ in the range. Common Core State Standards for Mathematics (CCSSM, 2010) specify that secondary school learners are expected to be able to solve quadratic equations using multiple methods; use their knowledge of quadratic functions to create and analyze graphs; and apply these skills, knowledge and cognitive skills to help them solve problems arising from a variety of contexts (Flanagan, 2017).

1.2 Background to the Study

Kenya secondary school mathematics aims at producing a person who will be numerate, orderly, logical, accurate and precise in thought. The person should also be competent in appraising and utilizing Mathematical skills in playing a positive role in the development of a modern society (KNEC, KCSE Mathematics syllabus, 2016 http://kcse-online.info/knec_kcse_mathematics-syllabus.html).

Learner's comprehension on threshold concepts in order for quadratic equations and functions knowledge to make sense. Threshold concepts are like a portal, opening up a new and previously inaccessible way of thinking. Threshold concepts in Mathematics focus on the cognitive (thinking) domain of learning rather than the affective (feelings, moods and emotions) or behavioural (physical/kinesthetic) domains of learning. In order to master a threshold concept, the theory suggests that learners may travel through a tunnel or ‘liminal space’ where they ‘get stuck’ and may be in a state of uncertainty. Subsequently, if satisfactory performance in Mathematics is to be realized, tremendous interests and efforts in conceptualizing and assessing the threshold concepts should be documented. There is need to acquire good strategies for effective teaching and learning of quadratic equations and functions that make better sense of teachers’ instructional preferences, design, reasoning, and decision-making (Flanagan, 2017).

Kotsopoulos, (2007) stated that quadratic equations and functions are one of the most conceptually challenging aspects of the high school curriculum. This is because many secondary learners lack comprehension on threshold concepts especially with basic multiplication table fact retrieval. Factorization concept is a process of finding products within the multiplication table; this directly influences students’ ability to engage effectively in factorization of quadratics. Furthermore, most secondary school learners were found to be confused about the concept of a variable and the meaning of a solution to a quadratic equation. For example, even if most students were able to obtain the correct solutions, $x = 3 \; and \; x = 5$ learners thought that the two *x*s in the equation $(x - 3)\,(x - 5) = 0$ stood for different variables. This showed that the students lack relational understanding and relied only on rote learning (Law & Shahrill, 2013; Pungut & Shahrill, 2014; Sarwadi & Shahrill, 2014; Vaiyavutjamai, 2004; Vaiyavutjamai, Ellerton & Clements, 2005; Vaiyavutjamai & Clements, 2006 as cited in Yahya & Shahrill, 2015).

Moreover, quadratic equations and functions in secondary school level usually starts in form two and end at form four. This gives less opportunity for learners to reason, make sense and build up a bridge in relating threshold concepts with solving problems. Hence, results in a gap between high- and low- achiever learners (Susac, Bubic, Vrbanc, & Planinic, 2014). Quadratic equations and functions emphasizes on applications of Mathematics to real life experiences and practical approaches to teaching and learning in an effort to address such contemporary issues as information technology, health, gender and integrity. Learners' Mathematics performance worldwide and Kenya in particular has been unsatisfactory. Mathematics examination results at the Kenya Certificate of Secondary Education (KCSE) for instance have been consistently dismal over the last decade. Findings of most studies globally and in Kenya attribute this unfavorable state of affairs to learner's gender, attitude, school type, teaching strategy among others.

Teaching strategy encompasses transparency, efficiency, generality, precision, since teachers would have varying strengths and preferences across the mathematical knowledge. Teaching routines, and teachers' knowledge and beliefs with varying sources and degrees of justification would exert different impacts on teaching, mathematics teacher preparation and professional development. Therefore, it should not only aim at strengthening teachers' mathematical knowledge for teaching, but also help teachers to develop the insights, skills, and flexibilities that are needed to make well-informed, thoroughly-reasoned, and balanced decisions in promoting mathematical proficiency among all mathematics learners (Moeti, 2016).

The current study, looked into an individual mathematics teacher's daily instructional activities in actual school and classroom settings to search for traces and indications of strategies used. Through this sequential mixed method explanatory study, the researcher generated richer, more subtle, and more authentic details of teacher's strategy that is demonstrated in and has influence on routines reasoning in teaching concepts in quadratic equations and functions. It was noted that teaching strategy influenced mathematics performance. However, performance in Kericho County has not been satisfactory compared to other counties in the country. For instance Kericho West Sub-County, no school appeared in the top ten in the KCSE in the year 2016, (MoE, 2016).

Education stakeholders and political leaders have noted this with a lot of concern. Some of the studies which have been conducted in the County include Mathematics teachers' perceptions on single and co-education schools based on KCSE results by (Barmao, Changeiywo, & Githua, 2015a). They found that Mathematics teachers' perceptions of their classes are positive irrespective of the class gender composition and no differences in the perceptions between single sex and mixed sex classrooms in both the sub county and county schools.

In solving quadratic equations and functions, learners must choose and employ a correct technique in order to get a correct solution. The correct and incorrect technique commonly employed by learners to solve problems includes changing the subject of a given formula, factorizing quadratic expressions and solving quadratic equations using the formula Yahya and Shahrill, (2015). Some of the concepts include the failure to manipulate operations correctly in changing the subject of a given formula, the incorrect selection of multiplication factors in the factorization of quadratic expressions, and the inability to recall correct quadratic formula in solving quadratic equations. There were other psychological factors noted as well, such as carelessness and participants' lack of confidence in answering the questions. Learner's performance was compared with gender and school type in order to describe their influence.

In Kericho County, parents have realized the importance of educating their children, and this was accelerated by the government when it introduced free tuition fees in primary and secondary schools. Therefore, most parents who could not have afforded fees for boarding schools have alternative choices for their children. Students attend all types of school like boys, girls, and mixed schools among other categorizations. Studies have been conducted to find out whether there is significant difference in student's performance in single sex and co-education schools. In recent years, there has been a belief in Kenya that girls are better off socially and academically in girls' only schools than in coeducational schools. Barmao, Changeiywo and Githua, (2015) found that boys enjoy co-educational environments, teachers find boys respond well in coeducational environments, and a number of key researchers and commentators identify the important role of diversity within schools and that this diversity includes, amongst a number of things, that offered by co-educational contexts. Moreover, motivation and achievement data clearly show that there is a great deal of overlap amongst boys and girls suggesting that inclusive and integrated contexts are not inappropriate for the bulk of the learner body.

Lower-cognitive quadratic equations and functions are more effective in promoting learners' performance, no difference between the two types of questions and their impact on learners performance, teachers' use of higher-level questions lead to greater learners performance and that there was little support for higher-level questioning enhancing learners performance (Shahrill & Mundia, 2014). In recommendation teachers should use both lower- and higher-cognitive questions in their lessons as this will enable students to review basic facts and skills and higher level questions to develop learners' critical thinking ability and skills. Therefore, questions was designed based on Bloom's cognitive domains of learning factoring in low-level and high-levels questions of quadratic equations and functions in order to determine whether there exist any difference in performance (Gall & Rhody, 1987, as cited in Shahrill & Mundia, 2014).

A considerable number of studies which have been conducted in Kericho County on mathematics performance in general based on KCSE examinations, attitude, and comparing performance of boys and girls in general. Learners' knowledge of solving techniques, connections, and justification of their answers was inquired which is the basic threshold concept in quadratic functions and equations. Hence, the objectives of the research study were twofold: one was to determine learner comprehension so that these sources can be eliminated through properly organized instructional methods, and to predict performance based on strategies teachers use, gender and school type.

1.3 Statement of the Problem

In Kenya, Mathematics is a core subject up to secondary school level of education. Every student has to take Mathematics as one of the subjects of study. It is for this basic reason that the Kenya government through TSC employs trained and experienced Mathematic teachers to teach in secondary schools. Therefore, learners should be able to comprehend all the threshold concepts in Mathematics since it is viewed as a gatekeeper to success in higher education, college preparatory and many career paths. In this regard, Gamoran and Hannigan (2000 cited in Chasanah, Zulkardi, & Darmawijoyo, 2015) claimed that Mathematics benefits all learners, regardless of their Mathematical abilities.

In contrast to the prominence of secondary school quadratic equations and functions, learners are reported to lack comprehension on threshold concepts. The Ministry of education, (MOE, 2015), while releasing the results of 2014 candidates, reported that the number of candidates with grade A went down from 3,073 in 2014 to 2,636 in 2015 and only 141 for 2016 candidates. However, Mathematics was among the subjects in which candidates performed poorly. The candidates who scored D+ and below were 40% (209,807 out of a total 525,802) in 2015, but in 2016 performance D plain and below were 60% (346,252 out of 577,000 candidates) which implies that the candidates scored less than 34% on average in the seven subjects, Mathematics included. Jupri, Drijvers and van den Heuvel-Panhuizen, (2014) noted that lack of learner comprehension and performance in quadratic equations and functions are related to how it is being taught the threshold concepts in secondary school. In this case, the common way of teaching often leads to learners' comprehension of variables, the arithmetic processes and the strategies to solve the algebraic problems.

In the teaching and learning of quadratic equations and functions, researchers have not yet developed a research-based framework identifying and describing learner's comprehension on threshold concepts in quadratics equations and functions. While individual teachers may have intuitive ideas of the comprehension their learners might encounter and leverage, there is no framework that can be used to make instructional curriculum development decisions, (Nielsen, 2015). In order to support secondary school Mathematics teachers, it is imperative to document these threshold concepts. Based on the aforementioned discussion, the present study posed the following research question:

What is the learner's comprehension and performance on threshold topics in Mathematics?

1.4 The Purpose of the Study

The purpose of this study was to establish the learner's comprehension and performance on threshold topics in Mathematics in Kericho County public secondary schools.

1.5 Specific Objectives

i. To discuss the influence of teaching strategy on learners' performance in quadratic equations and functions with one known.
ii. To describe learners' score performance in solving quadratic equations and functions with one known.
iii. To analyze learner's threshold concepts in solving quadratic equations and functions with one known that may attribute to gender.
iv. To determine gender difference if any, that may exist in the cognitive level and school type performance of quadratic equations and functions with one known.
v. To determine relationships if any between gender, teaching strategy and school type on one hand and performance in quadratic equations and functions with one known on the other hand.

1.6 Research Questions

In order to answer the research objectives (ii) and (iii) above, the following subsidiary questions in relation to students' threshold concepts in solving quadratic equations and functions with one known were utilized for the qualitative study:

i. Which strategy do teachers use that gives satisfactory performance in quadratic equations and functions with one known?
ii. What are the learner's score performance in quadratic equations and functions with one known?
iii. Which Mathematics connections do learners make in solving quadratic equations and functions with one known?

iv. Which methods of solving quadratic equations and functions with one known do learners use and how do they interpret the solutions?

v. What threshold concepts do learners face when solving quadratic equations and functions with one known?

1.7 Research Hypothesis

In order to respond to the research objectives (i), (iv) and (v), the quantitative research technique tested the following non-directional hypotheses:

H_{O1}: There is no significant difference in teaching strategy on learners' performance in solving quadratic equations and functions with one known.

H_{O2}: There is no significant difference in the cognitive level and school type performance of quadratic equations and functions with one known based on gender.

H_{O3}: There is no significant relationship between gender, school type and teaching strategy on one hand and performance in quadratic equations and functions with one known on the other hand.

1.8 Justification of the Study

The concept of quadratic equations and functions was chosen for this study for several reasons. Quadratics is often the first mathematics subject that requires extensive abstract thinking, a challenging new skill for many secondary school learners. Quadratics moves learners beyond an emphasis on arithmetic operations to focus on the use of symbols to represent numbers and express Mathematical relationships. Improving the teaching and learning of quadratics requires an instruction that moves students beyond superficial Mathematics knowledge and toward a deeper understanding of quadratics. Quadratic equations has many uses in career-related professions such as business, engineering and science where the concept is used for modeling ideal situations. In business it may be used to help in forecasting profit and loss. The U-shape of a parabola is incorporated in science in the construction of structures such as the parabolic reflectors of satellite dishes and car head lamps.

On crucial matters of security for instance in defense, military generally uses quadratic equations to help them target objects that are flying through the air. It is important to pinpoint where their artillery will fall or hit their target, so they use quadratic equations. This can be used for airplanes, bullets, missiles, and tanks. Police will use quadratic equations to figure out the trajectory of bullets and the speeds of cars. These are two very important things: if someone has been hurt by a bullet, the police need to know where the bullet was fired from and how fast it was going. This will help them figure out what type of gun was used and where the suspect was when they fired the gun. It is also important to be able to tell how fast a car was going, especially if there was an accident.

Engineers also use quadratic equations, along with many other types of advanced Mathematics. If they are designing an object with a curve, for example, a quadratic equation may be used to ensure that the piece is made properly. Almost every type of engineer uses quadratic equations, including automotive engineers, electrical, chemical, audio, and computer engineers. Many of the sciences also use quadratic equations, including astronomers, chemists and physicists. Additionally, agriculturists use these equations to produce bigger fields with the materials they are given, as well. Several people like managers and clerical staff requires quadratic equations. For example, production and engineering managers must know how to do these equations, because they have to check the work of the engineer or production employee that did the actual equation. If the work is incorrect and is not noticed by the manager, the product will be made incorrectly.

Quadratic equations and functions can also be used to determine how long products such as household appliances will last. These appliances become less safe over time, and it is important for manufacturers to have an idea of how long these appliances will work properly, so that they may include this information in user manuals. The quadratic equation can also help developers of products. If a new product is being created, the type of profit expected should be known. By using a quadratic equation, it can ultimately be figured out the profit based on the amount of money used to make and advertise the product, along with how many units made. The same concept is true if for example, to create a product that is twenty percent larger than it was previously. When a manufacturer claims that a product now offers 20 percent more, or is 15 percent larger, quadratic equations can play a big part in determining the new size of the box or container. Property surveyors must also use the quadratic equation, if they need to know the area of a property. If every piece of land is a perfect square, there wouldn’t be any trouble, but property isn’t divided into perfect squares.

Quadratic functions are equations containing a squared term. When plotted, a quadratic function describes a curve called a parabola. The U-shape of a parabola can describe the trajectories of water jets in a fountain and a bouncing ball, or be incorporated into structures like the parabolic reflectors that form the base of satellite dishes and car headlights. Quadratic equations and functions help forecast business profit and loss, plot the course of moving objects, and assist in determining minimum and maximum values. Most of the objects used in every day, from cars to clocks, would not exist if someone somewhere hadn't applied quadratic functions to their design.

1.9 Significance of the Study

The significance of this study lies in the fact that the purpose of the terminal examination in Kenya is twofold. Firstly, it aims at producing creative and problem solving individuals who can identify opportunities in their environment for self-reliance. Secondly, examinations are used as criteria for selection and elimination of learners, for further education and training as well as for job placement. In this respect therefore, the examination is expected to test various levels of cognitive domain as well as exhibit a deliberated bias in higher levels of thinking. This is supposed to be the case because form four learners are in formal operational stage and can apply logic more abstractly; hypothetical thinking develops (Bloom, 1956). This was undertaken by comparing performance in quadratic equations and functions along low and high taxonomic levels. Comparison was based on male/female paradigm and school type. The study identified specific cognitions which need to be strengthened, helped and reduce performance gaps by identifying where differences in performance occurred and why. It provided information to curriculum development institutes and textbook designers so that they can improve the way concepts and principles are identified and explained. It also provided information to curriculum developers, trainers and Mathematics teachers on specific content areas that need to be strengthened during teaching. Thus, if researchers can know and describe the learner's comprehension and performance on threshold concepts in Mathematics in detailed way, it would be easier for teachers and researchers to design effective strategies to improve learner's comprehension and performance.

1.10 Scope of the Study

Many factors influence learner's academic performance, but this study focused only on teaching strategies, gender and school type (boys, girls, or mixed schools) and the students' difficulties in solving quadratic equations and functions common which affects their performance. This was because there are suggestions that boys are more advantaged than girls since Mathematics is said to be boys friendly. This study only investigated teaching strategy, gender and school type performance and questions along the cognitive domain of learning (low and high levels) with respect to the performance of quadratic equations and functions. The Mathematics national examinations system is not structured to test affective and psychomotor domains of learning. The target population was form four learners from public secondary schools from Kericho County who were selected because they have completed the whole content of quadratic equations and functions, and their responses would be a reflection of what they have learned. Remarkably, since they had come to the end of secondary school Mathematics course and to establish whether the objectives set out in the syllabus on the learner's comprehension and performance on quadratic equations and functions with one known would have been achieved.

1.11 Limitations of the Study

This study was carried out in Kericho County, which is one of the 47 Counties in the republic of Kenya. Consequently, only a few selected schools and form four learners were used to make generalization and conclusion. Secondly, effective learning of Mathematics in secondary schools depends upon several factors, including learner related factors such as gender, level of intelligence, attitude, environmental factors such as adequacy of learning resources, teacher related factors such as mastery of the subject matter and the school status/category such as county and sub- county, day and boarding schools or boys, girls and mixed schools in Mathematics.

There are so many research approaches in research like qualitative and quantitative but the study took a sequential mixed explanatory approach. Therefore, more information could have been found if a pure qualitative or quantitative research design could have been employed. The research instrument contained only 9 questions, more questions could cover more content on threshold concepts on quadratic equations and functions and richer information could be documented.

1.12 Assumptions of the Study

The study was undertaken with the following assumptions:

a. Teaching strategy influence performance in quadratic equations and functions.
b. Threshold concepts are attributed to gender.
c. Gender, school type and teaching strategy linearly affects learners' performance in quadratic equations and functions.

1.13 Theoretical Framework

This study was based on Piaget's theory of learning which states that cognitive structures are patterns of physical and mental actions that underlies specific acts of intelligence and corresponds to stages of child development. Piaget (1957) proposed that children develop knowledge by inventing or constructing reality out of their own ideas about how the world works. Piaget viewed intelligence as the individual's way of adapting to new information about the world. They do this through a process of equilibration, which means balancing assimilation (fitting reality into their existing knowledge) and accommodation (modifying schemas to fit reality).

When learners are asked to solve a Mathematical problem, they immediately performed operations without thinking carefully about what the problem was asking for and whether the operations were appropriate. As a result, their answers frequently do not make sense. So, students are often forced to examine their reasoning and to connect the problem with their concrete experience before they can see their errors. Novice learners quickly select a solution strategy and then spent all their time executing it, rarely stopping to evaluate their work to see if it is leading to the goal. Lacking self-monitoring and self-regulation, they waste much time on “wild goose chases”. Even when they have adequate mathematical knowledge to solve the problem, they are unable to activate it constructively.

This study targeted form four learners, whom by Piaget’s theory are in the final stage of cognitive development. Consequently, the learners should possess the qualities and characteristics of this stage by having a good comprehension on threshold concepts as this reflects high order reasoning. Therefore, mental skills running through the six taxonomic levels are tested this study alongside the Piagetian theory so that the intellectual maturity of the learner is nurtured. Therefore, much of the examination questions should test the six levels; more emphasis ought to be placed on the application, analysis, synthesis and evaluation. The diagnostic test questionnaire contains questions constructed with the guide of Bloom’s and Piaget’s theories. This enabled the researcher to make judgments on the learners’ comprehension and performance on quadratic equations and functions.

The implication of this advanced theory is that, children learn Mathematics by doing, through experiences that should be presented to the learner in a systematic manner. The teacher therefore should introduce the learner to the new concept by relating them with previous related concepts. Secondly, the learner should be exposed to a variety of experiences that will help to master and widen his experiences about the new concept. For this particular theory the existing knowledge in every cognitive level is very important since the learner is either modifying the already existing or formed via experiences.

In support of the Piaget's cognitive theory of learning, Vygotsky, (1978), developed the social cultural theory, a new framework for conceptualizing educational dialogues, through which learners acquire new modes of handling knowledge and solving problems. Passing the child along the meaning that their culture assigns to objects and events and assisting them with the challenging task can promote their cognitive development. The argument is that child development first occurs through the social interaction. Second, development occurs within the learner from the knowledge, skills and experiences he/she has acquired through social interaction in the environment surrounding him or her. The basic premise of Vygotsky's theory is that all uniquely human's higher forms of mental development starts from social and cultural contexts which are shared by members of that context because those mental processes are adaptive.

The next aspect of Vygotsky's cognitive development is the Zone of Proximal Development (ZPD) that is the "distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers". Vygotsky further argued that a person cannot fully understand a learner's developmental levels without determining the upper boundary of that development.

The Vygotsky's ZPD describes how cognitive growth occurs in learners, rather than considering a learners' potential in terms of a static measure such as an IQ[2] score. Vygotsky felt that a developmental measure is needed to better assess learners' educative potentials. Thus, ZPD provides a conceptualization of how developmental potentials might be understood.

Vygotsky offered stages of concept development and regarded thing as concepts if they consist of ideas and parts of ideas that are linked together and to other ideas by logical connections that form part of a "socially-accepted system of hierarchical knowledge". The stages are:

a) Syncretic Heap Stage

In syncretic heap stage, a learner groups together ideas or objects that are grouped together on a page in a textbook, or because they are discussed on the same day in class, but this grouping takes place according to "chance, circumstance or subjective impressions in the learners' mind" (Nielsen, 2015, p. 23).

b) Complex stage

In complex stage Vygotsky describes it as a stage in which a learner begins to "unite homogeneous object in a common group, to combine them in accordance with the objective connections that the learner finds in the things themselves" (Vygotsky, 1987). The task of the learner is to decide if, how and/or why certain objects or ideas go together and what overarching idea unites them. As the learner matures, s/he begins to rely more on the characteristics of the objects themselves. In this stage, learners begin to link ideas together by associations or common attributes between the items. The learner begins to notice or abstract different attributes of the idea or object and starts to organize ideas that share particular properties into groups, creating a basis for more sophisticated generalizations that will come later. In this stage, the learner does not use standard Mathematical logic, but relies on "non-logical or experimental association". Berger, (2005 as cited in Nielsen, 2015) points out that this type of complex thinking may manifest in what is called "bizarre or idiosyncratic usage" of Mathematical objects, concepts or signs. The Mathematical signs and symbols learned in this stage will help the learners in communicating about them which gives them the opportunity to talk with others and even their teachers.

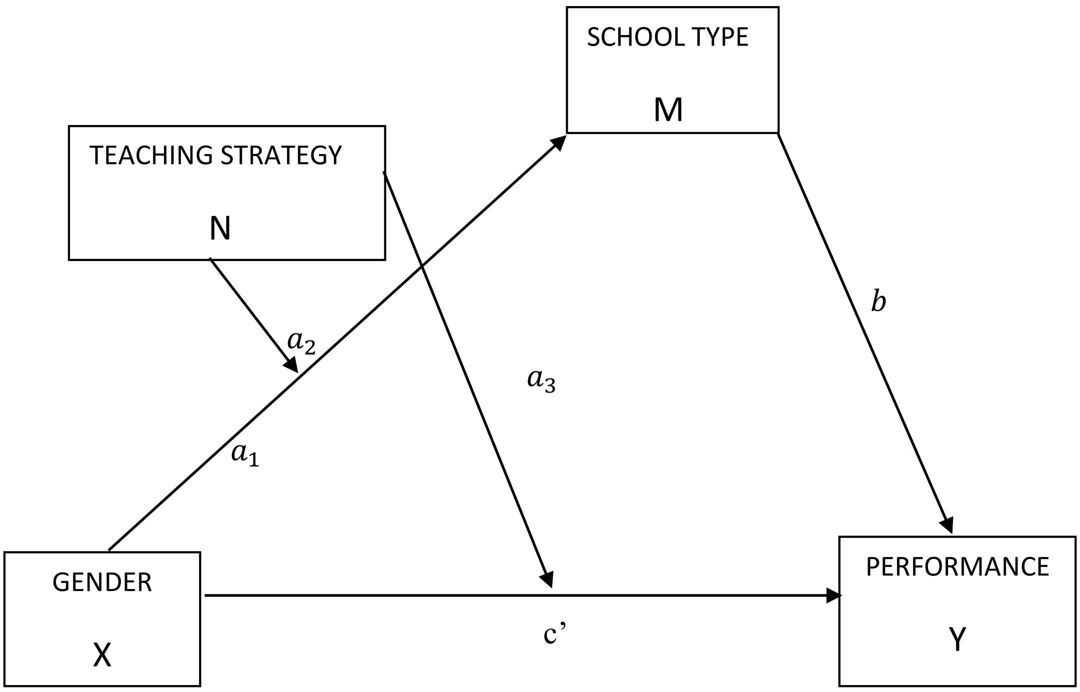
c) Pseudoconcept Stage

Pseudo concept is a stage which comes before learner forms a concept which Vygotsky envisioned it as a bridge to concept formation (Vygosky, 1987). Pseudo concepts resemble true concept in their use, but the thinking the student is doing is still complex in nature. The learner can use the mathematics without being able to understand what s/he is doing or explain it. At this point in the development of understanding, the learner can talk through the mathematics with the teacher and other learners and through these conversations, interventions and being engaged in meaningful problem solving situations the learner forms concepts.

The role of prior knowledge and learner understands is one of the main tenets of constructivist learning that everything a person learns is built upon what the learner already knows and understands. People continually try to understand and think about the new in terms of what they already know (Nielsen, 2015). Learners come to formal education with a arrange of prior-knowledge, skills, beliefs and concepts that significantly influence what they notice about the environment and how they organize and interpret it and therefore, the logical extension is that teachers need to pay attention to the incomplete understandings, the false beliefs and the naïve renditions of concepts that learners bring with them to mathematics classes.

1.14 Conceptual Framework

In Kenya, the major yardstick used to measure educational output is performance in examinations. This output, however, is achieved after the various inputs into the educational process undergo what is referred to as educational production process. The inputs into the educational production process include the teaching strategy, gender of the learner and the school type. Thus, the educational output, in this case denoted by performance, is a function of how these educational inputs interact. If the interaction is favorable then output (performance) should be satisfactory and vice versa. This study sought to establish the kind of interaction taking place in schools with regard to teaching strategy, gender and school type based on the learners' comprehension and performance on quadratic equations and functions with one known. Figure 1.1 depicts all the essential prototypes of variables influencing learners' comprehension and performance on threshold concepts in quadratic equations and functions and how these variables relate.



Independent Variable **Direct Effect** **Dependent Variable**

Figure 1.1: Conceptual Model for the Moderated Macro Process

Figure 1.1 is an intervening variable model; variable X (gender) in the study was postulated to exert an effect on an outcome variable Y (performance) indirectly through M and N intervening variables (moderators). The moderated model focused on the estimation of interactions between the moderators and the pathways that define the indirect effect ($a\ and\ b$). This model conceptualizes an interaction between X and a moderator variable N on Y as carrying its influence through an intervening variable M. Therefore, the path from X to M is moderated by a third variable N, whereas the path from M to Y is un-moderated. Hence, the study focused on the estimation of the indirect effect of the product of X and N on Y through M.

1.15 Operational Definition of Terms

The following terms ran through the study and therefore needed to be defined as they were used.

Cognitive domain: Involves knowledge and the development of intellectual skills in performing mental skills and are arranged hierarchically in order of complexity.

Critical thinking: Ability of a learner to be imaginative, objective and pragmatic in solving problems and disappointments so as to arrive at unbiased solutions.

Function: A relation for who each value from the set the first components of the ordered pairs is associated with exactly one value from the set of second components of the ordered pair.

Gender: Refers to the learner being either male or female.

High order questions: Based on Bloom's (1956) taxonomies of cognitive objectives which refer to questions testing on the third, fourth, fifth, and sixth levels.

High order thinking: Instances in which a learner takes new information in memory and interrelates and/or rearranges and extends this information to achieve a purpose, comprises of application, analysis, synthesis, and evaluation.

Low order questions: Refers to questions testing on simple recall and knowledge.

Performance: Refers to the quadratic concept formation and expressed by learners' scores in an examination.

Teaching Strategy: method a teacher uses in teaching quadratic equations.

Threshold topics: topics with fundamental understandings that sit at the heart of a body of knowledge.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter explicates the reviewed literature on learner's comprehension and performance of threshold concepts and teaching strategies employed in quadratic functions and equations. Mathematics gender performance, school type performance, low and high cognitive levels performance of quadratic functions and equations as illustrated by the research literature map in appendix F.

2.2 Quadratic Equations and Functions with one Known

It is commonly believed that the first true algebra text is the work on Al-jabr and Al-muqabala by Mohanmmad ibn Musa al-Khwarizmi (780-850), written in Baghdad around 825, Katz (2007) cited in (Amidu, n.d.). The word algebra came from the title of this work. The word al-jabr means restoration or reestablishment that is to eliminate negative terms from the through adding the same terms to both sides of equations. The word of Al-muqabalas means balance, meaning to divide every term in an equation by the coefficient of the terms (Amidu, n.d.)The nth-degree equation in *x* is called a polynomial; a polynomial of second degree is called a quadratic. Thus $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots\ldots\ldots a_1 x^{n-1} + a_o x^0 = 0$ where a_n, a_{n-1}, a_{n-2}, a_1, a_0 € R are the coefficient of x^n, $x^{n-1,}$ $x^{n-2} \ldots x, x^0$ respectively.

Mu'awiya, (2013), while conducting a study on analysis of problem-solving difficulties with quadratic equations among senior secondary schools students in Zaria, Nigeria, used the Jackson-Ashmore model. A total of 126 Senior Secondary 2 Mathematics learners randomly selected from three private schools in Zaria with a mean age of 17 constituted the sample size for the study. The Mathematics achievement tests (MAT), Mathematics competence test (MCT) and Problem-Solving Test in Quadratic equation (PSTQ) were used for the study. The learners were classified as high achievers and low achievers using the categorization test score designed by the investigator. Data were analyzed using facility values (FV), mean, simple percentages (%) and chi-square statistics. The findings from the study showed that learners performed poorly in Mathematical problems involving quadratic equations.

Nielsen, (2015), while conducting a study entitled “understanding quadratic functions and solving quadratic equations: An analysis of learner’s thinking and reasoning” sought to learn what high school learners who have completed an Algebra 2 or Pre-calculus class understand about quadratics. This qualitative study employed cognitive interviews of 27 learners in grades nine through eleven. This study took place in a high school in the northwestern United States. Learners in this high school take a series of mathematics courses starting with Algebra 1 in 7th, 8th or 9th grade and then continuing with Geometry, Algebra 2, and possibly Pre-alculus and Calculus. Most learners at this school complete at least Algebra 2. The selection of classrooms and schools was a purposeful convenience sample. Moreover, several Mathematics teachers in the circuit where the study was conducted complained that learners were not performing well in this topic during examinations. Very little is known about learners’ understanding of the behavior of quadratics and how the graphs and equations of quadratic functions are related.

However, the learner’s comprehension and performance on threshold concepts in quadratic equations and functions in Kenya has not been adequately studied in order to establish learners’ difficulties so as to arrest the situation. A very common and easy-to-understand application of a quadratic function is the trajectory followed by objects thrown upward at an angle. In these cases, the parabola represents the path of the ball (or rock, or arrow, or whatever is tossed). If distance on the *x*-axis and height on the *y*-axis is plotted, the distance of the throw will be the *x* value when *y* is zero. This value is one of the roots of a quadratic equation, or x-intercepts, of the parabola.

2.3 Teaching Strategies used in Quadratic Equations and Functions

In teaching quadratic equations and functions, learners often give the appearance of having understood, because they believe they have understood. Their teachers also believe they have understood only for some learners to fail spectacularly when confronted by threshold concepts in an examination questions. In order to master a threshold concept, the theory suggests that learners may travel through a tunnel or 'liminal space' where they 'get stuck' and may be in a state of uncertainty. Threshold concept theory proposes that there are a number of concepts that are central to the mastery of quadratic equations and functions, as originally described by Meyer and Land (2003 as cited in Hoadley, Wood, Kyng, & Tickle, 2015). These concepts have five key characteristics:

Characteristic	Description	Example
Transformative	A shift occurs in the learner's perception. New comprehensions are assimilated into a learner's biography, becoming part of which they are, how they look at a problem.	In a quadratic equations and functions students learn the square in the variable implies 2 solutions; a positive and a negative. For instants, $2^2=4$ and $(-2)^2=4$ so 2 and -2 are solutions.
Irreversible	Once understood the learner is unlikely to forget it. New patterns and connections are recognized and earlier patterns of comprehension cannot easily be retrieved. Subsequent variation or a rejection of the new concept is still possible	In quadratic equations and functions learners are required to understand a standard form of the equation in order to identify a correct method of solving. Reflective and critical thinking become inherent in getting the solution.
Integrative	Exposes inter-relatedness, enabling students to coherently integrate what were previously seen as unrelated aspects of the subject. Things start to click into place	In solving quadratic problem by factorization, zero products are a threshold concept. This calls upon their knowledge of the roles of other algebra like multiplication.
Bounded	Bordering with other thresholds or new conceptual spaces. The more interdisciplinary a subject, the more complex this will be	In a quadratic word problem like question 9 in the questionnaire, learners require graphical knowledge, time, scale drawing and tangents.
Troublesome knowledge	Difficult to grasp concepts - this may include concepts that: clash, compete or interact; appear illogical, unfamiliar or alien; counter-intuitive and initially very difficult for learners to accept	In a word problem like question 6 the learners' desire for the 'correct answer' by linearizing is counter intuitive to the modeling process that emphasizes alternatives used to support problem solving which is Pythagoras theorem.

Concerning teaching and learning, Swan, (2006a) conducted a study whose aims were to help learners to adopt more active approaches towards learning and to develop more challenging, connected, collaborative orientation towards their teaching. The research results show that many learners view mathematics as a series of unrelated procedures and techniques that have to be committed to memory. Instead, they are required to engage in discussing and explaining ideas, challenging and teaching one another, creating and solving each other's questions and working collaboratively to share methods and results.

According to the report by Kesianye, Durwaarder and Sichinga (2001, as cited in Mamba, 2012) the traditional, formal approach to teaching quadratic equation is to look at it as a purely mathematical discipline with no emphasis in linking quadratic to day-to-day circumstances. At the end of the quadratic course, students will have done quadratics without really realizing a necessity for it, resulting in numerous difficulties. The emphasis here is doing mathematics recognizing connections and modeling real-life situations and noting that algebra is a powerful tool in the hands of the learners on conditions that they understand its uses and the limitations of the tools at hand.

Traditional, 'transmission' methods in which explanations, examples and exercises dominate do not promote robust, transferrable learning that endures over time or that may be used in non-routine situations. They also demotivate students and undermine confidence. In contrast, the model of teaching we have adopted emphasizes the interconnected nature of the subject and it confronts common conceptual difficulties through discussion. We also reverse traditional practices by allowing learners opportunities to tackle problems before offering them guidance and support. This encourages them to apply pre-existing knowledge and allows us to assess and then help them build on that knowledge. This approach has a thorough empirically tested research base (Swan, 2005).

Teachers in schools use different strategies while teaching quadratic equations and functions with one known for instance group work, exposition and explanatory, problem solving, practical work, direct instruction; this may be the application of constructivist approach in teaching and learning of Mathematics. The key element in the approaches is for learners to use mathematical skills and knowledge confidently in solving Mathematical problems. Learners are evaluated thereof to on their understanding of the concepts in mathematics and the quality of teaching, and this helps to reveal learners' errors and misconceptions (Borasi, 1994 & Riccomini, 2005, as cited in Makgakga, 2016a).

Makgakga, (2016a) while citing Sorensen, (2003), reported that teachers should know their learners' Mathematical thinking to be able to structure their teaching of new ideas to work with or correct those ways of thinking, thereby preventing learners from making errors. The way learners think about a concept depends on the cognitive structures students have developed previously (Battista, 2001 as cited in Makgakga, 2016a). Consequently, if learners cannot develop concepts by themselves, they will have a narrow understanding of those specific concepts, and will not be able to engage themselves in problem solving. Learners who do not have background knowledge in mathematics usually display numerous errors in solving mathematical problems, and this therefore results in most of learners grappling with quadratic equations by completing a square. Conceptual knowledge works hand in hand with procedural knowledge.

Sibuyi, (2013) conducted a qualitative research approach which focused on effective teachers' pedagogical content knowledge in teaching quadratic functions in mathematics using a case study method The research site was located in a certain school circuit of the Mpumalanga Department of Education in South Africa. The population for this study comprised all mathematics teachers who came from schools that had obtained an average of 80% overall pass rate and an average of 80% in the National

Senior Certificate Grade 12 mathematics examinations for the past three (or more) consecutive years.

The results of the study found that TB demonstrated that he has insufficient knowledge of teaching strategies to teach quadratic functions in grade 11. Furthermore, as TB asked questions, he pointed at learners who raised their hands and mostly left out those seemed to know the correct answers only and did not engage those who did not. He mainly used the lecture method where he was observed as the main imparter of information to learners to present his lessons.

In summary, the two teachers; Teacher A and Teacher B, used the telling method to present most of their lessons on quadratic functions. Teacher A, usually asked recall type of questions during lesson presentation whereas Teacher B sometimes posed questions that required his learners to speak out their mathematical thinking regarding topics on quadratic functions. The two teachers assessed their learners at the end of each lesson and gave them an additional opportunity to learn the concepts through home work.

Shulman, (1987) in an article on knowledge and teaching: Foundations of the new reform builds his foundation for teaching reform on an idea of teaching that emphasizes comprehension and reasoning, transformation and reflection. To articulate and justify this conception, Shulman responds to four questions: What are the sources of the knowledge base for teaching? In what terms can these sources be conceptualized? What are the processes of pedagogical reasoning and action? Lastly what are the implications for teaching policy and educational reform? The answers — informed by philosophy, psychology, and a growing body of casework based on young and experienced practitioners — go far beyond current reform assumptions and initiatives.

Gess-Newsome, (1999) explored a conceptual framework to describe and analyze the challenges around preparing teachers to create, sustain, and educate in a ‘community of learners’. This conception allows us to understand the variety of ways in which teachers respond in the process of learning to teach in the manner described by the ‘Fostering a Community of Learners’ (FCL) programme. The model illustrates the ongoing interaction among individual student and teacher learning, institutional or programme learning, and the characteristics of the policy environment critical to the success of theory-intensive reform efforts such as FCL. An accomplished FCL teacher is not only ready, willing, and understanding of FCL teaching, but he or she is also able to perform this kind of teaching, which is enormously complex in its practice. An accomplished professional is not only someone who is inspired, enlightened, and motivated; he or she must also be skilled in the varieties of practice.

Gess-Newsome, (1999) further noted that, FCL teaching makes great demands on the performance of teachers in the design and adaptation of curriculum, the management of multiple rotations occurring simultaneously in classrooms, the formal and informal assessment of complex understandings and processes among diverse learners, the integration of deep disciplinary understanding with sustained motivation and interactions among learners, the uses of technology in the everyday life of the classroom, etc. Such skill will develop slowly over time. Teacher educators and professional developers need to analyze how such skill development can be identified, fostered, measured, repaired, and sustained. This is due to the fact that learning is influenced from both internal and external forces to the learners interacting with one another. Furthermore, it is reasonable that the learning already existing in the learner, not so much the teaching that have a crucial impact on new learning. Equally, Ausubel (1968 as cited in Mamba, 2012) commended: if I had to reduce all of educational psychology to just one principle, I would say this “the most important single factor, influencing learning is what the student already knows. Ascertain this and teach the learner accordingly.”

According to Brodie (2007), the new curriculum that has been recently introduced into South African schools calls for learners to participate in mathematics lessons and to express their mathematical ideas. Teachers are encouraged to make their lessons more learner-centered by encouraging learners to contribute to the lesson. The choice of the instructional strategy to be used by the teacher is very important. Different lessons require different teaching methods, (Li, 2011) while conducting a class observation described the sequence of Mathematical practices that a teacher designed and enacted during three consecutive lessons about four algebraic routines for solving quadratic equations, and focuses on the Mathematical knowledge that is entailed in the teacher’s actions and decisions.

According to Shulman, (1999), reported that the correct choice of such an instructional strategy does not depend on the teachers’ knowledge of the subject matter only but also on the teacher’s knowledge of the learners’ level of understanding. Since this research investigated the use of instructional strategies during lessons, it is important to know what “good” teaching strategies the teacher used in mathematics teaching.

Lima, (2008) in his study of the characteristics of Mathematics teaching in Shanghai, noted that the success of a teacher in teaching a specific Mathematics topic depends on the depth and breadth of the individual teacher’s pedagogical content knowledge because, prior to the commencement of a lesson, a Mathematics teacher needs to: plan the lesson, choose a teaching strategy and select content that will suit the learners’ level of understanding. These three activities are all assumed to be elements of pedagogical content knowledge. Teachers with a sound knowledge of the elements of pedagogical content knowledge, always select teaching strategies that are appropriate for the level of development of their Mathematics learners. Cockburn, (2008) asserts that, although content knowledge is central to an educator's effectiveness in teaching mathematics, the method of teaching plays an equally important role if any learning is to take place. In the case of this study the teaching strategies that the effective teachers used when they taught quadratic functions was also investigated.

Tanner (2003) posits that good instructional strategies should: actively engage the learners, assist them in using their prior knowledge and skills to solve problems in mathematics, motivate the learners to participate during the lesson; and also create an appropriate learning environment. According to Ingvarson, Beavis, Bishop, Peck and Elsworth (2004) excellent teachers of mathematics are aware of a wide range of effective teaching strategies and techniques for teaching and learning mathematics that promote the learners' enjoyment of the subject. Furthermore, such teachers usually choose teaching strategies that tend to create the best learning experience for every learner. The pedagogical content knowledge of teachers according to De Miranda (2008) involves:

> .. knowing how to take advantage of different teaching approaches that make a learning experience most appropriate for the learners. This includes being flexible and adjusting instruction that takes into account various learning styles, abilities and interests. Knowing how to best teach a concept so that the learners will receive the best learning experience speaks to the essence of PCK. The different teaching approaches employed will vary from teacher to teacher and in differing contexts, but invariably will revolve around similar principles for each approach.

Westwood, (2004) asserts that, "studies have indicated that although expert teachers differ in their actual style of teaching and management, they all use instructional strategies that maximize students' time and engagement in learning tasks and encourage students' active participation during lessons. In addition, they ensure that students understand the work they are required to do; and, they set tasks and activities at the right level to ensure high rates of success. Expert teachers also create a positive and supportive classroom environment, they are good managers of behaviour; and are skilled in motivating learners to learn". This study too, investigated how the teachers used their teaching strategies to benefit the learners. The teaching strategy that the participating teachers used during lesson presentation was investigated by checking the method used such as telling method, group work and self-discovery teaching method.

Baumert et al., (2009), in their study involving teachers' Mathematical knowledge, cognitive activation in the classroom and learner progress, mention three components of instructional strategies that are crucial for initiating and sustaining insightful learning processes in mathematics lessons. These three components are: Cognitively challenging and well-structured learning opportunities, learning support through monitoring of the learning process and individual feedback and adaptive instruction, and efficient classroom and time management.

Still on teacher factors, Baumert et al., (2010) posit that the pool of alternative mathematical representations and explanations given by teachers to learners in the classroom are largely dependent on the breadth and depth of the teachers conceptual understanding of the subject, and that insufficient understanding of the mathematical content, limits the teachers' capacity to explain and represent that content to learners in a sense-making way. This is a deficit that cannot be offset by pedagogical skills alone. Anecdotal evidence suggests that efforts of teachers with limited conceptual understanding of the Mathematics topics that they teach fall short of providing students with powerful mathematical experiences.

This includes encouraging learners to make connections between quadratics concepts and the procedures present in problems, and helping learners recognize how the placement of the quantities relative to the operations in problems impacts the solution strategy, (Star et al., 2015). Teaching quadratic equations might be regarded as a difficult task. However, using hands on visual models can make quadratic equations accessible to learners. Some learners simply memorize procedures and formulas that lead to solutions of quadratic equations but little understanding of their meaning. Students will have a deeper understanding and a better chance of deriving quadratic formula if they are comfortable with the process of completing square. However, when this process is given only as a sequence of algebraic steps, it sometimes makes little sense to learners (Vonogradova & Wiest, 2007).

Moeti, (2016) in a research project, studied two competent, qualified, experienced secondary teachers' choice and use of examples from two contrasting South African school contexts [i.e., urban (fee-paying school) and former informal settlement (no-fee school)] using qualitative methods. The two contrasting South African school contexts [that is, urban (fee-paying school) and former informal settlement (no-fee school)] in which these teachers were teaching were purposely selected because (a) the two schools have acceptable functional administrations and high standards of governance (as determined by a Dr. Kenneth Kaunda District official of North West Department of Education and Training) and (b) the matriculation performance percentage was above the benchmark stipulated by North West Department of Education and Training, that is, 70%. The fee-paying school is situated in a mining area and was a former mining-sponsored school. The school fee is R700 per annum. The two competent, qualified, experienced teachers and their respective schools were purposively selected to learn and develop a detailed understanding (Creswell, 2012) of their choice and use of examples in their different socio-economic settings of their schools, so as to illuminate the roles of examples in those contexts.

The study found that both TA and TB chose to use spontaneous examples. TA changed a planned example that might have confused the learners. TB chose and used a spontaneous example because the learners got the planned example wrong or was difficult for the learners. He therefore chose and used an example to attend to learners' errors. This example was not a planned example but chose and used example that had the same object of learning with the difficult one. Competent, qualified, experienced teachers rely heavily on their experience and textbooks to choose and use examples at the moment of teaching. The different socio-economic settings of the two teachers (fee-paying and no-fee school) understudy seemingly do not have a bearing on their choice and use of examples. The time-table and time allocation (the length of a period) may constrain or afford the choice and use of examples. The length of the period is not affected by school's socio-economic setting but the length of a period is determined by official approaches to teaching and learning contact time. The current study considered learners to have similar socio-economic status and therefore not affected a strategy the teachers used in teaching quadratic equations and functions.

Benning and Agyei, n.d., studied the effect of using spreadsheet in teaching quadratic functions on the performance of senior high school students and seventy four (74) learners of average age 16 years in SHS 1 who participated in the study, were from 2 different high schools. The schools were purposively selected for the study to ensure that they belonged to the same category based on rankings by the Ghana Education Service. This was to ensure that all other factors that could affect the result of this study, except for the approach of teaching were held constant. One intact class consisting of

32 learners (16 males and 16 females) participated in quadratic spreadsheet supported lesson, while the other intact class consisting of 42 (25 males and 17 females) were taught the same lessons with the conventional approach and served as the control group. The question required learners to transfer their knowledge of the nature of the parabola for the algebraic function ($y = ax^2 + bx + k$). The results from the analysis showed that out of 32 learners in the experimental group who answered this question, 26 representing 81.3% were able to determine the right answer from the list of options whiles 28 out of 42 (66.7%) of learners in the control group had it correct. Apparently, the use of the spreadsheet unlike the CM, gave students greater opportunity to make links between spreadsheet formula, algebraic functions and graphs, analyze and explore number patterns which promoted their concept formation much better.

McCarthy, Sithole, McCarthy, Cho and Gyan (2016), in their study on teacher questioning strategies in Mathematical classroom discourse in two grade eight teachers in Tennessee, examined the questioning strategies used by two grade 8 teachers, selected at random, from twelve middle school teachers each handling quadratic mathematical modeling as one of their lessons in a project. Each class was videotaped over six-month period but only a section from each of the two selected classes, on quadratic modeling, was watched for about 45 minutes long for the purpose of this paper. The strategies include: probing and follow-up, leading, check-listing and learner-specific questioning. The need to develop appropriate questioning techniques is an important part of teaching and assessment for the Mathematics learner. Research studies in recent years have seen a surge of interest in the relationship between teacher questioning and students' knowledge levels; but student's level of understanding can be evaluated by teacher questioning strategies as an assessment tools. The use of alternative forms of assessment in Mathematics such as combination of questioning and observing has grown in popularity as result of the standards movement and other calls for reform in Mathematics education.

Mathematics teachers are encouraged to ask questions that help them to work together with their students and make sense of Mathematics; to learn to reason mathematically; to learn to conjecture, invent, and solve quantitative problems; and to connect Mathematics, its ideas and its applications (NCTM, 1991 cited in McCarthy, Sithole, McCarthy, Cho, and Gyan, (2016b)Verbal interactions and performance-based assessments are seen as important parts of teaching and learning process in mathematics. Questioning and explaining have also been used as important means of diagnosing students' misconceptions and error patterns in Mathematics. Teacher questioning and learner explanation have the potential of ascertaining "the nature and extent of student's knowledge about a particular domain by identifying the relevant conceptions he or she holds and the perceived relationships among those conceptions" (Ashlock, 2002, in McCarthy et al., 2016b). Teacher effective questioning and student's explanations in mathematical conversations rely on verbal communication as the primary means for eliciting this information from the participants.

McCarthy et al., (2016b), found that observing the two teachers' questioning strategies in their questioning practices, it was evident that the probing and follow-up, the leading as well as the check listing and the learner-specific questioning strategies portray Mathematics classroom discourse as a system that moves learning forward. From the findings above, Joshua and Kola brought a variety of questioning strategies to their Mathematical classrooms discourse to bring forth a world of significance to their learners. Their use of appropriate questioning strategies is important skill to develop for such Mathematics classroom discourse: The teachers used the follow-up questions, particularly, to question learners for both correct and incorrect responses; they used the follow-up strategies to specifically focus on learners' thinking and to probe learners for their right or wrong answers; the probing and follow-up as well as the learner-specific strategies were used to ascertain the nature and extent of learners' knowledge about the concept on the table for discussion - in this case "factoring quadratic equations", "the roots of quadratic equations", and "solving with and or without the use of the calculator". Furthermore, the questioning strategies helped to evaluate learners' understanding of solving quadratic equations with or without the graphing calculator.

Benning and Agyei, n.d. found that the mean gains of the learners taught with spreadsheet instructional method (SIM) shows that out of a score of 45, the mean scores of the achievement test before and after the use of the spreadsheet were 9.812 (sd = 4.610) and 27.094 (sd = 3.325) respectively. The results also showed that an overall significant product - moment correlation (r = 0.55, p = 0.001< 0.05) was strong indicating that learners' pretest score had a strong correlation with their post test scores. The substantial difference observed seems to suggest how much impact the SIM might have had on the learners' learning. Although this is expected, it is worth knowing whether the distance travelled was significant. A paired sample t-test was therefore used to test the null hypothesis at 5% significance level that H_0: Spreadsheet as instructional tool has no effect on the performance of the learners.

The results revealed that the difference in performance was significant [sig. (0.0001) <0.05]. The eta square statistic (0.956) indicated a large effect size. This is a clear indication that learners progressed in their understanding of quadratic functions after the lessons. Apparently, the use of the spreadsheet instructional approach gave learners greater opportunity to explore quadratic concepts better by helping them to make links between spreadsheet formula, algebraic functions and graphs. A second Hypothesis explored the impact of the CM approach. The mean scores of the achievement test before and after using the CM to teach were 12.405 (*sd* = 4.169) and 22.131 (*sd* = 4.170) respectively indicating an increase in the achievement test. It shows the difference between the pre-post-tests scores of the distance travelled for this group of learners.

A paired sample *t*-test at 5% significant level indicated that the difference in performance was significant [sig. (0.0001) < 0.05]. The eta squared statistic (0.812) indicates a large effect size which suggests a substantial difference in the achievement test scores obtained. This is an indication that the students' performance had increased. Although this would be expected in a normal lesson after instruction, the pronounced pre-posttest difference is worth noting. This result is an indication that a well-planned CM of teaching can improve learners' performance in learning quadratic functions.

Another observation made in the item analyses was the difficulty portrayed in solving questions that applied to real life situations especially by learners in the CM group. Only 13 out of 42 (31.0%) of the conventional method group could solve this problem correctly. Whiles more than 50% of the learners from the CM were unable to solve this question, 87.5% (28 out of 32) of the spreadsheet instructional method group showed success in solving this problem. This seems to suggest that the SIM group showed mastery of applying knowledge from lessons taught to realistic settings as compared to their counterparts in the CM.

2.4 Threshold Concepts in Solving Quadratic Equations and Functions

Threshold concepts in solving quadratic equations and functions with one known can be identified when learners used the methods of factorization, completing the square, quadratic formula or the graphical method.

2.4.1 Factorization

There seems to be agreement in the field that when solving equations, students tend to use procedures without understanding and that students have difficulties with aspects of solving quadratic equations such as factoring, applying the zero-product property, and solving equations that are not in general form (Didis & Erbas, 2015). Learners' approach to solving quadratic functions and equations prefer factoring as a solution method when the quadratic is obviously factorable, and yet, factoring can be tricky for learners, particularly when the leading coefficient does not equals to 1 (Nielsen, 2015).

Zakaria and Maat (2010), while conducting a case study that used a survey method on 30 Form Three students grouped in three different category of achievement: low, medium and high. The school has classified learners' achievement based on placement test. Three (3) male and four (4) female learners belong to high category, the medium category has 5 males and 7 females and the low category has 6 males and 5 females. The results of the study was that, most of the types of errors made by learners in using factorization to determine the root of a quadratic equation were transformation errors followed by process skill errors. Factoring can be problematic for students as claimed by (Nielsen, 2015). Some learners have difficulties with their multiplication facts, which make it difficult for them to quickly find factors for expressions in the form Kotsopoulos, (2007). These difficulties increase when the parameter *a* does not equal one (for example in expressions such as $6x^2+3x+2$ and become even more challenging when *a* and/or *c* have multiple factors, leading to many possible factor pairs in expressions such as $20x^2+63x+36$. It is worth noting that the research literature on factoring quadratics attends to factoring when *a*, *b*, and *c* are integers resulting in expressions that can be factored into binomials with integer coefficients.

Mamba, (2012) conducted a research study which sought to get a deep understanding of why learners continue to perform poorly, and what the factors are which contribute to poor performance. This research entailed a detailed error analysis of four items of the 2008 Mathematics paper 1 senior certificate examination scripts, to see the trends and patterns of written responses with regards to the types of errors made by learners. The study was aimed at investigating South African Grade 12 learners’ errors exhibited when solving quadratic equations, quadratic inequalities and simultaneous equations. The four items analyzed in the study comprised of questions from three important areas of algebra namely: quadratic equations, quadratic inequalities and simultaneous equations. The scripts were analyzed for carelessness, conceptual and procedural errors.

The learner misconceptions were discovered in learners’ work; these comprised the notions of equality and inequality, the construct of the variable, order of operations, factorization, and solution of equations instead of inequalities. From this, the researcher noted that learners' learning difficulties are usually presented in the form of errors they show. Not all the errors that learners had are the same; some errors in procedures can simply be due to learners' carelessness or overloading working memory. The results obtained indicated a number of error categories under each conceptual area, namely, quadratic equations and inequalities and simultaneous equations. Under the conceptual areas indicated above, the main reason for misconceptions seemed to be the lack of understanding of the basic concepts including numbers and numerical operations; functions; the order of operations; equality; algebraic symbolism; algebraic equations, expressions and inequalities; and difference between equations, expressions and inequalities.

Didis and Erbas, (2015) in their research on the performance and difficulties of students in formulating and solving quadratic equations with one unknown utilized mixed method research design. It draws on both qualitative and quantitative data to describe and analyze learners' performances and difficulties with quadratic equations in symbolic and word-problem contexts. The participants were 217 tenth grade learners from three public Anatolian high schools in Turkey; one is located in Ankara (n_1 = 84) and the other two are in Çorum (n_2 = 78 and n_3 = 55). The participants' ages ranged from 14 to 16 (mean = 15.95 and *sd* = 0.46). The quantitative analysis of the data revealed that only about ten percent of the learners (N = 217) solved all of the symbolic equation questions correctly. The data shows that the percentage of correct solutions ranged from 25.8% to 80.6%.

Makonye and Shingirayi, (2014) on their research on obstacles faced by the learners in the learning of quadratic inequalities, discovered that learner errors on quadratic inequalities tasks lie in their lack of competency on basic algebraic processes. The research also found that in most cases learners made different kinds of errors in response to a single problem. This indicates that the process of error making is not static. More than 80% of the learners were not in a position to solve inequalities due to failure in algebraic processes, such as factorization, transposing inequalities so that one side of the inequality became zero. In other cases, learners failed to determine the critical values of the quadratic equations inherent in the inequality. Where learners were able to transpose, they had challenges in assigning the correct signs. In some cases learners relegated the inequalities to equations, which equations they failed to solve because they had not yet mastered the factorization procedure.

The quadratic factorization procedure is one that needs to be done with some understanding. Learners had many conceptual and procedural errors in factorization. Learners showed clearly that their understanding of factorization was in the main incomplete. Therefore far from exploring errors in solving quadratic inequalities per se, the researchers found themselves lodged in exploring errors in the “pathway processes” to solving inequalities such as factorization and dealing with solution to quadratic equations. In as much as learners could not do these we had little hope to study the errors and misconceptions on the target task; solution of quadratic inequalities.

On strategies for factorizing quadratic expression, Yahya and Shahrill, (2015) found out that majority of the participants used trial and error to factorize quadratic expressions. Only two learners used the splitting method. For the second part of the written test, there were four fundamental errors that participants made that instigated them to make errors in answering the given problems. And quite a number of the participants were unable to define the term factorization when asked for its definition during the interview. A lot of them could not relate factorization with distributive law, which is, putting the common terms or linear expression in brackets. A number of participants were inept when asked to state the general formula for quadratic expressions.

Students made errors on multiplication of factors, namely, the wrong use of the third term multiplication factors when doing the splitting method, and the incorrect terms used in finding the multiplication factors used to solve the quadratic expression. Factorizing $2x - 2$ did not yield $2(x + 1)$ as was written by a learner. Hence the sum of $3x$ and $2x$ could not be used to replace $5x$ in this case. Furthermore, $3x$ and $2x$ were not the correct multiplication factors for $-6x$. The correct factor multiplication of $-6x^2$ to be used here should be $6x$ and $-x$. Didiş, Baş, and Erbaş, (2011), observed that students employed different approaches to factorization depending on the kind or structure of the quadratic equation to be solved, and thus experienced difficulties in different stages of the process.

2.4.1.1 Incorrect Factors

Certain errors made by the learners revealed they had factorized the quadratic equation into two linear factors incorrectly, and determined the roots incorrectly because they had made false guesses while using the cross-multiplication method. This kind of error occurred mostly, where many students used cross-multiplication as a factoring technique to find the roots of the quadratic equation. The most common error emerged when students used a cross-multiplication method, based on a kind of "guess-and-check" approach, while factorizing the quadratic equations. For example, learners guessed the factors of the constant term incorrectly. Similarly, although learners guessed the factors of the coefficient of x^2 correctly, they were not able to determine the factors of the constant correctly.

In recent years Li, (2011) focused his work on a particular content theme in algebra – basic routines and procedures, which include the algebraic rules, algorithms, and formulas that can be applied to given inputs and yield desired outcomes through finite steps (e.g., the distributive property of multiplication over addition and its extensions, such as the so-called FOIL formula $(a + b)(c + d) = ac + ad + bc + bd$ and identities $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)(a - b) = a^2 - b^2$; various established methods or formulas for solving linear and quadratic equations, such as the balancing and backtracking methods, factoring, completing the square, and the quadratic formula).

Makonye and Shingirayi, (2014), on their research on obstacles faced by the learners in the learning of quadratic inequalities used non-probability purposive sampling. The study was conducted at Lamula Jubilee Secondary School in Soweto, Johannesburg, South Africa where one of the researchers taught mathematics. Participants were drawn from a Grade 11 mathematics class consisting of 27 learners of whom were 12 boys and 15 were girls. The average age of the learners was 17.2 years. Learners struggled to obtain the correct cognitive structure to solve quadratic inequalities. Learners were at a quandary as they were dealing with many interrelated concepts that they were supposed to sort out; correct factorization, determining the critical values, determining and writing the values of x in inequality form. Sometimes they were required to write the given inequalities in standard form first in which transposing and collecting like terms was necessary. Some learners ignored the inequality signs and handled these as if they were equations. Some learners changed the signs; for example from greater than to less than without any specific logical reason. It was thus quite clear that learners were very anxious and could not handle the problems with inequality signs in a logical manner. Learners clearly lost control of their reasoning and all their work seemed to be guesswork.

Yahya and Shahrill, (2015) while investigating the strategies used by secondary school learners in solving algebraic problems in one of the secondary schools in Brunei Darussalam, conducted in one of the secondary schools in the Belait District (one of the four districts in Brunei Darussalam). The target sample for this study was a class of 21 learners, repeaters who participated in the initial stages of the study. The main reason why the repeaters were chosen to be the sample was because the first author herself taught half of the repeaters when they were in Year 9 and Year 10. Therefore, she knew their weaknesses in the selected areas of algebraic topics. From the 21 learners, only ten repeaters were randomly selected for further participation further in this study. All the learners were similar in socioeconomic status, with the majority of them coming from middle-income families. In the study learners' difficulties in solving questions of changing the subject of a given formula was the most frequent errors made by the learners which included manipulating operations, factorizing linear expressions and in the use of cancellation.

2.4.1.2 Zero-Product Property

Once learners have factored an expression and work to solve it using the zero-product property (if the product of two numbers is zero, one of the numbers must be zero), they run into additional obstacles. When working to solve an equation such as $x(x+2) =0$, learners sometimes "cancel" the x from both sides (divide by x) leaving $x +2 =0$ and $x =0$. They do not see that by doing so, they lose track of the root (Didis & Erbas, 2015; Kotsopoulos, 2007).

The data revealed that some of the learners did not correctly judge whether the quadratic equation to be solved was factorable over some domain, such as rational numbers. For example, some learners attempted to factor the equations; "$x^2 + 2x - 1 = 0$" and "$x^2 + x - 1 = 0$," although they are not factorable, over the rational numbers. For the quadratic equation $x^2 + 2x - 1 = 0$, some learners tried to factorize it as $(x^2 - 1)^2$, $(x + 1)^2$ or $(x - 1)(x + 1)$. For this quadratic equation, all students who had successfully found the roots used the quadratic formula.

2.4.1.3 Incorrect Factorization in Non- Standard Form

Some learners could not correctly apply the algebraic identity $a^2 - b^2 = (a - b)(a + b)$ to factorize quadratics. Some learners initially moved the term $6x$ to the left side of the equation. They then identified the greatest common factor of the polynomial; $3x$, and rewrote the polynomial using the factored terms. However, although learners put the common term in front of the parentheses correctly, they put the resulting expression inside the parentheses incorrectly. Therefore, when they equaled the factors to zero, they ended up obtaining one of the roots of the quadratic equation incorrectly.

On learners' reasoning in quadratic equations with one unknown (Didiş et al., 2011) sampled 113 learners in four 10th grade classes, and this study was performed in a high school in Antalya, Turkey during the spring term 2009-2010. The study result revealed that factoring the quadratic equations was challenging when they were presented to students in non-standard forms and structures. After looking at the examples of learners' solutions, it can be said that the learners knew some rules (or procedures) related to solving quadratics. However, they tried to apply these rules thinking about neither why they did so, nor whether if what they were doing was mathematically correct. These results give some clues about learners' instrumental understanding of solving quadratic equations with one unknown. Although most of the learners were aware of the correctness of the result, they did not explain the underlying null factor law used to solve the quadratics by factorization. The responses also reveal their misunderstanding of the unknown concept in a quadratic equation and was concluded that the learners' understanding in solving quadratic equations is instrumental (or procedural), rather than relational (or conceptual).

2.4.2 Completing Square Method

2.4.2.1 Dividing by the Coefficient of $\mathbf{x^2}$

Star et al., (2015) while citing Laridon et al., (2011) advised that when adding half the coefficient of x, learners should ensure that the coefficient of x^2 is 1. They also further stated half the square of the coefficient of x should be added both on the left hand side and the right hand side. The current study was carried out to diagnose difficulties and errors students make without true understanding of the underlying concepts and prevent them in learning other concepts.

Makgakga, (2016a), in his study reported that the first component of the analysis of learners' scripts in solving quadratic equations by completing a square was to identify the errors and misconceptions learners made. Those common errors were characterized by conceptual errors and procedural errors. Most of the common errors found were dividing by the coefficient of x^2 if the equation was greater than 1 or less than zero. Other learners did not find the additive inverse of a constant -1 before completing a square, which was also wrong for them to solve the equation in that fashion. More errors were found when some of them failed to factorize the equation after completing a square which revealed that learners lacked knowledge of factorization.

Some learners rewritten their equations in the form of $ax^2 + bx + c = 0$ and then factorized instead of factorizing the equation without writing it in standard form. Some of them found the additive inverse of the equation but only completed a square on the left hand side and failed to do it on the right hand side. Another common error was that some of the learners had failed completely to attempt to complete a square in solving quadratic equations, such as dividing the coefficient of x^2 if was greater or less than 1.

2.4.2.2 Adding $\left(\frac{b}{2}\right)^2$ on both sides of the Equations

A qualitative study on errors and misconceptions in solving quadratic equations by completing a square that was conducted by Makgakga, (2016) in five South African schools, Limpopo Province in Capricorn district diagnosed errors learners made in solving quadratic equations by completing a square and found the reasons why those errors occurred. The study used a diagnostic test followed by focus group interviews to understand the reasons behind the errors and misconceptions learners had in solving quadratic equation by completing a square. It was conducted under qualitative research approach in which the information collected was analyzed through description and not statistically (Rule & John, 2011). The study had been conducted in five schools of same circuit which comprises of 11 secondary schools.

Zakaria and Maat, (2010) reported that, there are a total of 16 comprehension error, 46 transformation error, 55 process skill error, 4 encoding error and 1 carelessness while solving using completing the square method. Most type of errors made by learners was process skill errors. Tularam and Hulsman, (2013) found out that, no learners mentioned any of the uses or applications of a completed square form. About 26% ($\frac{34}{133}$) of learners presented a worked example of completing the squares to solve a quadratic equation. Only 6% ($\frac{8}{133}$)) annotated the worked example to explain the process, whereas the majority of learners (74% i.e. $\frac{99}{133}$) did not use or even mention the completing of the square as a method of graphing parabolas-finding vertices of a parabola or solving any quadratic equation for $y = f(x) = 0$.

2.4.2.3 Failed to Complete a Square

Makgakga, (2016) reported that "in focus groups, some indicated that this concept is challenging to them as they compared with factorization and using quadratic formula. In the discussion, learners revealed that some of their teachers don't give them an opportunity to participate and only the fast learners were always given a platform. In some schools learners were unable to participate as their teachers praised those who gave correct answers".

The data in Didiş et al., (2011) showed that, among all questions, learners applied the completing square method only for solving $x^2 + 2x - 1 = 0$. Indeed, only eleven learners attempted to use completing the square method to solve the quadratic equation, while the majority of the learners applied the quadratic formula. Learners' incorrect responses revealed that each learner who attempted to solve the quadratic equation by completing the square method encountered a different challenge that led them to failure. For example, some of the learners had difficulties adding the numbers correctly on both sides of the equations to balance it and converting the left hand side of the equation to its squared form. On the other hand, several learners did not complete their solutions although they correctly converted the left hand side of the equation to its squared form. On the other hand, the interview data showed that some learners found the use of the completing square method challenging, and as such, they did not attempt to use it to solve the quadratic equations. The current study conducted an interview to find reasons why learners would prefer not to use the completing the square method over the other methods.

2.4.2.4 Square Root Property

The data also revealed that, in solving the two-term equation $9x^2 - 25 = 0$, some learners used neither factorization nor the quadratic formula. Rather, they tried to use the square root method. In this case, in order to isolate the squared variable, learners initially moved 25 to the right side of the equation, and then put it underneath, to get and took the square root. That is, the exponent passed to the other side as a square root. However, the majority of the learners who followed this procedure found only one correct root, and they neglected one of the roots of the quadratic equations, particularly the negative root (Didis et al, 2011).

2.4.2.5 Imposing Linear Structure

Research suggests that some learners try to apply their understandings from linear equations to quadratics when solving quadratic equations. In a study of 80 learners in Brazil who had studied quadratic equations, (Lima, 2008) asked the learners to make concept maps, solve equations, and complete questionnaires that were comprised of problems involving quadratic equations. The researchers found that learners take “rules” that they have developed from solving linear equations and either erroneously apply them to quadratics or use them to try to “linearize” quadratic equations. Working to isolate the variable by adding or subtracting terms from both sides is an example of the misuse of these rules, as is dividing both sides by x in the expression in the example above.

2.4.3. Quadratic Formula

2.4.3.1 Incorrect Discriminant

The quadratic formula is presented to all pupils at some stage of their Mathematical life. Very few of these pupils will actually have had the formula derived for them. Most learners will have had it presented to them as some magic formula which you use to find the roots of a quadratic when you cannot factorize the quadratic. The teacher would then possibly work through one or two examples to show the learners how the formula is used. It is not that the derivation of the quadratic formula is difficult, but for most pupils it is an awkward piece of algebra for them to work through and most teachers would possibly omit the derivation of the formula, (Didiş et al., 2011).

The quadratic formula, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is derived by completing the square on the general form of a quadratic equation: $ax^2 + bx + c = 0$, where $\boldsymbol{a}\neq\mathbf{0}$. The formula can be used to solve any quadratic equation and is especially useful for those that are not easily solved by using any other method (i.e., by factoring or completing the square). The term $b^2 - 4ac$ is called the discriminant. The discriminant is important because it tells how many roots a quadratic function has. Specifically, if $b^2 - 4ac < 0$, there are no real roots and the parabola it represents does not intersect the x-axis. Since the quadratic formula requires taking the square root of the discriminant, a negative discriminant creates a problem because the square root of a negative number is not defined over the real line. $If\ b^2 - 4ac = 0$ there is one real root, that function has exactly one real root and crosses the x-axis at a single point, and if $b^2 - 4ac > 0$ there are two real roots (Zakaria & Maat, 2010).

Didiş et al., (2011) on learners' reasoning in quadratic equations with one unknown showed that students encountered the following challenges while applying the quadratic formula to find the roots of the quadratic equations: learners either computed the discriminant incorrectly because of calculation errors, or could not compute it at all; learners computed the discriminant correctly, but applied the quadratic formula incorrectly, since they had misremembered it; students computed the discriminant incorrectly but they applied the quadratic formula correctly. Particularly, for questions 2, 3, 4, and 6, where the learners mostly used the quadratic formula to find the roots, learners' incorrect solutions were mainly based on either the incorrect calculation of the discriminant or incorrect use of the quadratic formula.

Most of the learners were not able to solve quadratic equation correctly, because they made calculation errors while they were finding the discriminant of the quadratic equation. On the other hand, most of learners calculated the discriminant correctly in questions 3, 4, and 6, but they did not use the correct form of the quadratic formula. For example, many of the learners misremembered the quadratic formula and applied the following forms to solve the equations. Furthermore, the learners' explanations in the interviews support the possibility that learners either misremembered the quadratic formula or totally forgot the correct form of the formula leading to an inability to solve the quadratic equation, especially when the equation was not factorable into binomials with rational coefficients.

Leslie, (2015) found that, most of the equations that learners solve did not require the quadratic formula, and most of the learners preferred to factor whenever possible. Many of the learners had the quadratic formula written in the upper right hand corner of the white board of their classroom. I found that when they were trying to remember the formula, they would glance up at that corner of the interview room as they worked to remember it. Very few learners were able to correctly solve using the quadratic formula. However many of them were able to determine that the equation was not factorable, and thought that there was some formula they could use.

2.4.3.2 Ignoring the Square Root

Solving quadratics equation using quadratic formula**,** most of the learners made process skill errors. The study concluded that the errors made in learning quadratic equations consists of error in comprehension, transformation, process skill, encoding and carelessness (Zakaria & Maat, 2010). However, most of the errors made were transformation and the process skill errors.

Tularam and Hulsman, (2013) while studying 1st year tertiary learners' Mathematical knowledge conceptual and procedural knowledge, logical thinking and creativity, shows that learners simply presented the quadratic formula in isolation; that is, not defining the terms in the quadratic formula or relating it in any way spatially to the general form. Around 65% ($\frac{87}{133}$) of the learners did not present information regarding factorizing being possible a method to solve a quadratic equation. A number of learners experienced difficulty in solving question involving square root, showing that they were unable to deal with equations involving the square root. They tend to ignore the square root or unintentionally forget about it. Furthermore, some participants who attempted this question made errors in the cancellation method. The participants made errors as early as when they were manipulating the operation (Yahya & Shahrill, 2015).

From the study conducted by Yahya and Shahrill, (2015), they found that two main factors caused learners to make errors in answering questions in the third section of the test. Namely, learners applied the quadratic formula incorrectly, and were prone to make careless mistakes in the substitution of negative integers in the quadratic formula. Two out of the ten participants totally forgot the quadratic formula. One of them said that he could not recall the quadratic formula, whereas the other used the trial and error method to solve questions to compensate for his inability to remember the quadratic formula at that time. Based on the researchers' observations, learners who relied on memorizing the quadratic formula without really understanding it were prone to make a lot of errors. Furthermore, having not been taught to understand the quadratic formula increased the possibility of students deriving the incorrect formula.

2.4.4 Graphical Method

The graph looks a little like a cup, and the bottom of the cup is called the vertex. The mouth of the cup keeps getting larger to infinity. For the most part, the region around the vertex is of interested. The cup is upright (vertex down) when $a > 0$, upside down (vertex up) when $a < 0$. There are three important cases of quadratics depending on where the graph crosses the x-axis (these points are called roots or zeros of the equation). In case I, two distinct, real roots, the vertex lies on the opposite side of the x-axis from the rest of the graph and so the curve must cross the x-axis exactly twice.

One can see exactly where the roots are from the graph, and they are clearly real numbers. Analytically, this case corresponds to the portion of the quadratic formula under the radical (the discriminant) being strictly positive:

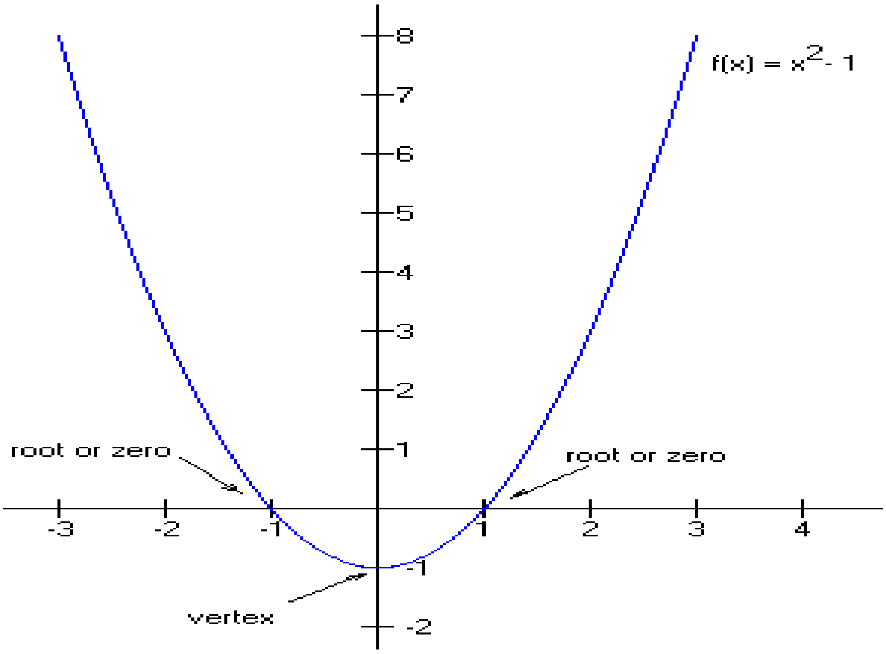


Figure 2.1: Parabola

A quadratic function is graphically represented by a parabola with vertex located at the origin, below the *x*-axis, or above the *x*-axis. Therefore, a quadratic function may have one, two, or zero roots. This method can be used to derive the quadratic formula, which is used to solve quadratic equations. In fact, the roots of the function, $f\ (\mathrm{x}) = \mathrm{ax}^2 + \mathrm{bx} + \mathrm{c}$ are given by the quadratic formula. The roots of a function are the *x*-intercepts. By definition, the *y*-coordinate of points lying on the *x*-axis is zero. Therefore, to find the roots of a quadratic function, we set $\mathrm{f}\ (\mathrm{x}) = 0$, and solve the equation $\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c} = 0$. The vertex is an important coordinate to find because we know that the graph of the parabola is symmetric with respect to the vertical line passing through the vertex. The coordinate of the vertex of a quadratic equation in standard form ($y = a\mathrm{x}^2 +$

$bx + c)$ is $(-\frac{b}{2a}, f(-\frac{b}{2a})$, where $x = -\frac{b}{2a}$ and $y = f(-\frac{b}{2a})$.This means that to find the x-value of the vertex in the equation, $y = -3x^2 + x + 1$ use the formula that $x = -\frac{b}{2a}$. In this equation, "b" is the coefficient of the x-term and "a", like always, is the coefficient of the x^2 term (Mathematics Curriculum, 2014).

The study findings by Amidu, n.d., showed that 3 items were presented and analyzed in the content category of ‘recognition of quadratic function for given graphs by using their roots and intercepts’. The average percentage of correct responses for the three items was 35.8%. This shows that the performance of the pre-service teachers in this category was below the 50% average mark and was not very encouraging. Recognizing a quadratic graph with negative co-efficient of and having two distinct roots was categorized by the researcher as the easiest of the three items but only 42.5% $(n = 17)$ of the pre-service mathematics teachers were able to supply correct responses. The participants’ worse performance was on ‘recognizing quadratic graph with positive co-efficient of and having two distinct roots’ which only 27.5% $(n = 11)$ of the pre-service mathematics teachers was able to supply correct responses.

The results show that pre-service Mathematics teachers were not comfortable with the items in this content category, and thus necessitate the study to find out if secondary school learners in Kericho County were able to comprehend threshold concepts in solve quadratic equation and functions with positive coefficient and having 2 distinct roots. Furthermore, the mean percentage score of the participants making the correct match was 39.2%. The question, which required pre-service Mathematics teachers to recognize a new quadratic graph, when the value of the co-efficient of is tripled, had the highest correct responses score (65%, n=26). However, only 27.5% (n=11) and 25.0% (n=10) of the participants were able to recognize the new quadratic graphs when the values of the co-efficient of was halved and became negative. That implies that a few of the respondents were able to get the clue that when the co-efficient of in a quadratic function becomes negative the graph opens downwards.

Parent, (2015) while studying learners' understanding of quadratic functions which was mainly an investigation of the effects that traditional and multiple representation tasks have on how students think about the quadratic function, specifically the axis of symmetry, vertex, the location of roots, whether the parabola opens up or down, the maximum/minimum point, the y-intercept and the main translations of the function itself when graphed. The specific methods of factoring for roots were not a part of this study. Utilizing a "think-aloud" protocol, each pair participated in the same four tasks. The tasks varied, with one being more traditionally worded, one focused on more multiple representations, and then a combination of the two for two mixed methods tasks. Learners participated in the study tasks over a four-day period (before or after school) for a maximum duration of 45 minutes each day. All six study participants were enrolled in the high school (grades 9-12) where the researcher taught that is located in northern Vermont with a population of approximately 1150 learners. There were four males and two females in the study, two of the males and one female were sophomores (10) and the other three were juniors (11) in high school, making an even split.

While analyzing the data, Stokes noted the following strategic and misconception observations to be key: Participants preferred the standard form over the vertex form, participants confused the y-intercept of the standard form versus the y-coordinate of the vertex when the function was in vertex form, participants preferred algebraically solving a problem versus tabular or graphical, the linear function term of "slope" came up when learners were discussing the transformations of the quadratic graph, and the learners interpreted the maximum/minimum point of the quadratic function to be the entire (x, y) point of the vertex instead of solely the y-coordinate of the vertex.

The data from this study reveals that the participants were limited in both their conceptual and procedural understanding of the quadratic function. The participants illustrated a variety of misconceptions when presented with standard problems related to the quadratic function. But, when given hints through graphs, a function, a formula etc., they were more successful in solving the problem.

In addition, the participants had higher confidence in their answers if the problems were presented in the quadratic standard form where they could algebraically solve for the answer. One does have to remember that the learners were involved in this study during or just after the time period that they were initially introduced to the quadratic function. The fact that they were not all sure of themselves in every situation is to be expected, and the reason why the study occurred when it did in their curriculum. The researcher did not want rehearsed and finely tuned answers; the study was an attempt to capture learners' initial thoughts about the quadratic function. With quadratic functions being such an important piece of the mathematical puzzle, it is important that learners have the background knowledge to do the more mundane mathematical tasks when recognizing and solving these functions, which will then bridge to other functions.

On the occasion that learners did understand the function presented to them, they may not have a complete understanding of all of the elements or be able to transfer the function between different representations of it – ordered pairs, table, equation, graph, etc. If a learner only understands a particular form of function, due to that being the only one used in a course, that a learner will only retain that particular form. Procedural knowledge can allow a learner to pass a class, but conceptual knowledge combined with the procedural knowledge will allow the learner to be prepared for the next mathematical level. As noted by the researchers learners preferred to convert the vertex form of the quadratic to the standard form in order to solve the problem. This is primarily due to the fact that learners see the standard form of most functions more than any other form while taking quadratic functions.

2.5 Learners' Threshold Concepts in Solving Word Problems

Didiş et al., (2011) found that only one learner, who was also successful in solving all the symbolic equation, was successful in solving all the questions in the word-problems context. While the learners were mostly successful in solving word problem $2(46.1\%)$, they were least successful in solving the word problem $4(3.7\%)$. On the other hand, the learners who provided the correct answer for the word problems did not necessarily solve the problem using quadratic equations. In particular, while some learners solved the first problem with a guess-and-test strategy, some learners attempted to solve the third problem by initially making a drawing and then examining it from a different point of view, without formulating a quadratic equation to represent the relationship. Learners attempted to use guess-and-test strategy in order to find the number of days that they initially planned to get the order ready.

In other problems, the learners initially drew a triangle to show the data and to see what was going on. Then, these learners recognized that the triangle was familiar with the 3:4:5 triangles, and concluded that the dimensions of the right triangle in the diagram must be a 30-40-50 triangle. Analysis of students' incorrect solutions and the interview data revealed that the reasons for low performance in forming quadratic equations stated as word problems are threefold: learners did not fully comprehend the problem, learners understood the problem, however, they did not know how to represent the information as a quadratic equation (or approach the problem differently), and learners understood the problem and represented the information as a quadratic equation, however, they had difficulty solving the problem and thus, also with interpreting it.

A study by Egodawatte, (2011) on secondary school learners' misconceptions in algebra used a mixed method research design and reported several types of errors and they were categorized into six major groups: Reversal error; most common error was the reversal error: (48%). The majority of learners (84%) used the equal sign to denote equality without considering the proportional relationship of the variables, used the letters as labels instead of a varying quantity. majority learners considered symbols as labels and formed the equation by mapping the sequence of words directly into the corresponding sequence of literal symbols. Guessing without reasoning errors resulted when students apparently solved a problem by guessing-that is, when there was no overt evidence that the stated information was the result of a mathematical operation, performed a mental operation; hence, unsubstantiated outcomes rather than guessing and not able to verify the realness of their answers by use of meta-cognitive abilities such as verification or looking back.

Forming additive or multiplicative totals from proportional relationships, learners attempt to connect the variables in an equation as an additive total. Learners can understand the problem statement; however, they do not know how to represent the given information as a quadratic equation Didiş et al., (2011). Thus, instead of applying Pythagorean theorem and getting a quadratic equation that represents the relationship among the distances in the situation, the learner set up an incorrect relationship; $2x + (2x + 10) = 50$, as if the sum of the distances traveled after two hours was 50 km. It is safe to say that the problem can be solved easily; $2x = 30 \; or \; 2(x + 5) = 40$, upon recognizing the triangle as $30 - 40 - 50$. However, the learner failed to apply the Pythagorean Theorem to set up the correct linear relationships. Similarly, in other word problem, some learners did not formulate the correct algebraic relationship between the side length of the square and the width of the rectangle. Although they did comprehend the problem statement, they could not interpret the information presented, in order to form the quadratic equation, or could not set up the quadratic equation in the correct form.

Threshold concepts in grasping the relationship between two or three varying quantities, learners are expected to understand the relationships among the variables, form equation(s), and solve them. Many of the answers indicated that learners used arithmetic methods, working backwards, or guessing to find solutions rather than algebraic methods. Only $5\%\ and\ 14\%$ of the learners used algebraic methods to solve the problems. Didiş et al., (2011) also found that although some learners set up the quadratic equation correctly in the word problems, they made mistakes while solving it. The most common error for the second word problem was that the students constructed the algebraic relationships and formulated the quadratic equation correctly as; $2a^2 + 6a - 176 = 0$ however, they made calculation errors while using the cross-multiplication method and zero product property. On the other hand, some learners set up the quadratic equation correctly but they could not solve the equation and as such, their solutions were incomplete.

Incorrect reasoning in word problems with a familiar context, learners have to think beyond the given data by constructing an equation from the given data and prove that the rule does not always work by explaining the relation between variables. (14%) of the learners used only the given data to arrive at incomplete or wrong conclusions and another 14% extended their thinking beyond the given data but they could not grasp the relationship between the two variables at the same time. It was difficult for them to understand the changing relationship between two variables and they lack proportional or relational reasoning.

Didiş et al., (2011) while analyzing comprehension of the word problems; the students' solutions revealed that, since a large proportion of students either did not comprehend or miscomprehended the text in the word problems, they could not formulate the related equation. Although the length of the paperboard is twice its width, before cutting and making it into an open box (i.e., a rectangular prism), the student symbolizes the dimensions of the open box as x for the width and $2x$ for the length, and then forms the equation by way of a volume formula for a rectangular prism. Here the student correctly symbolizes the relationships and formulates the quadratic equation, however, his misinterpretation leads to an incorrect solution. The interview data supported the factor that forming quadratic equations were quite challenging for students due to their difficulty comprehending the problem statement. During the interviews in the current study, students will be asked to express why they did not comprehend the problem statement, nor the information presented within. Miscellaneous forms of incorrect answers given, was significant (22%) showing that the learners' tendency to misinterpret the operation as a multiplication when it is actually a division.

Although learner performance regarding solving quadratic equations stated in symbolic equation was not high, their performance depended on the structural properties of the symbolic form of the quadratic equation, Didiş et al., (2011). Their performance also depended on how effectively they used factorization, completing to the square, and quadratic formula for solving quadratic equations. Although the quadratic equation in a question is factorable, have integer coefficients and its roots are all rational numbers, the quadratic equation in some questions is not factorable; its second coefficient is a rational number, and its roots are irrational numbers.

Data suggest that learners displayed high performance in solving the quadratic equation when it is easy to factor, has rational roots, and students have had more practice and greater procedural abilities in solving these types of equations. However, when it came to the non-factorable structure of the quadratic equation, students are unsuccessful, due to their limited procedural algebraic and arithmetic abilities. This question required more algebraic symbol manipulations and arithmetic operations, with rational and radical numbers, while applying either a quadratic formula or complete square.

2.6 Gender Performance

While studying a single subject research incorporates an experimental design that documents causal or functional relations between independent and dependent variables Strickland, (2011), conducted an interview and established connections between the area context and her previous knowledge, abstract symbolism, and the concept of area as a quantity. Additionally, a disconnection was observed between the area contexts and factoring. First, the area context provided a connection to learner's previous knowledge of area as length multiplied by the width using discrete numbers. The area context also served as an anchor to the abstract symbols. After discussing the area context through the word problem and the tabular data, a student developed a generalized algebraic expression to represent the area context and was able to perform the symbolic manipulation to transform the factored-form (i.e., dimensions) into the standard-form (i.e., the area). However, when presented with a non-contextualized quadratic in factored-form, she struggled to associate the symbolic manipulation as "the answer" or the final result to the 197 task. Although we continued to write the symbolic-equation using the area formula, there was no story situation to anchor her response.

Gender difference exists in the field of mathematics, Stoet and Geary, (2013), in their study in Across River estate in Nigeria found that, women's participation in the workforce and pursuit of higher education has increased substantially, but there continue to be striking sex differences in college majors and career choices. Sex differences are particularly notable at the highest levels of scientific achievement; for example, fewer than 3% of Nobel laureates in science are women, and no women have so far received one of the top three awards in mathematics (the Fields Medal, the Abel Prize, and the Wolf Prize).

Solomon Four-quasi-experimental research design was used in the study conducted by Githua and Njubi, (2013) on the effects of practicing mathematical creativity enhancing learning/teaching strategy (MCETS) during instruction on secondary school students' mathematics achievement by gender in Kenya's Nakuru Municipality. It had four nonequivalent groups of Subjects: the experimental group one (E_1) the experimental group two (E_2), the control group one (C_1) and the control group two (C_2) which were used. Group E_1 and E_2 formed the experimental groups while C_1 and C_2 formed the control groups. Group E_1 and C_1 received a pre-test (O_1 and O_4) to ascertain whether or not the groups under study had comparable characteristics while E_1 and E_2 got treatment (X), that was an exposure to mathematical creativity enhancing strategy. All groups in this study received a post-test that facilitated comparisons between them. The target population was form two mathematics learners aged 16 years from provincial public secondary schools within Nakuru Municipality. The findings of hypothesis (HO_2), however show that gender differences in performance scores in experimental groups were not conclusive.

The results also indicated that both male and female learners taught through the MCETS performed significantly better than corresponding groups taught through the conventional teaching methods. Performance by gender at the secondary school level differ in that girls perform better than boys in language subjects and relatively poor in mathematics and key science subjects. Girls in single sex schools perform equally and even better than boys whereas those in mixed secondary schools perform poorer. The disparity of performance by gender has been the trend in Kenya for a long time. The study findings reveal that there is a significant difference in the performance of Mathematics and Chemistry between boys and girls. This means that a significant relationship exists between the student's gender and performance in Mathematics and Chemistry in mixed secondary school.

The reports by Shahrill and Mundia, (2014) also found that teachers asked boys more questions than they ask girls were true for US and Australian schools that the author had observed. In US, 30 male students were more engaged in teacher questions compared to only 21 female learners, whereas in Australia, the 'teacher question' ratio of male to female learners was 2:1 (38:19). The overall results have shown that male learners volunteered, and were called on more to teachers' questions compared to the female learners. The data in this study suggest that gender biased teacher questioning do occur in the six Year 8 Mathematics classes. Since teacher questions were mostly directed to male learner in the Mathematics classroom, thus, receiving more teacher attention, more teacher interaction and more feedback. Therefore, this will likely to cause an effect on the female learners' class expectations and their achievements in

Mathematics recommended that stated that teaching behaviour is the most potent, single, controllable factor that can alter learning opportunities in the classroom. It is important for teachers to recognize and understand that preconceived attitudes and expectations about boys and girls are likely to have an effect on children, particularly in the mathematics classroom US and Australian female students in this study should be included equally in classroom interactions and given equal encouragement in mathematics as their male classmates.

An effective teacher, balance responses from volunteering and non-volunteering learners and, a vigilant teacher; employ both higher-level and lower-level questions in the classrooms and the performance on both genders will be high as students are able to apply their abstract knowledge in solving problems of higher order in which this study will establish based on good or poor performance. To explore why girls are less confident than boys in their math abilities, Stoet and Geary, (2013), found that girls use different strategies and have different motivations to do math. Boys, tend to use memory to retrieve sums and are motivated by a sense of competition to get the answer fast, even if they sacrifice accuracy. Girls care less about speed than accuracy and more often rely on “manipulative” — counting on their fingers or a counting board. “Girls will use manipulative even when they might be able to retrieve [the answer], they need an added push that boys don’t need to start using cognitive strategies”, Azar, (2011). That’s important because while using manipulative is an excellent strategy when students first learn math, it slows them down as problems get more difficult. In fact, becoming fluent, and therefore faster, at basic math is directly linked to math performance.

The study also found that girls were less fluent than boys. And if all students are fluent, most gender differences would be eliminated. A research on the effects of problem-based learning approach on junior secondary school students' achievement in algebra, conducted by Emmanuel, Abonyi, Okafor and Omebe, (2015), employed a quasi-experimental design. The study specifically employed a pre-test post-test non-equivalent control group design. The result obtained from this study revealed that the students taught algebra with the problem based learning method performed better than the students taught algebra with the conventional chalk-talk approach. The results also showed that the interaction effect of gender and instructional approach on the students' mean achievement in Algebra is not significant. The current study employed a descriptive research design to investigate gender performance across low and high cognitive levels as well as testing for the interaction effects on both cases.

The majority of research papers and interventions in this area have focused on reducing the mathematics achievement gap, which favors boys at the highest levels of achievement Stoet and Geary, (2013). The focus on this issue may have resulted in a relative neglect of boys' overall academic underperformance, especially among socially and economically disadvantaged groups. The present results demonstrate that initially small differences in school performance between low-income boys and girls increase across the course of schooling, ultimately leading to large differences in secondary school performance. One implication of the findings is that well-timed, early interventions may reduce the gender gap in achievement that emerges later in schooling.

Focused interventions during the elementary years, such as increasing maternal school involvement and learner reading skills, may help protect children at academic risk from experiencing declining school performance after the transition to secondary school. The continuing decline of boys' performance during secondary school suggests that ongoing supports, including those that target the social/peer atmosphere within schools, may be necessary for those boys who are at highest risk of failure.

Finally, attention to current stressors and precipitating factors (e.g., situations emerging in secondary school that lead to the decision to leave school) may also be required, as other work suggests that a subgroup of adolescents do not follow a clear identified path to school dropout. Preventive intervention is likely to be complex at that level but may be necessary to prevent high-risk children from leaving school without completing their secondary studies. Gender bias in high schools has been a decreasing problem as teachers, administrators, and community members work to combat it (Kingdon, Serbin, & Stack 2017). The relationship between gender and math has been explored extensively throughout the last decade but only recently have studies shown that girls have begun narrowing the gender gap in Mathematics. This review will highlight various factors that researchers have found to explain gender inequality in math such as internalizing negative messages and the larger societal structure that may explain teacher and students behavior in the classroom.

2.7 School Type and Performance

Studies conducted to determine the effect of this policy on factors related to learners reveal conflicting results. Booth and Nolen, (2012) conducted a study using a sample of English fifteen year old students from coeducational and single sex schools to examine the role of nurture in explaining why women may shy away from competition. They found that girls in single sex schools are significantly more likely to be competitive. The behavior of boys and girls attending single sex and coeducational schools was also compared. The researchers found that girls attending single sex schools behave more competitively than their counterparts in coeducational schools. For boys they found that neither attendance in single sex nor coeducational school influences whether they choose to compete. This finding suggests that class type has no effect on the competitive nature of boys while girls become competitive in single sex classes.

Eisenkopf, Hessami, Fischbacher and Ursprung, (2015) analyzed the impact of female only Mathematics classes on Mathematical performance of girls. The researchers randomly assigned girls into single sex and coeducational classes in a Swiss secondary school. Their finding indicated that girls' performance in Mathematics improved in single sex classes and that this improvement was greater when taught by a male teacher. This could be an indication that apart from the single sex setting girls' also thrives if taught the subject by male teachers. However, a report published by the American Association of University Women in 1998 contrasts (Younger & Warrington, 2002).

The report noted that though girls' achievement improved in single sex schools the same did not happen for girls in single sex classes within coeducational schools. It further noted that in single sex classes for boys, the teachers often failed to notice their reading and writing problems, handled inappropriately their emotional and social needs and tended to interpret their behavior as discipline problems. The report concludes that teachers generally failed to adjust `their teaching methods to take into account boys' unique learning styles.

Younger and Warrington, (2002), carried out another study on teaching of learners in single-sex classes in a coeducational comprehensive school in England entitled "An Evaluation of Single Sex Teaching based upon Learners' Performance and classroom Interactions." The researchers interviewed teachers and learners of one coeducational school where single sex teaching had been the practice from the time the school was started. The findings of the study indicated that both male and female learners benefited from having their own learning space. The teachers indicated that they explicitly adjusted their teaching styles when teaching boys' or girls' classes. The study's findings further indicated that girls consistently achieved better results than boys in most subjects and that the improvement levels of both girls and boys were similar and significantly higher than the national average. This study's findings may imply that single sex teaching has potential of raising students' achievement levels especially for girls provided that different teaching approaches are planned and implemented for males and females.

The study on the influence of gender streamed (boys' and girls' only) classes on coeducational secondary schools' mathematics teachers perceptions done by Barmao et al., (2015a) employed an *ex-post facto* (causal-comparative) research design. This study compared mathematics teachers' perceptions between those who teach gender streamed (boys' and girls' only) and mixed sex classes in sub-county and county schools. Findings show that mathematics teachers' perceptions were higher in boys' only classes with a mean of 3.83 followed by mixed sex classes with a mean of 3.66 and lastly girls' only classes with a mean of 3.52 in sub-county schools out of the highest possible score of 5. In county schools teacher perception scores were higher in mixed sex classes with a mean of 3.95 followed by boys' only classes with a mean of 3.88 and lastly girls' only classes with a mean of 3.85.

From the findings, it is clear that Mathematics teachers' perceptions scores were lower in girls' only classes for both district and county schools. Furthermore, the findings show that there are no statistically significant differences in mathematics teachers' perceptions of gender streamed (girls' and boys' only) classes and mixed sex classes in both the sub-county and county schools. The findings indicate that from the Mathematics teachers' perspectives, there are no special benefits that may accrue to learners' learning the subject in boys' only, girls' only and mixed sex classes. To them boys and girls can excel in the subject irrespective of the class type. These findings are in agreement with those of the American

2.8 Low and High Cognitive Levels Performance

Research on cognitive skills indicated that facilitating learners' higher order thinking skills in the learning process helps to make them more aware of their own thinking and also fosters their learning performance and cognitive growth Saido, Siraj, Bin Nordin and Al Amedy, (2015). In addition, these HOT skills are activated when students encounter unfamiliar problems, uncertainties, questions, or dilemmas. Successful application of these skills in the science classroom result in explanations, decisions, performances, and products that are valid within the context of available knowledge and experience and that promote continued growth in these and other intellectual skills.

The 21st century skills has focused largely on promoting students' higher order thinking skills (HOTS), through doing different activities that require them to use these skills (Vernez, Culbertson & Constant, 2014). Besides, as Iraq is not involved in any international assessment program such as International Association for the Evaluation of Educational Achievement (IEA) and Programme for International Student Assessment (PISA), it assess the students' performance in these skills to provide empirically grounded information on how far the new science curriculum has achieved its objectives, which will then inform policy decisions. Therefore, this study aimed at assessing the level of HOT skills among secondary school students in the county of

Kericho besides identifying any association between students' level of cognitive skills, school type and their gender. In order for the students to develop higher order thinking, it is imperative that teachers emphasize HOTS; Saido et al., (2015) elaborated this when he conducted a case study with 66 prospective mathematics teachers (PMT) who were undergoing their practical teaching in secondary schools, Malaysia, in the seventh semester of their 4-year degree Mathematics Education Programme. These PMT had taken all mathematics education courses except the Reflection Seminar Course and had completed all of their mathematics required for the programme.

The concept of higher order thinking (HOT) is derived from the Bloom taxonomy of cognitive domain introduced in 1956 (Forehand, 2010). The cognitive domain involves knowledge and the development of intellectual skills (Bloom, 1956). This includes the recall or recognition of specific facts, procedural patterns, and concepts that serve to develop intellectual abilities and skills. There are six major categories of cognitive processes, starting from the simplest to the most complex. Bloom categorized intellectual behavior into six levels of thinking: knowledge, comprehension, application, analysis, synthesis and evaluation (Yahya, Toukal, & Osman, 2012).

The categories in the Bloom taxonomy for cognitive development are hierarchically ordered from concrete to abstract (Pappas, Pierrakos & Nagel, 2013). The hierarchical progression identifies the lower level to higher level of cognitive processing Forehand, (2010); the first three levels of Bloom's taxonomy require basic recognition or recall such as knowledge, comprehension and application and these have been regarded as lower level of thinking skills. In contrast, the other three levels of Bloom's taxonomy require students to use higher order thinking skills hence fostering their learning performance (Yahya et al., (2012). Based on research into the cognitive domain among secondary school students, the first three categories of the Bloom taxonomy, knowledge, comprehension and application measure the learners' lower level of thinking skills (LOTS), whereas the other three levels of analysis, synthesis and evaluation measure the higher levels of thinking skills or HOTS Pappas et al., (2013).

In the revised version of Bloom's taxonomy modified by Tutkun, Güzel, Köroğlu and Ġlhan, (2012), the version is not hierarchal, but two-dimensional. They devised a chart consisting of cognitive processing skills (remembering, understanding, applying, analyzing, synthesis and evaluating).

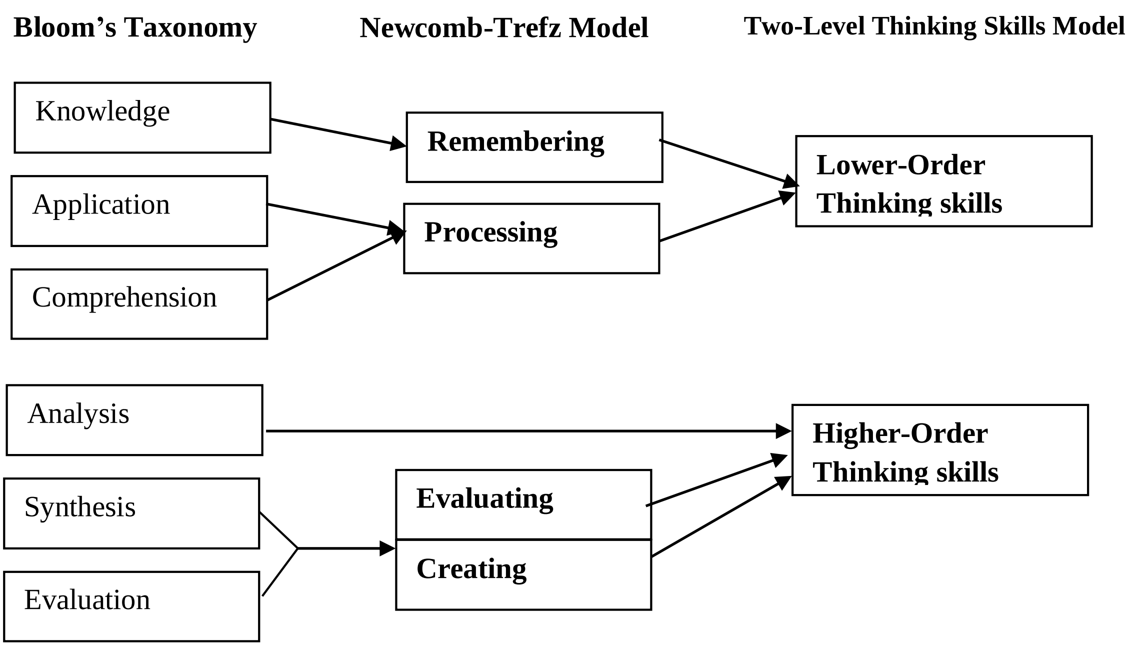


Figure 2.2: Conceptualization of Bloom Taxonomy and Newcomb-Trefz's

Learning Model, and a Two-Level Thinking Skills Model (from Whittington, 1995). Yahya et al., (2012), reported that the majority of the teachers have been using rote memorization or drill. They assumed that this type of instruction is an efficient teaching approach. Sometimes, considering the demands of administrators, teachers have to forgo teaching actively and creatively to develop higher-order knowledge. Sparapani

(1998 cited in Yahya et al., (2012) detailed six challenges that hinder higher-order thinking and learning in educational settings: learners do not have enough time for reflection, discussion, interaction, and providing feedback due to the short time of the class schedule; learner attitudes reflect the status quo of the classroom. Learners are satisfied with the teachers asking questions and them answering the questions; Teachers' attitudes are a major issue because higher-order thinking requires more time, energy, and creativity to prepare challenging learner learning activities; sufficient resources must be provided. Both students and teachers will lack motivation if they receive limited or no resources;

The classroom atmosphere directly reflects upon the learners and a stimulating classroom can stimulate students' thinking and imagination, which can promote HOTs; Authentic assessment practices and learning can reflect students' current intellectually capacity. However, traditional objective-testing forms of assessment may not support creative thinking. High-stakes examinations assess lower-order knowledge (e.g., recall, comprehension) instead of higher-order skills and creating levels) and knowledge dimensions (factual, conceptual, procedural, and meta-cognitive).

There are several concepts associated with higher-order thinking: critical thinking, problem solving, creative thinking, and decision-making. Lewis and Smith (1993 cited in Murray, 2011) define higher order thinking as instances in which "a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations". When people use higher-order thinking they decide what to believe and what to do. They create new ideas, make predications, and solve non-routine problems. Educational researchers correlate higher-order thinking with creative and abstract thinking, decision-making, analyzing theories, and active mental construction. All of these concepts are important and necessary to help learners gain a deeper understanding of school Mathematics.

Mathematical achievement improved and gaps diminished when students experienced instruction focused on problem solving, conjecturing, and explanation and justification of ideas. Learners had a better attitude toward Mathematics, were more likely to take more Mathematics and higher-level classes, and had higher test scores when they were encouraged to use multiple representations and make connections between new and previous knowledge (Boaler & Staples, 2008 cited in Murray, 2011). Some teachers implement tasks that promote higher-order thinking with specific populations of learners. They found that some teachers used higher-order thinking only with high-achieving learners because they believed either that tasks that required higher-order thinking were too difficult for low-ability learners or that such tasks would be too frustrating for these students to solve.

Other teachers believed that higher-order thinking was an appropriate goal for all learners, and worked to modify their instruction to provide the necessary support and guidance for low-ability learners to engage in higher-order thinking. For example, these teachers reported: breaking up a complex task into several simpler components; leading learners through a sequence of steps necessary to solve a problem; giving clues; adding more examples; and letting learners work in groups of mixed ability so that peers can learn from each other. Although these teachers felt that higher-order thinking was an appropriate goal for low-ability students, they were actually reducing the cognitive demand of tasks by making many of these adjustments.

The cognitive demand of a task is the type and level of thinking required to solve the task. Therefore, it is important to investigate how teachers select and implement high-demand tasks and maintain that demand during implementation to understand how they can facilitate students' engagement in higher order thinking. Teachers need to understand learners' common conceptions, preconceptions, and misconceptions in order to teach higher-order thinking skills. "Many studies have shown that students often make sense of the subject-matter in their own way which is not always isomorphic or parallel to the structure of the subject-matter or the instruction" (Murray, 2011).

It is important therefore, that teachers understand students' knowledge construction in order to guide learners toward more sophisticated conceptions of mathematics. According to Stein and Kaufman, (2010) it is important for teachers to spend time understanding mathematics in order to help them be able to provide effective lessons. By understanding mathematics, teachers are better able to focus on advancing their students' conceptual understanding, supporting their learners' thinking, and maintaining the level of the tasks in order to facilitate higher-order thinking. Professional development can support teachers as they learn how to implement pedagogical strategies that promote higher-order thinking and understand the mathematics they are expected to teach.

Teachers in countries whose learners outperformed U.S learners implemented cognitively challenging tasks in ways that maintained students' opportunities to engage in high-level thinking and reasoning (Stigler & Hiebert, 2004 cited in Boston & Smith, 2009). Although teachers in the United States used percentages of high-level tasks consistent with the percentages of high-level tasks used in higher performing countries, the most striking and significant difference between the United States and higher performing countries in the study was the inability of U.S. teachers to maintain the high-level cognitive demands of mathematical tasks during instruction.

Similarly, in the sample of Mathematics classrooms Collins, (2014), only 15% of observed lessons were classified as providing opportunities for thinking, reasoning, and sense-making. Improving students' opportunities to learn Mathematics with understanding will require sustained opportunities for students to engage with cognitively challenging Mathematical tasks. To provide such opportunities, Mathematics teachers will need to: select high-level tasks for instruction and implement high-level tasks that maintain the cognitive demands. Hence, increasing students' exposure to and sustained engagement with high-level tasks will require changes in the knowledge and instructional practices of mathematics teachers.

2.9 Chapter Summary

In teaching quadratic equations and functions, learners often give the appearance of having understood, because they believe they have understood. Their teachers also believe they have understood only for some learners to fail spectacularly when confronted by threshold concepts in an examination questions. In order to master a threshold concept, the theory suggests that learners may travel through a tunnel or ‘liminal space’ where they ‘get stuck’ and may be in a state of uncertainty. Threshold concept theory proposes that there are a number of concepts that are central to the mastery of quadratic equations and functions:

- **Transformative** – once understood, a threshold concept may potentially cause a significant shift in the perception of a subject or part thereof; sometimes it may even transform one’s personal identity.

- **Irreversible** – it is unlikely that a threshold concept is forgotten or unlearned once acquired due to transformation.
- **Integrative** – a threshold concept is able to expose ‘the previously hidden interrelatedness of something’.
- **Bounded** – a threshold concept can have borders with other threshold concepts which help to define disciplinary areas.
- **Troublesome** – threshold concepts may be counter-intuitive (moving against and beyond a common-sense understanding towards an expert understanding).

Threshold concepts in solving quadratic equations and functions can be identified when learners used the methods of factorization, completing the square, quadratic formula or the graphical method.

In solving word problem learners were least successful in solving the word problem. On the other hand, the learners who provided the correct answer for the word problems did not necessarily solve the problem using quadratic equations. Incorrect reasoning in word problems with a familiar context, learners have to think beyond the given data by constructing an equation from the given data and prove that the rule does not always work by explaining the relation between variables. It was difficult for them to understand the changing relationship between two variables and they lack proportional or relational reasoning.

From the literature, girls in single sex schools are significantly more likely to be competitive. The behavior of boys and girls attending single sex and coeducational schools was also compared. The researchers found that girls attending single sex schools behave more competitively than their counterparts in coeducational schools.

For boys they found that neither attendance in single sex nor coeducational school influences whether they choose to compete. This finding suggests that class type has no effect on the competitive nature of boys while girls become competitive in single sex classes. Mathematics teachers' perceptions scores were lower in girls' only classes for both district and county schools. There are no special benefits that may accrue to learners' learning the subject in boys' only, girls' only and mixed sex classes. To them boys and girls can excel in the subject irrespective of the class type.

Research on cognitive skills indicated that facilitating learners' higher order thinking skills in the learning process helps to make them more aware of their own thinking and also fosters their learning performance and cognitive growth. In addition, these HOT skills are activated when learners encounter unfamiliar problems, uncertainties, questions, or dilemmas. Successful application of these skills in the science classroom result in explanations, decisions, performances, and products that are valid within the context of available knowledge and experience and that promote continued growth in these and other intellectual skills.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

The main methodological constructs employed in various stages of the study was explained in this chapter in accordance with the following research objectives: To discuss the influence of teaching strategy on learners' performance in quadratic equations and functions with one known, to describe learners' scores in solving quadratic equations and functions with one known, to analyze learner's difficulties in solving quadratic equations and functions with one known that may attribute to gender, to determine gender difference if any, that may exist in the performance of quadratic equations and functions with one known and finally to determine relationships if any between gender and school type on one hand, and performance in quadratic equations and functions with one known on the other hand. This discussion includes a review of the research paradigms; research design; independent and dependent variables, study area, target population, sample size, sampling procedure and techniques, validity and reliability of the research instruments, data collection instruments, data analysis methods, and ethical considerations.

3.2 Research Paradigms

Research involves systematic investigations undertaken to discover resolutions to a problem. According to Hale, (2011), the general purpose of research is to contribute to the body of knowledge that shapes and guides academic and/or practice disciplines. Therefore, there are two main research paradigms: scientific and naturalistic. Synonyms for the scientific approach are the objectivist or the positivist. In the scientific approach, quantitative research methods are employed in an attempt to establish general laws or principles. This approach assumes that social reality is objective and external to the individual. Alternatively, synonyms for the naturalistic approach are the subjectivist or the anti-positivist. This method emphasizes the importance of the subjective experience of individuals with a focus on qualitative analysis.

Creswell and Creswell, (2017), argued that there are eight compelling reasons to undertake a qualitative study; this study addressed four of them. Creswell's first rationale is to select a qualitative study because of the nature of the research questions. For example, qualitative researchers often start with how or what questions while quantitative researchers start with why questions. The second rationale is to choose a qualitative study when variables cannot be easily identified or theories are not available to explain the behavior of the population. The third one is to choose a qualitative approach in order to study individuals in their natural setting. In the fourth rationale, Creswell and Creswell, (2017), stated that:

> *Employ a qualitative approach to emphasize the researcher's role as an active learner who can tell the story from the participant's view rather than as an expert who passes judgment on participants.*

These purposes are not entirely mutually exclusive. However, the researcher paid attention to all of these four areas.

The scientific and naturalistic division between quantitative and qualitative research is still prevalent, at the same time, mixed methods research is drawing increasing attention in educational circles. This paradigm systematically combines ideas from both quantitative and qualitative methods. Mixed methods researchers believe that they can get richer data and strong evidence for knowledge claims by mixing qualitative and quantitative methods rather than using a single method (Johnson & Christensen, 2008). This idea is further reinforced by the belief that social phenomena are extremely complex and in order to understand them better, there is need to employ multiple methods.

In addressing the purpose of mixed method inquiry, the researcher used the qualitative phase to inform the quantitative phase. In this case the study selected learners for interviews based on their performance in the learner's questionnaire ranging from a very good performer to a low performer. The two phases was integrated together to get better explanations about the main focus of this study on learners' threshold concepts in solving quadratic equations and functions with one known. In this study, the quantitative part helped in understanding learner's performance of quadratic equations and functions with one known numerically and other factors affecting the performance while the qualitative part helped to deepen the study focus to threshold concepts which could enabled the learners correctly/incorrectly solve quadratic problems and be able to explain more about those threshold concepts through interviews.

3.2.1 Sequential Explanatory Design

The study employed a sequential explanatory design which is characterized by the collection and analysis of quantitative data in a first phase of research followed by the collection and analysis of qualitative data in a second phase that builds on the results of the initial quantitative results Creswell, (2009). In the quantitative phase, the study used diagnostic test instrument to identify and classify learners' performance in solving quadratic equations and functions with one known and be able to diagnose learners' comprehension and performance of threshold concepts. Based on the study questions at this stage, the researcher observed and interviewed to expose learners' comprehension of threshold concepts in the qualitative phase of the study. There are four main stages in the sequential explanatory design which the researcher utilized as indicated in the following schematic diagram.

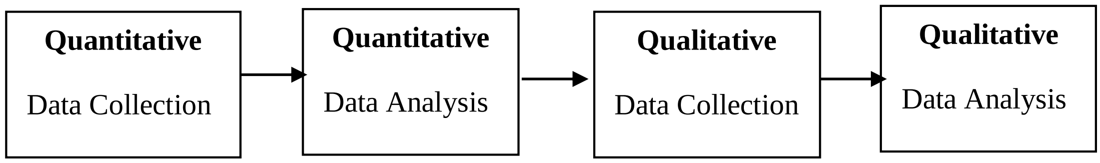


Figure 3.1: Sequential Explanatory Design

Source: Adapted from (Creswell, 2009)

Bean (2007), identified five particular research purposes for which qualitative studies are especially suited. They are: to understand the meaning of the events, situations and actions involved, to understand the particular context within which the participants act, to identify unanticipated phenomena and to generate new grounded theories, to understand the process by which events and actions take place, and to develop causal explanations. Sometimes, more than one of the above purposes would likely be achieved in one study, but this study employed the qualitative study in order to develop causal explanation to learner's threshold concepts in solving quadratic equations and functions with one known.

From a constructivist point of view, learner's interviewing was necessary because reflective ability is a major source of knowledge on all levels of mathematics. Students should be allowed to articulate their thoughts and to verbalize their actions which will ensure insights into their thinking processes. During such mental operations, insufficiencies, contradictions, or irrelevancies in forms of threshold concepts would likely to be spotted (Creswell, 2009). Learner's thoughts opened up a way to explain why a particular difficulty or error occurred while performing quadratic equations and functions with one known.

However, these interviews were not clinical in nature. Instead, learners were given a chance to elaborate their comprehension on threshold concepts through thinking such as in think aloud methods. The researcher prompted them with “explain more”, “go ahead” and “how” or “why” questions whenever necessary. Sometimes, the researcher would ask further questions or provide examples for further explanations but this was limited to the cases that needed more elaboration. Therefore, the study method of interviewing was a mix of think aloud procedures with a lighter version of interview questions regarding quadratic equations and functions with one known.

3.3 Research Design

Research design refers to the overall strategy that a researcher chooses to integrate the different components of the study in a coherent and logical way, thereby, ensuring it will effectively address the research problem; it constitutes the blueprint for the collection, measurement, and analysis of data. The research problem determines the type of design a researcher should use, not the other way around Trochim, (2005). Consequently, descriptive survey research design would provide answers to the questions of who, what, when, where and how associated with a particular research problem; a descriptive study cannot conclusively ascertain answers to why. The subject is being observed in a completely natural and unchanged natural environment. True experiments, whilst giving analyzable data, often adversely influence the normal behavior of the subject (a.k.a., the Heisenberg effect whereby measurements of certain systems cannot be made without affecting the systems). Descriptive research was used as a pre-cursor to more quantitative research designs with the general overview giving some valuable pointers as to what variables are worth testing quantitatively (Given, 2007).

This research design was the most appropriate approach for this study because the existing situation, of learners' comprehension of the threshold concepts in solving quadratic equations and functions with one known was described. It also described the gender performance (IV) on the overall performance (DV) of learners in secondary schools. The other characteristics of the independent variables was school type, and teachers' strategies used against the same DV. This design also enabled the study to get information relating to the recurrent poor Mathematics performance in Kericho County by finding out exactly the threshold concepts learners are facing, and identify a teaching strategy which can be used in teaching quadratic equations and functions with one known. In this design, questions were administered to participants through interviews and questionnaires and there after the researcher scored and described the responses given. This survey was both reliable and valid since the questions were constructed properly and well written so that they are clear and easy for learners to comprehend.

3.3.1 Research Variables

In order to facilitate the qualitative data analysis using electronic software (Nvivo Pro 11), the data comprised of 3 sources: documents (interviews, observation and document analysis), memos (methodological and procedural note memos) and PDF (sketches and excerpts). In addition to this, 24 cases were involved: 18 learners (2 good, 2 average and 2 poor performing learners from each school type) and 6 teachers (2 from each school type). The study also constituted 2 case classifications (teachers and learners) at the same time 13 attributes, 8 for learners (3 for age, 2 for gender and 3 for school type) and 5 for teachers (3 for teaching experience and 2 for gender). Lastly there were 5 theme nodes which were equivalent to the research questions.

The quantitative research study constituted sixteen variables; gender was taken to have two independent variables (male and female). Six dependent variables (DVs) which corresponded Bloom's taxonomy: knowledge questions, comprehension questions, application questions, analysis questions, synthesis questions, and evaluation questions. Each of these six dependent variables were defined as the percentage of questions or sub-questions from the total number of a given examination question that was classified in the same category of the taxonomy. Two intervening variables which moderated the effect exerted by the gender on academic performance were teaching strategy-N (problem solving, use of example and problem solving) and school type-M (single-gender and coeducation). Besides the six intervening predictor variables two additional intervening criteria variables were included because they might have deferred or isolated with one or more of the six dependent measures: low and high cognitive levels. These two variables were intervening because they were associated with one or more of the six dependent variables. Especially, low cognitive levels were created by collapsing of the first three levels of the taxonomy (knowledge, comprehension and application). While high cognitive levels included the last three levels (analysis, synthesis and evaluation) of the cognitive domains of learning.

3.4 Study Area

The study took place in one of the largest and most diverse County out of the 47 Counties of Kenya; Kericho County. It borders by Kisumu County on the west, Nyamira County on the south-west, and Bomet County to the south-east, Nakuru County to the east, Baringo County to the north and Uasin Gishu County to the north-west. Kericho County was selected because mathematics performance in this county has been far much below expectations. The classroom instruction was not carried out, and most interactions with participants took place outside the classroom. The performance of students in Mathematics in KCSE in Kericho East Sub County has also been low.

3.5 Target Population

The study targeted all the form four learners in Kericho County. According to county Director of education, there were 152 secondary schools of which 140 were public schools and the rest were private. There were 10,466 form four learners in public secondary schools.

3.6 Sampling Techniques and Sample Size Determination

3.6.1 Sample Size Determination

Since the target population was known and the study set significance level at 0.05, the sample size was determined, using the formula (Cochran, 1963):

$$n = \frac{Z^2}{d^2} pq$$

Where:

n represents the desired sample size (if the population is greater than 10,000).

p represents the proportion in the target population estimate to have characteristics being measured.

Z represents the standard normal deviate at the required confidence level.

q represents 1-p

d represents the level of statistical significance set.

where: $z = 1.96$ $q = 50\%$

$p = 50\%$ $d = 0.05$

Therefore; $$n = \frac{1.96x1.96x0.05x0.05}{0.05x0.05} = 384$$

3.6.2 Sampling Techniques

Sampling refers to the process of selecting the sample from a population to obtain information regarding a phenomenon in a way that ensures that the population was well represented (Cohen & Arieli, 2011). The objective in qualitative research of this study was to select participants who were best able to give the researcher access to a special perspective, experience or condition which the researcher wishes to understand (Tavakol & Dennick, 2011).

3.7 Sampling Procedure

Qualitative sampling is theory-driven because the selection of participating entities, settings and interactions are determined by the conceptual question and not a concern for representativeness (Cohen & Arieli, 2011). This study employed purposive sampling as its typically informal and based on the expert judgment of the researcher or some available resource identified by the researcher and the selected cases offers rich data from which the most can be learned (Teddlie & Yu, 2007). The sample consisted of very high performing students and very low performing students in order to be able to provide the researcher with rich information.

Regarding sampling, identifying units of analysis is an important part of the research. Once a research question has been identified, selecting units of analysis is part of the process of deciding on a research method and how it will be operationalized (Polanin, Maynard, & Dell, 2017). The unit of analysis is the object which is to be studied in terms of research variables that constitute the construct of interest. In this study, the learners was the unit of analysis, which is a construct located in the sample. The written test provided evidence of the learners' comprehension and performance in solving quadratic equation and functions with one known. Probabilistic sampling technique was applied and all the schools were categorized into three strata based on school type. Hence, the county had 24 boys' schools, 23 girls' school and 93 mixed schools. Participating schools were proportionately distributed depending on the number of schools in each stratum as illustrated in table 3.1 below.

Table 3.1: Proportional Distribution of Schools

School Type	Boys	Girls	Mixed
Schools	24	23	93
Proportion	1	1	4

Participating schools therefore, was randomly selected based on the registration list in Kericho County Education office. Each school was assigned a number corresponding to its serial number in the registration list, folded and shuffled in a basket, and then participating school in each stratum was picked without replacement. The procedure was repeated until the required number of schools in each stratum was attained; 5 boys, 5 girls and 20 mixed schools participated in the study.

Participating learners were selected for the quantitative data based on the proportion of the number of learners in each stratum. Since there were 2990 boys, 1756 girls and 4720 learners in boy's, girl's and mixed schools respectively, a proportion was calculated by dividing each stratum with the smallest number, 1756, as shown in table 3.2.

Table 3.2: Distribution of Participants

School type	Boys	Girls	Mixed
Leaners	2990	1756	4720
Dividing with the smallest	$\frac{2990}{1756}$	$\frac{1756}{1756}$	$\frac{4720}{1756}$
Results in decimal fraction	1.7	1	2.7
Proportion	3	2	5

The study distributed participating learners in each stratum disproportionately using 3:2:5, (boy's school contributed $\frac{3}{10}$ of the participants, girl's school $\frac{2}{10}$ and mixed school $\frac{5}{10}$ learners) from the sample size. This gave a total number of boys from boy's school to be 115, girls from girl's school were 77 and total participants from mixed schools were 192 where equal number of boys and girls were selected equally. A form four class index numbers from each eligible school was used and computerized table of random number was used in selecting learners.

Consequently, the selection of informants for qualitative data was identified as they attempted the questions from the diagnostic instrument through observation and with the assistance from the Mathematics teacher picked 3 learners from a boy's school, 3 learners from a girl's school and 2 boys and 2 girls from a mixed school whose performance above average. These informants were interviewed in order to get in-depth information on learners' threshold concepts in solving quadratic functions and equations with one known.

3.8 Pilot Study

A pilot study was conducted in 3 schools in Nandi County since it has almost the same educational standards as Kericho County. A pilot study was done in a mixed school in order for the researcher to get used to the logistical requirements of the actual study for instance getting an adequate and a manageable number of learners for example selecting learners for interviews. The test items in the research instrument went through a rigorous process. Firstly, the researcher asked Mathematics teachers from the pilot studies to verify if the items were consistent with their own analysis discussed and worked through the problems with the researcher.

Test items were prepared by considering three main aspects. There were items that were directly related to the basic concepts of quadratics. In them, students needed to explain some basic properties in quadratics or they had to identify patterns or relationships and represent or interpret them quadratically. Some of them contained quadratic manipulations. Problems without a specific context pertaining to simplification of quadratics expressions, evaluating expressions, and solving equations formed examples of this group. The next type of questions was the word problems that learners needed to represent quadratically in order to solve. These items usually appear in day-to-day life. Most of them were contextual problems. In some of the short-answer problems in the test, students had to provide and justify their answers by using Mathematical language or other representations. In this way, the lapses of their difficulties were identified.

For each problem, the researcher asked teachers to describe how learners might solve the problem and to anticipate different solution strategies, including alternative correct strategies (different from the teacher’s own strategy), incorrect strategy, and mistakes. The researcher asked teachers to consider the incorrect strategies and difficulties they anticipated learners making and to identify what a student who uses that strategy knows about quadratic equations and functions with one known. These discussions helped to understand threshold concepts in solving regarding the study. The teachers' information helped refine the research instruments and to design follow up questions that the teachers thought the learners might use.

3.9 Reliability and Validity of the Research Instruments

Trustworthiness in qualitative research is evaluated differently from quantitative research as the widely considered concepts of validity and reliability are inapplicable. Partly due to the association of the terms with positivist positions which states that there is an objective reality or truth out there that can be attained, a position qualitative researchers contest (Mason, 2000 as cited in Jwan & Ong'ondo, 2011). Summarizing the rationale of the terms used in qualitative research, Creswell & Miller pointed out that the constructivists believe in pluralistic, interpretative, open-ended, and contextualized perspectives towards reality. Since this research study takes the form of a sequential explanatory mixed method approach, the researcher enhanced the trustworthiness of the research instruments as illustrated below.

3.9.1 Validity of the Research Instruments

The validity of a test instrument is equally important as its reliability. If a test does not serve its intended function well, then it is not valid. According to Brinberg and McGrath, (1985), validity is not a commodity that can be purchased with techniques instead; it's like integrity, character and quality to be possessed relative to purposes and circumstances. Validity in broad sense is concerned with the relationship between an account and something external to it, that is, the phenomenon that the account is about and whether that phenomenon is interpreted as objective reality or a variety of other possible interpretations for the same phenomena. Validity concerns in this study were addressed by the thick description provided as an essential component of both quantitative and qualitative research enterprise. Therefore the establishment and maintenance of validity in this research were critical and of paramount importance. The credibility is involved in establishing that the results of the research are believable. This is a classic example of ‘quality not quantity’. It depends more on the richness of the information gathered, rather than the amount of data gathered (retrieved from http://www.angelfire.com/Trustworthypaper.pdf).

In order to ensure trustworthiness in qualitative study and agreement with constructivist-interpretivist paradigm, its credibility was evaluated. Credibility is the extent to which the study actually investigates what it claims to investigate and reports what actually occurred in the field (Creswell & Miller, 2002 as cited in Jwan & Ong’ondo, 2011). Basing on the expositions in research literature, the researcher addressed the question of credibility, the key concept in this study by exercising triangulation. Triangulation was used in qualitative research to mean the use of multiple approaches, methods, techniques and/or source of data and is noted as a way of strengthening the trustworthiness of a study, while also facilitating a researcher to gain a deeper understanding of the phenomenon under study.

Mason, (2002) as cited in Jwan and Ong’ondo, (2011) concurs that triangulation aids a researcher in exploring diverse aspects of the entity under study, answering various research questions or the same question but different perspectives, trying to uncover all aspects of a phenomenon and seeking to “collaborate one source and method with another, or to enhance the quality of data”. In relation to the current study, a test, interviews and observation was used to generate detailed data about students’ difficulties in solving quadratic equations and functions with one known. Each qualitative research study is unique and cumulative in nature. The more a researcher interviews participants and observe them in natural social settings, the more ideas one generate for a study (Saldana, 2009).

3.9.2 Reliability of the Research Instruments

Jwan and Ongondo, (2011) defined reliability as the extent to which a researcher provides sufficient detail and clarity of the research entire process in a way that would make it feasible for a reader to visualize and appreciate and for a researcher to replicate the study, if necessary. The reason for reliability in qualitative study, according to Yin (2003, as cited in Jwan & Ongondo), is "to be sure that if a later investigator followed the same procedures as described by an earlier investigator and conducted the same study all over again, the later investigator should arrive at the same findings and conclusions ….the goal of reliability is to minimize the errors and biases in a study". If a test is reliable, all the items should correlate with one another. If the items are highly correlated with each other, the whole test then should correlate highly with an alternate form (Nunnally & Kotsch, 1983). Measurements are reliable if they reflect the true aspects but not the chance aspects of what is going to be measured Gilbert, (2000). Thus, internal consistency of a test is essential for it to serve its purpose.

To check the dependability of the qualitative study, the researcher used dependability audit which is a major technique in which the supervisors reviewed the activities of the researcher to see how well the techniques for meeting the credibility and transferability standards have been followed. In order to develop a detailed audit trail, a researcher maintained a log of all research activities, develop memoirs, maintain research journals, and document of all data collection and analysis procedures throughout the study (Creswell & Miller, 2000). Carcary (2009), while citing Lincoln and Guba, (1985) discusses six categories of information that need to be collected to inform the audit process as raw data, data reduction and analysis note, data reconstruction and synthesis products, process notes, materials related to intentions and dispositions and preliminary developments information. This enabled the researcher to reflect on how a study unfolded. Further, it helped readers to follow each stage of the process and trace through the research logic and helps other researchers determine whether the study's findings may be relied upon as a platform for further inquiry and as a basis for decision making.

In this regard, dependability ensures that the research findings are consistent and could be repeated and measured by the standard of which the research is conducted, analyzed and presented. Each process in the study should be reported in detail to enable an external researcher to repeat the inquiry and achieve similar results. This also enables researchers to understand the methods and their effectiveness. Furthermore, audit ability' implies that another investigator could come to equivalent findings given the same data and research context. The auditor considers the whole process of research along with the product, data, findings, interpretations, and recommendations (Fives & DiDonato-Barnes, 2013).

Consequently, reliability of the quantitative research study was done by determining Cronbach alpha coefficient using the data collected from the pilot study. The number of test items, item interrelatedness and dimensionality affect the value of alpha (Tavakol & Dennick, 2011). There are different reports about the acceptable values of alpha, ranging from 0.70 to 0.95. A low value of alpha could be due to a low number of questions, poor interrelatedness between items or heterogeneous constructs. So from the pilot study, results which had low alpha due to poor correlation between items then were revised. A Cronbach alpha coefficient (α) of 0.85 was found and the research instrument would measure what it purported to measure.

The easiest method the study employed to calculate construct reliability was to compute the correlation of each test item with the total score test. This was possible by performing item analysis in the process of standardizing the diagnostic research instrument. Based on this statistics, the researcher was able to; identify and eliminate test items that were too easy or too difficult by calculating difficulty index. A difficulty index of 0.54 was found which gives the proportion of the participant who answered an item correctly and lies between 0.9 and 0.1; items with low correlations (approaching zero) was deleted. A discrimination index indicates how well an item discriminates between participants who performed well and who performed poorly. This was calculated in order to get a correlation between an item and the total score on the construct. A discrimination index belongs and should be higher than 0.2. It identify shortcomings in items such as bad wording. The pilot study results found a discrimination index of 0.39 which was therefore regarded as a good instrument. If alpha was too high it would have suggested that some items are redundant as they were testing the same question but in a different guise. A maximum alpha value of 0.90 has been recommended (Nunnally, 1972).

3.10 Data Collection Instrument

Data was generated using interview guides and questionnaires as detailed below:

3.10.1 Teachers Pre-lesson Interview

The teachers were interviewed before each lesson to find out more about their instructional content. The purpose of these interviews was to find out how teachers organized the lessons and the teacher’s knowledge of key concepts to be taught. Also observed were the teaching strategies used, how the teacher planned to assist students with difficulties, assessment tasks, any expectations of learner’ misconceptions that the teacher might have had and also to triangulate lesson observation data. The questions that were used during the interviews are presented in appendix A.

3.10.2 Questionnaire for Mathematics Teachers

This questionnaire sought to find out how the mathematics teachers acquired the mathematics teaching knowledge that they had which helped them to teach effectively; by producing good results. The questions that were used are tabulated in appendix B.

3.10.3 Learner's Diagnostic Test Instrument

Diagnostic tests instrument was used to provide the researcher with essential information used to make decisions about learners' difficulties. A table of specification was used to help frame the decision making process of test construction and improved the validity based on tests constructed for use in the study. The table of specification provides a two-way chart to help teachers relate their instructional objectives, the cognitive level of instruction, and the amount of the test that should assess each objective (Nortar et al., 2004, as cited in Fives & DiDonato-Barnes, 2013).

Table 3.3: Table of Specification

Levels	**Lower Levels**			**Higher Levels**			**Total**
	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation	
Distribution	1	3	2	1	1	1	9
Questions	1	2,3,4	5,6	7	8	9	9
Marks	6	7	7	10	10	10	50

Typically, because longer tests can include a more representative sample of the instructional objectives and learner performance, they generally allow for more valid inferences. However, this is only true when test items are good quality. Furthermore, learners are more likely to get fatigued with longer tests and perform less well as they move through the test. Therefore, the ideal test is one that learners can complete in the time allocated, with enough time to brainstorm any writing portions, and to check their answers before turning in their completed assessment. Every objective does not need to be assessed in every assessment. A table of specification can help to make sure that the most relevant objectives are assessed and that a sampling of less prominent ones is also included. A learner when preparing for a test studies everything and gains an understanding of the content. What can actually be assessed is only a sampling of the students' knowledge at a particular point (Moeti, 2016).

3.11 Data Collection Procedure

Data was generated in two phases: quantitative and qualitative data.

3.11.1 Quantitative Data

The first phase involved the quantitative data in which the participants attempted a written task of about 40 minutes consisting of nine questions on solution of quadratic functions and equations with one known. The tasks were selected from previous examination questions as well as from revision secondary Mathematics text books. The questions varied in nature as well as the degree of their complexity as was designed based on (Bloom, 1956) cognitive domains of learning, (appendix A). The participants were instructed to show all the working to enable analysis as well as to make thorough observations of the learners' threshold concepts. Strict control measures were taken to ensure that no participants accessed to reference material during the writing of the test. This was easily attained by administering the test in a room different from their classroom. The learners also requested to sit at least a meter apart from each other in order to minimize interactions among themselves. During the examination, the researcher purposefully selected four participants based on their performance rated from very good, good, average, and weak. The selected learners were interviewed in the second phase of the research to explain the difficulties observed in their answer script.

3.11.2 Qualitative Data

The second phase of data generation used interviews and observation as the major data generation technique. The emphasis in this study was that words that make up qualitative data ought to be as much as possible, repeat the voices of the participants involved in the study. Alongside other sources of data generation techniques used were interviewing. Interviews were intended to get what a participant in research thinks, the attitudes of that person, and/or explore person's reasons for thinking in a certain way or for carrying particular perceptions or attitudes. Interview schedule was used and the agenda was totally pre-determined by the researcher who works through a list or a set of questions in a pre-determined order (Jwan and Ong'ondo, 2011). Interview was conducted using semi-structured interviews that involved image taking of student's work, to enable the researcher to have a set of well-structured interview questions while allowing flexible use of the questions and responding to what learners have (Merriam, 2009).

Jwan and Ong'ondo, (2011), citing (Cohen et al., 2007), reported that in qualitative research, semi-structured interviews allow deeper exploration of responses by participants-probing and exploring emerging dimensions that may not have been previously considered pertinent aspect of a study. The researcher interviewed each learner independently for about ten minutes. The interview questions were mostly generated from the learners' various responses from the written tasks as described in the interview schedule in appendix D. For each question, learners were asked to describe their understanding and how they solved the problem. The researcher would offer to be asked questions that the learners were uncertain about.

The following elements were assessed during lesson observations:

a) Knowledge of the content topic in which the teacher is engaged, that is, conceptual knowledge and procedural knowledge where the following were used to assess this element: accuracy of Mathematical facts presented; flexibility of presentation; sequential representation of facts; flow of ideas and hierarchical presentation of facts,

b) Teachers' knowledge on quadratics was investigated in the following aspect: how they planned and teach using various teaching strategies, how they engaged learners through questions and assessment tasks, how they used the chalkboard during lesson presentation on topics of quadratic functions. Data were collected via observation of the teachers' lesson presentation, and one-on-one interviews with each Mathematics teacher to find answers to the research questions (appendix B for format and scoring of each instrument).

c) Knowledge of learners' conceptions (misconceptions and pre-conceptions) about the topic under discussion where the following were used to check the teacher's knowledge of this element: assessing learners' concept formation; identifying threshold concepts learners makes and determining sources of such threshold concepts; identification of misconceptions and elimination of them by probing questions; and using appropriate tasks.

Observation is one of the methods of data generation technique in this study. Observation means getting data through critically watching a person or persons as they participate in a particular activity with a view to obtaining deeper understanding about the activities the person under study are engaged in, the observer "gathers data 'live' from 'live' situations" (Cohen et al., cited in Jwan & Ong'ondo, 2011). Just like in interview, observation employed semi-structured format thus involved paying attention to the whole event as it took place and taking field notes on what were considered salient issues which are developed as transcripts to be later analyzed qualitatively, (Borg, 2006; Richards, 2003; Mason, 2002; Cohen et al, 2007, cited in Jwan & Ong'ondo, 2011). The current study involved classroom observation of the teaching and learning of quadratic equations and functions with one known and employed focused observational procedure. The observation protocol schedule assessed how teachers presented their lessons in order to assist learners in comprehending the topic; how the teachers assessed their learners after the lessons; the teaching strategies employed; and how the teachers dealt with learners' difficulties during lesson presentation. Appendix B depicts the classroom observation protocol that was used during lesson observations.

3.12 Data Analysis

Since this study utilized a mixed method research paradigm data was analyzed in two stages; qualitative data first then followed by quantitative data as illustrated in table 3.4 below.

Table 3.4: Data Analysis Table

	Objective	Data Type	Analytical Technique
i.	To discuss the influence of teaching strategy on learners' performance in quadratic equations and functions with one known	Quantitative	One-Way ANOVA
ii.	To describe learners' scores in solving quadratic equations and functions with one known	Qualitative	Thematically
iii.	To analyze learner's difficulties in solving quadratic equations and functions with one known that may attribute to gender	Qualitative	Thematically
iv.	To determine gender difference if any, that may exist in the performance of quadratic equations and functions with one known		One-Way MANOVA
v.	To determine relationships if any between gender, teaching strategy and school type on one hand, and performance in quadratic equations and functions with one known on the other hand	Quantitative Quantitative	2X2 Factorial ANOVA Moderated Multiple Regression Analysis

3.12.1 Qualitative Data

Jwan and Ong'ondo, (2011) citing (Yin, 2003 & 2009) defined data analysis in qualitative data to involve looking at the data, assigning categories and putting together emerging issues into themes in an attempt to answer the research questions. But they defined it as a systematic process of transcribing, collating, editing, coding and reporting the data in a manner that makes it sensible and accessible to the reader and researcher for purposes of interpretation and discussion.

Therefore, qualitative data was analyzed thematically first by reading and listing the categories of experiences from the transcribed data and field notes then identifying all the data that illustrate the categories. Related categories were then combined into themes that emerged from the informants' responses which were pieced together to form a comprehensive picture of their collective experiences, which the researcher built a valid argument for choosing the themes by reading and making inference from the literature (Aronson, 1994 cited in Jwan & Ong'ondo, 2011). In the current study, qualitative data analysis took a deductive approach in which themes were derived from pre-determined objectives conceptualized from the literature during the process of reviewing literature.

Qualitative data is characterized by its subjectivity, richness, and comprehensive text-based information. Analyzing qualitative data is often a muddled, vague and time-consuming process. Qualitative data analysis is, the pursuing of the relationship between categories and themes of data seeking to increase the understanding of the phenomenon. Traditionally, researchers utilized colored pens to sort and then cut and categorized these data. The innovations in software technology designed for qualitative data analysis significantly diminish complexity and simplify the difficult task, and consequently make the procedure relatively bearable. NVivo Pro 11 (Computer

Assisted/Aided Qualitative Data Analysis) software, the qualitative data analysis software developed to manage the ‘coding’ procedures is considered the best in this regards (AlYahmady & Alabri, 2013).

Qualitative data analysis is a “process of bringing order, structure and meaning to the mass of collected data” (Marshall & Rossman, 1999). Such process is not an easy task since it is disordered, hard, and time consuming, even though it is an innovative and captivating method. Qualitative data analysis is, in fact, pursuing the relationship between categories and themes of data seeking to increase the understanding of the phenomenon. Thus, rather than being strict and procedures-based, the researcher is required to be alert, flexible and positively interact with data collected (Corbin & Strauss, 2008).

Since the qualitative data are text-based, the corner stone of analyzing these data is the coding process. Codes according to Miles and Huberman, (1994) are “tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study”. Codes often adhere to chunks of words, phrases, sentences or the entire paragraph. Coding involves pursuing related words or phrases mentioned by the interviewees or in the documents. These words or phrases are then combined together in order to realize the connection between them.

The manual method of data analysis from qualitative study normally follows a six point procedure involving transcribing the data, re-familiarizing with the data, first phase coding, second phase coding, third phase coding, and producing a research report and mainly questions will be analyzed as illustrated below. Although the stages used in the analysis of the data look sequential, they were iterative and built up on the previous stage as Judger, (2016) citing Braun and Clarke (2006) highlighted, ‘Analysis is typically a recursive process, with movement back and forth between different phases. So it's not rigid, and with more experience (and smaller datasets), the analytic process can blur some of these phases together.'

Given the innovations in software technology, electronic techniques of data coding are gradually being more employed to obtain rigor in dealing with such data. Moreover, using a computer basically "ensures that the user is working more methodically, more thoroughly, more attentively" Bazeley, (2013). But it should be made clear that Nvivo does not actually code the data and therefore, it is the researcher's responsibility and/but the software efficiently stores, organizes, manages, and reconfigures the data to enable human analytic reflection (Saldana, 2009). Cited in Judger, (2016), Ishak and Bakar (2012) states that:

> Nvivo is just another set of tools that will assist a researcher in undertaking an analysis of qualitative data. However, regardless of the type of software being used, the researcher has to dutifully make sense of all the data him or herself, without damaging the context of the phenomenon being studied. Inevitably, the software cannot replace the wisdom that the researcher brings into the research because at the back of every researcher's mind lies his or her life history that will influence the way he or she sees and interpret the world.

The software indeed reduces a great number of manual tasks and gives the researcher more time to discover tendencies, recognizes themes and derives conclusions (Wong, 2008 cited in Judger, (2016). Therefore, Nvivo was used in this research to manage data, manage ideas, query ideas, modeling visually and in reporting the findings, the following procedures modified from Bazeley, (2013) was employed.

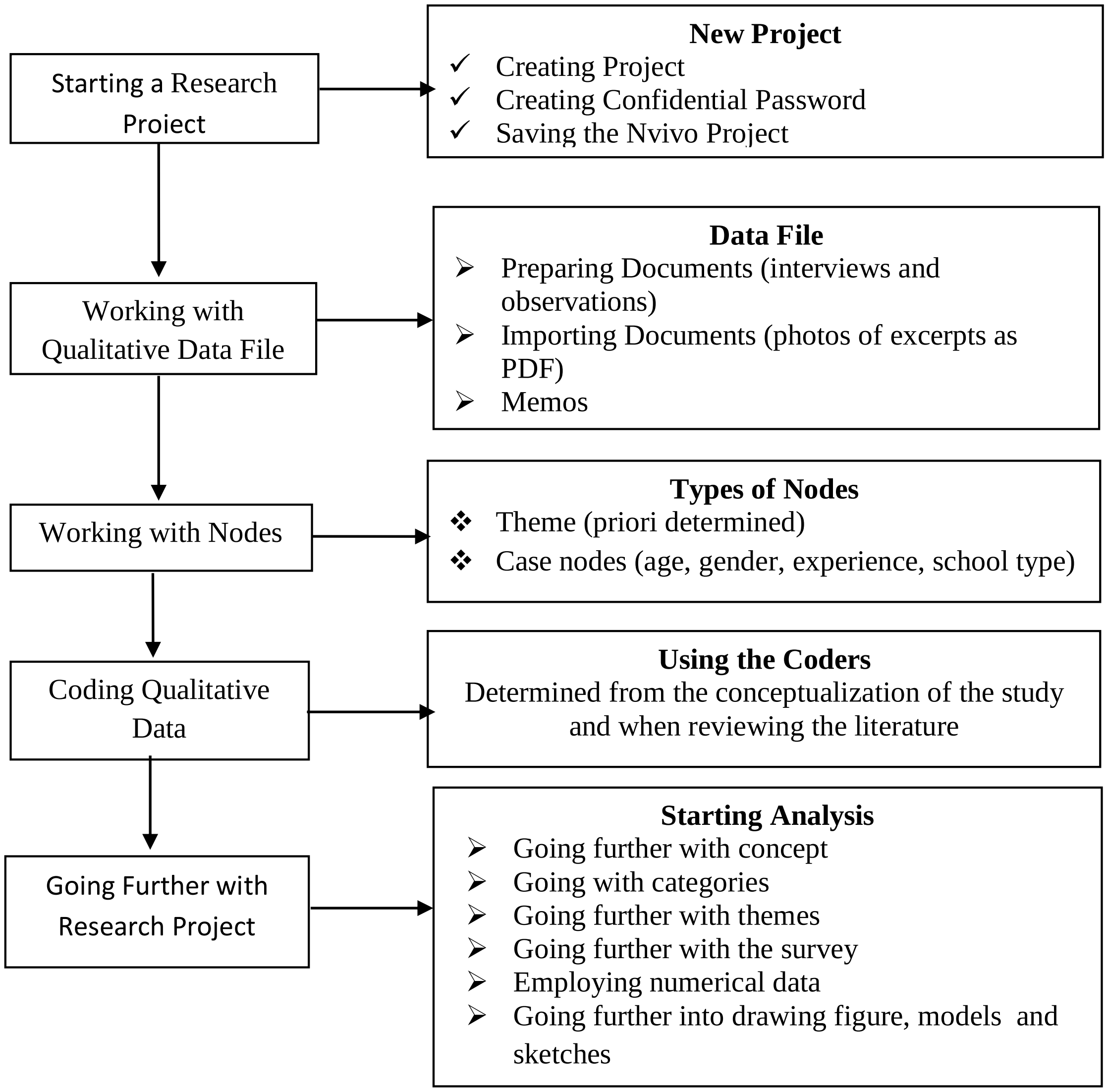


Figure 3.2: Procedure followed in Applying Nvivo Pro 11
Source: Modified from Bazeley, (2013)

3.12.2.1 Threshold Concepts attributed to Gender.

The researcher engaged in preliminary analysis during the interview of the participants. To aid in the formal analysis of the data, an organizational system was devised, by coding student's transcripts, looking across students and questions to identify themes and patterns. The initial set of codes resulted from the researcher's conceptual framework and the review of the literature and included: (a) solving code which describes moves that participants made as they solve problems and was further divided into solving techniques and graphing approaches. (b) Connections code which were mainly checked on the participant's knowledge between equations and equations and diagnosing the difficulties they made. (c) Justification code which sought to determine the quality of explanation participants provided. (d) Solutions code which gathered for the initial responses to the problem. These codes incorporated four categories; Correct, Incomplete, Incorrect, and Blank. This was coded in a 4-point Likert Scale, with 0 corresponding to blank response and 4 corresponding to a correct solution response.

The scores for each participant was entered in a table such that, the average score per problem as well as for each participant was found. The resulting scoring system which was used is shown in table 3.5 in order to make connections with respect to the codes described above.

Table 3.5: Participants Response Code

Participant Response Code	**Description**	**Score**
Correct	Solved using correct method and got correct answer	4
Incomplete	Correct method could get correct answer if moved 1-2 steps further	3
Incorrect	Attempted but used wrong method	2
Blank	Not able	1

Data collected from the observation was analyzed according to the following categories as indicated in appendix A:

a) Knowledge of the subject matter (checking for the teacher's conceptual understanding of the topic; display of skills in problem solving/procedural knowledge).

b) Knowledge of teaching strategies (checking for use of appropriate activities; use of real-life examples; and use of different teaching strategies).

c) Knowledge of the learners' conceptions (checking for the teacher's ability to address the learners' misconceptions; expectation of possible learners' difficulties; discussion

3.12.2 Quantitative Data Analysis

Stage two of data analysis involved mainly hypothesis testing as illustrated below.

3.12.2.1 Teaching Strategy and Performance

In order to determine the proportion of variability to each of several components of the teaching strategy used to teach quadratic equations and functions, one-way ANOVA was performed. One-way ANOVA compared the means of learners' performance that vary on a single independent variable (strategy).

3.12.2.2 Performance

To test the null hypothesis that there is no significant difference between low level cognitive performance and high level performance, a paired-sample t-test was used to compare means because the two variables were from the same dependent variable (performance). Paired–sample t-test was used to compare observations from 2 measurement occasions for the same group. The sample should be drawn from a continuous underlying distribution and groups should be from normally distributed and from independent population (Cronk, 2016). Low cognitive performance was formed by collapsing knowledge, comprehension and application performance, while analysis, synthesis and evaluation scores were collapsed to form high cognitive performance.

3.12.2.3 Gender Performance

In order to analyze for learners' gender and performance, a factorial design was used since it involves more than one independent variable (Cronk, 2016). It was described with a numbering system that simultaneously identified the number of independent variables and the number of levels of each variable. Thus, this study hypothesized that there was no significant difference between gender and students' performance. Since this was a 2 × 2 factorial design with 2 independent variables; gender had 2 levels (male and female), similarly performance had 2 levels (low and high).

3.12.2.4 Teacher's Strategy, Gender and School Type versus Performance

The proportion of independent variables explained by the dependent variable was analyzed using a moderated multiple regression technique in the ratio data between gender and the school type. Regression analysis allowed the researcher to make statements about how well one or more independent variables predicted the value of dependent variable (Cronk, 2016). Using the same SPSS macro process, the original moderated mediation model was tested, controlling for the effect of gender on learners' performance. Consequently, a moderator analysis was used to determine whether the relationship between learners' performance and gender (continuous dichotomous variable) is different for boys', girls' and mixed schools (the continuous dependent variable is students' performance, the continuous independent dichotomous variable is gender and the polytomous moderator variable is school type, consisting of three groups).

3.13 Ethical Considerations

Prior to conducting the research, an approval from Moi University, School of education was obtained. In the preliminary preparation, research authorization permit from the National Council of Science, Technology and Innovation (NACOSTI) was acquired. It was also mandatory to seek approval from the Ministry of Education, through Kericho County Director of Education to conduct research in their schools. Informed consent of public secondary school principals and teachers was obtained using relevant documentation (Appendix I).

These documents included informed invitation letters to the principals to conduct the research in their schools, and informed invitation letters to Mathematics teachers and learners for their participation in the study. Self-introduction to the participants prior to the test and the interviews, they felt more comfortable during the interviews by knowing that they could communicate freely with the researcher. Participation was voluntary and participants had the right to withdraw from the study at any time. During data collection, none of the participants, or schools was identified (pseudonyms was used) and participants were not judged or evaluated on their participation.

3.14 Chapter Summary

For the purpose of mixed method inquiry, qualitative phase was used to inform the quantitative phase. The two phases were integrated together to get better explanations about the main focus of this study on learner comprehension and performance in quadratic equations and functions with one known. The study employed a sequential explanatory design which is characterized by the collection and analysis of quantitative data in a first phase of research followed by the collection and analysis of qualitative data in a second phase that builds on the results of the initial quantitative results. In the quantitative phase, the researcher used diagnostic test instrument to identify and classify learners' comprehension and performance in quadratic equations and functions with one known and be able to diagnose student's difficulties. Consequently, descriptive survey research design helps provide answers to the questions of who, what, when, where, and how associated with a particular research problem; a descriptive study cannot conclusively ascertain answers to why.

In order to facilitate the qualitative data analysis using electronic software (Nvivo Pro 11), the data comprised of 3 sources: documents (interviews, observation and document analysis), memos (methodological and procedural note memos) and PDF (sketches and excerpts).

The quantitative research study constituted sixteen variables; gender was taken to have two independent variables (male and female). Six dependent variables two intervening variables which moderated the effect exerted by the gender on academic performance were teaching strategy-N (problem solving, use of example and problem solving) and school type-M (single-gender and coeducation). Besides the six intervening predictor variables two additional intervening criteria variables were included because they might have deferred or isolated with one or more of the six dependent measures: low and high cognitive levels.

The study targeted form four learners in Kericho County having 152 secondary schools of which 140 were public schools and the rest were private. There are 10,466 form four learners who were in public secondary schools. Hence, the county had 24 boys' schools, 23 girls' school and 93 mixed schools. For a matter of convenience, 5 boys, 5 girls and 20 mixed schools participated in the study.

Participating learners were selected for the quantitative data based on the proportion of the number of students in each stratum. Since there were 2990 boys, 1756 girls and 4720 learners in boys', girls' and mixed schools respectively. Therefore, a total number of boys from boy's school to be 115, girls from girl's school were 77 and total participants from mixed schools were 192 where equal number of boys and girls were selected equally. This study employed purposive sampling as its typically informal and based on the expert judgment of the researcher or some available resource identified by the researcher and the selected cases offered rich data from which the most can be learned. The sample consisted of very high performing learners and very low performing learners in order to be able to provide the researcher with rich information.

A pilot study was conducted in 3 schools (boys, girls and mixed) in Nandi County since it has almost the same educational standards as Kericho County. In order to ensure trustworthiness in qualitative study and agreement with constructivist-interpretivist paradigm, its credibility was evaluated. So from the pilot study, results which had low alpha due to poor correlation between items then were revised. A Cronbach alpha coefficient (α) of 0.85 was found and the research instrument would measure what it purported to measure.

The teachers were interviewed before each lesson to find out more about their instructional content. The first phase involved the quantitative data in which the participants attempted a written task of about 40 minutes consisting of nine questions on solution of quadratic functions and equations with one known. The researcher purposefully selected four participants based on their performance rated from very good, good, average, and weak. The selected learners were interviewed in the second phase of the research to explain the difficulties observed in their answer script. The second phase of data generation used interviews and observation as the major data generation technique. The qualitative data was analyzed thematically first by reading and listing the categories of experiences from the transcribed data and field notes then identifying all the data that illustrate the categories.

In order to determine the proportion of variability to each of several components of the teaching strategy used to teach quadratic equations and functions, one-way ANOVA was performed. To test the null hypothesis that there is no significant difference between low level cognitive performance and high level performance, a paired-sample t-test was used to compare means because the two variables were from the same dependent variable (performance). In order to analyze if there was learners' gender and performance, a factorial design was used since it involves more than one independent variable. Since this was a 2×2 factorial design with 2 independent variables; gender had 2 levels (male and female), similarly performance had 2 levels (low and high). Moderated multiple analysis was conducted to determine whether the relationship between learners' performance and gender (continuous dichotomous variable) was different for boys', girls' and mixed schools (the continuous dependent variable was learners' performance, the continuous independent dichotomous variable is gender and the polytomous moderator variable was school type, consisting of three groups).

CHAPTER FOUR

DATA ANALYSIS, PRESENTATION AND DISCUSSIONS

4.1 Introduction

This chapter focuses on information of the respondents. The respondents included form four Mathematics teachers, and form four learners from public secondary schools. It further presents the research findings, makes interpretation and discusses them based on research objectives. The purpose of the study was to establish learners' comprehension and performance on threshold topics in Mathematic in public secondary schools in Kericho County.

To attain this purpose, the analysis was conducted in the light of the following research objectives: To discuss the influence of teaching strategy on learners' performance in quadratic equations and functions with one known, to describe learners' scores in solving quadratic equations and functions with one known, to analyze learners' difficulties in solving quadratic equations and functions with one known that may attribute to gender, to determine gender difference if any, that may exist in the performance of quadratic equations and functions with one known and to determine relationships if any among teaching strategy, gender and school type on one hand, and performance in quadratic equations and functions with one known on the other hand. The study findings are presented using tables of frequencies, percentages, means, standard deviations and bar graphs for visual impressions based on the research objectives (ii) and (iii) in the case of qualitative data as illustrated below. While the research findings for objectives (i), (iv) and (v) were presented using one way ANOVA, one way MANOVA, and moderated multiple linear regression analysis respectively.

4.2 Background Information

4.2.1 Demographic Characteristics of the Learners

Learners' ages with respect to their gender was studied and found to be 17.6 years. Majority of the learners aged 17 – 18 years and a few aged less than 16 years and more than 19 years old.

4.2.1.1 Gender and Ages

The study investigated demographic characteristics of the learners and 2.1% male and 3.9% female learners aged less than 16 years representing minority of the learners. Male and female learners aged between 17 and 18 years were 46.9% and 35.9% respectively. Majority of learners who were 19 years and above were male whose representations were 6% compare to 5.2% female learners, as shown in the following figure.

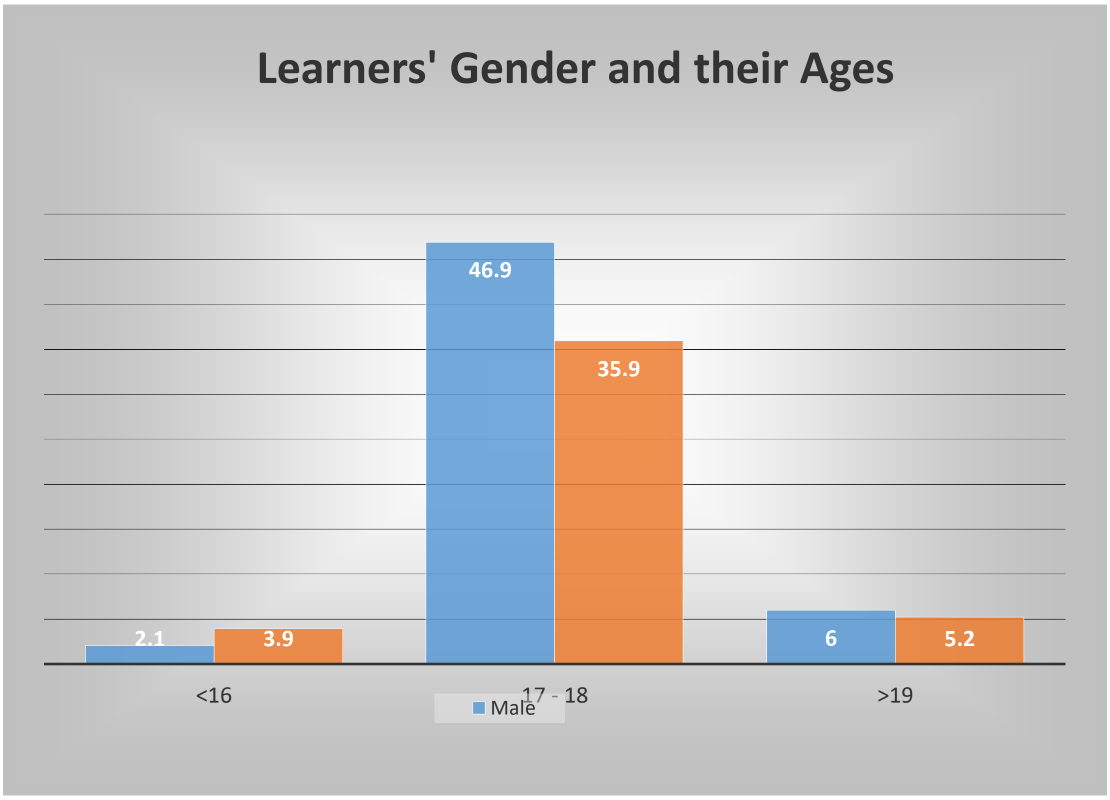


Figure 4.1: Learners' Gender and their Ages

4.3 Findings of the Study

4.3.1 Teaching Strategy

When designing and implementing a lesson, a teacher must draw upon the knowledge of the difficulties learners are experiencing in order to help them comprehend the threshold concepts being taught. The threshold concepts highlighted by the study should help Mathematics teachers in the process of planning instruction. If teachers know about potential concepts to understanding before teaching a quadratic lesson, they can devise well-developed lesson plans and use modified teaching strategies. The primary intention for teaching secondary school Mathematics is to assist learners to learn and appreciate Mathematics. This cannot be achieved without employing a correct teaching strategy that yields good performance.

The study interviewed the Mathematics teachers in a live classroom session before administering a test. The study was able to identify three majorly used teaching strategies; problem solving, lecture method and use of example. Learners who were taught using lecture method scored 30% (mean score = 28.94), the lowest marks among the strategies employed by the teachers. Mathematics teachers mostly assume that learners have understood the concept as expected without employing relevant teaching strategies which identify learners' threshold concepts in solving quadratic equations and functions. Teacher D taught the method of solving quadratic equations using quadratic formula and in his lesson, he asserted that:

> *Quadratic formula is found at the back of the mathematical table and you are required only to make substitution for the terms into the formula and solve for x.*

Completing square method normally precedes the method of solving quadratic method by formula method. Surprisingly no single teacher was found deriving the formula using completing square method. Teachers could have killed two birds using one stone rather than telling the learners that the formula was found at the back of the Mathematical tables, thus encouraged rote learning and creating more difficulties. Learners 3 liked using quadratic formula method in solving quadratics compared to factorization which she reported to have a difficulty in finding factors. She said when their teacher first introduce the formula, she crammed and she used to forget *a* in the common denominator but divide only by 2. She also reported that getting the determinant proved difficulty especially when c was negative.

In high school the quadratic formula was introduced to class by employing four kinds of representations in a row: symbolic, verbal, rhythmic, and metaphorical. The mathematics teacher first presented the formula for the general form of quadratic equations, $ax^2 + bx + c = 0$, then asked a learner to read the formula aloud, and asked the entire class to sing the formula as a rhythm: "x equals negative b, plus or minus the square root, b squared minus 4ac, all over 2a." the Mathematics teacher also told the story of the sad little bee three times as a phonetic cue:

> *The bee is sad (negative), and he is feeling wishy-washy, maybe he will go or maybe he won't (plus or minus). It's about going to the radical party. He's feeling a little squared, about the four awesome cheerleaders. The entire party was over, however, by 2am.*

Didiş et al., (2011) while citing Vaiyavutjamai and Clements (2006) proposed that learners' difficulties with quadratic equations stem from their lack of both instrumental understanding and relational understanding of the specific Mathematics associated with solving quadratic equations. They suggest that while lecture method of instruction with strong emphasis placed on the manipulation of symbols, rather than on the meaning of symbols, increases learner performance regarding solving quadratic equations, their (relational) understanding would still be quite low, and they could develop misconceptions.

Swan, (2005) reported that the approach a teacher takes when teaching a concept in Mathematics is influenced by their own conception of those concepts, as well as what s/he wants the learners to be able to do with those concepts. For example, if the teacher has a conception that mathematics is just about doing procedures correctly, s/he will teach a Mathematical formula and show the learners how to use it and then will expect the learners to be able to apply it. On the other hand, if the teacher has a conception that Mathematics is a reasoning science, the teacher expects the learners to be able to analyze the Mathematical formula, decide whether it has a solution and apply that formula only if necessary.

Likewise Manly and Ginsburg (2010 cited in Mamba, (2012) stated that the teaching of quadratic equations and functions is likely to focus on fundamental issues of symbol manipulation, simplifying expressions, and solving equations. This quick 'fix' approach is largely reliant on rote learning of progressions of actions and does not deliberately represent a coherent picture of quadratic equations and functions. Learners scarcely ever reach the kind of conceptual understanding and reasoning competence essential for the successful search of further goals. It is not unexpected then that this minimal teaching strategy is also inadequate to provide learners with enough information to select being in/out of developmental quadratic equations and functions. However, all teachers in Kericho County are professionally qualified to teach Mathematics and furthermore teaching quadratic equations and functions. The only challenge is that they focused on symbols and the apparently in-comprehensible rules that show procedures using them in the abstract domain. For instance, lecture method produced the lowest performance is a theoretical approach in which 'real' problems are inserted into some procedural lessons to provide practice in the new skill.

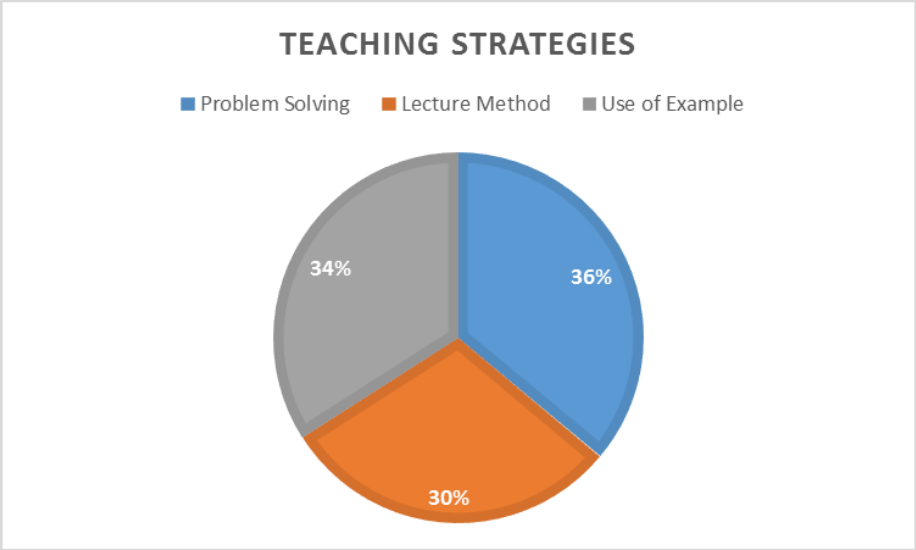


Figure 4.2: Teaching Strategies and Performance

The second teaching strategy used by most of the teachers in this topic was the use of examples in which the average score was found to be 34% (mean score = 28.96). Traditional, 'transmission' methods in which explanations, examples and exercises dominate do not promote robust, transferrable learning that endures over time or that may be used in non-routine situations, Swan, (2006b).

The average performance of the learners taught using problem solving strategy got a mean score of 36% (mean score = 34.94), the highest score compared to the other strategies used. Learners should be allowed an opportunities to tackle problems before the teacher interject with his pedagogical skills as this encourages them to apply pre-existing knowledge and their threshold concepts shall be noticed thus the teacher would be able to help them build on that knowledge, Swan, (2006b). A good teaching strategy encourages reasoning rather than ‘answer getting’; often, learners are more concerned with what they ‘have done’ than what they ‘have learnt’. It is better if teachers can use strategies that encourage learners’ reasoning in order to aim for depth than for superficial ‘coverage’. Swan, (2006b), asserted that if learners cannot develop concepts by themselves, they will have a narrow understanding of those specific concepts, and will not be able to engage themselves in problem solving. Concept formation works hand in hand with procedural knowledge and this can only be achieved through problem solving.

Rowland (2008 cited in Moeti, 2016) distinguished two roles and uses of examples in Mathematics teaching as; example of the general principle of a concept and example for doing exercise or practice. The role of example of a concept is to reflect a general principle of a concept by employing examples as particular instances of the general concept. Teacher A used topical examples because she wanted to develop and enhance abstraction and conceptualization using graphical method of solving simultaneous quadratic functions. She wrote the example, $f(x) = x^2 – x – 3; -3 \leq x \leq 3$ in order to demonstrate to the learners how to draw the graph of a quadratic function. The examples chosen and used by Teacher A were within the scope and definition of the parabola for learners to understand the concept. Just like Teacher A, Teacher B used the lesson specific example ($x^2 – 3x – 4 = 0$) to demonstrate how to solve quadratic equations using completing square method. She proceeded well using her procedural knowledge but her learners questioned how she arrived at $(x – 3/2)$ in her second last step. This was a clear indication that learners had a difficulty of taking the square root on both sides of the equation. Examples of doing exercises/practice are used as examples that facilitate procedural fluency and enhancement of conceptual understanding.

4.3.2 Learners' Scores

In order to describe the learners' performance in quadratic equations and functions with one known, a system of scoring was developed. Learners' scores were then categorized according to their responses in the answer scripts. The four categories included; Correct, Incomplete, Incorrect, and Blank. Then scores were coded in a 4-point scale in which blank score was coded 1, Incorrect = 2, Incomplete = 3 and Correct = 4. Gender performance across the categories was found and recorded in the following contingency table.

Table 4.1: Categorical Distribution of Scores

Gender	**Scores (%)**				
	1	**2**	**3**	**4**	**Total**
Male	2226 (45.87)	873(17.99)	263(5.42)	1491(30.72)	4853(100)
Female	1985(49.89)	815(20.48)	108(2.71)	1071(26.92)	3979(100)
Total	4211(47.68)	1688(19.11)	371(4.20)	2562(29.01)	8832(100)

After scoring the learners' answer scripts, the total number of learners across the scores was found to be 8832; there were 23 (questions) items in the questionnaire multiplied by the total number of learners (384). Every item was categorized within the organization scheme indicating that 47.68% (4211) of the learners had a blank score across both genders. Incorrect and incomplete scores were scored by 19.11% (1688) and 4.20% (371) of the learners respectively. The rest of the learners whose scores were correct were 29.01% (2562). This implied that over 50% of the female learners left the questions incomplete or blank. So most of the learners have not comprehend the threshold concepts and this might have affected their performance in quadratic equations. This made it possible to describe learners' score performance and across the question as illustrated in the following table.

Table 4.2: Learners' Scores

Question	Scores				
	Blank	**Incorrect**	**Incomplete**	**Correct**	**Total**
1a	13	300	36	836	3.09
1b	10	332	63	748	3.00
1c	31	278	27	820	3.01
2	71	226	6	792	2.85
3	41	402	102	432	2.54
4	46	332	39	636	1.78
5	219	192	15	256	1.78
6	169	382	27	60	1.66
7a	143	60	18	820	2.71
7b	176	68	12	680	2.44
7c	161	68	12	740	2.55
7i	290	166	9	32	1.29
7ii	299	152	6	28	1.26
8a	55	38	105	820	3.45
8bi	286	18	96	228	1.64
8bii	152	28	21	844	2.72
8biii	214	84	9	500	2.11
8biv	347	54	81	40	1.15
9a	244	32	66	408	1.95
9b	304	54	6	204	1.48
9c	298	84	126	176	1.45
9d	317	86	12	80	1.29
9e	325	78	9	68	1.24
Total Scores	**4,211**	**3,554**	**987**	**10,248**	**19,000**
Average	**1.00**	**2.11**	**2.66**	**4.00**	**2.44**

Learners' performances were scored from their answer scripts using the coding scheme mentioned above. Since all the questions were compulsory, it was possible to calculate average scores for each question and for every score {for instance 1a;$[\![(13x1) + (150x2) + (12x3) + (209x4) \div 384 = 3.09]\!]$ as shown in table 4.1 above. Average score per score was also calculated by dividing the total score by the number of learners in that score; that is blank average score was calculated as $4211 \div 4211 = 1.00$. The average score for this diagnostic test was found to be 2.44 out of a score of 4; this implies that most learners' scores were incomplete and incorrect. This might have affected the overall performance in Mathematics. Learners with incomplete scores according to Vygotsky (1970) regarded them to be in a complex stage. Learners, who are in this stage, are guided by complex thinking and therefore attend to one particular aspect of Mathematical expression and not see the whole.

4.3.3 Connections Learners Make

In order to describe well the connections learners make in the performance of quadratic equations and functions with one known, the study considered: connections which lead to incorrect solutions of the symbolic questions and those which lead to incorrect solutions of the word and graphical problems.

4.3.3.1 Erroneous Connections

The first method of solving quadratic equations is factorization. Learners factorized for example a question like Q1a: $2q^2 - 6q - 8 = 0\, factors\, (q-1)(q+4) = 0$ gets either $q - 1 = 0\, or\, q + 4 = 0$. Learners tend to make the following error $p^2 + 2p - 35 = 20\ factors\, (p-5)(p+7) = 20$. So learners would have, either$(p - 5)\, or\, (p + 7) = 20$. That error was witnessed with learner 146 and 287 respectively as shown in their excerpts below.

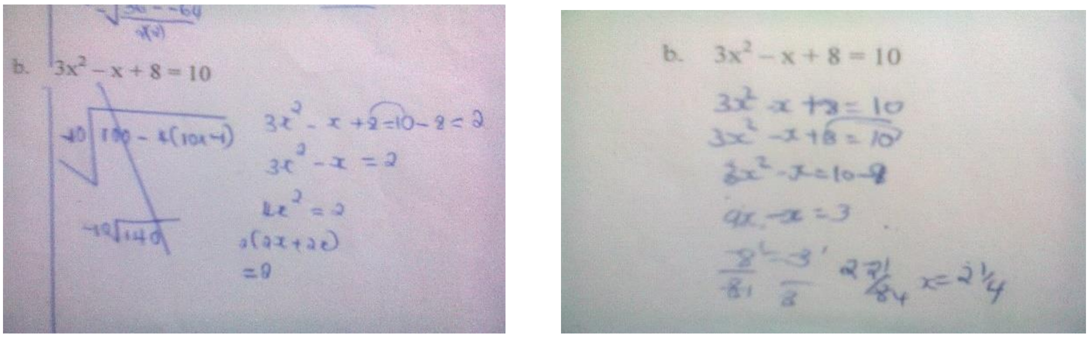


Figure 4.3: Excerpts of Erroneous Connections of Learners 146 and 287

In this question 1b, the learners were required to factorize the equation as follows: $3x^2 - x + 8 = 10$; $3x^2 - x + 8 = 10\ to\ get\ 3x(x-1) + 2(x-1) = 0. (x-1)(3x+2) = 0\ \ so\ that\ either\ (x-1)\ or\ (3x+2) = 0$ and the answer was $x = 1\ or\ x = -\frac{2}{3}$. Both learners collected the like terms in the quadratic equation by subtracting 8 from both sides and learner 146 got a correct answer 2 and unfortunately interpreted $4x^2$ to be the same as$(2x + 2x)$, but learner 287 got 3 instead of 2. The lack of comprehension by both learners was equating terms on the left hand side to the value on the right hand side; this made the equation to lose the concept of quadratic equation. Mamba, (2012) referred this type of difficult to be very hard to eradicate permanently. Even with performing learners, receiving excellent teaching emphasizing the special role of zero, this error often continues to crop up in learners' work. In spite of careful explanations of why it is an error and despite temporary elimination of the error, it keeps cropping up.

4.3.3.2 Linearizing Quadratic Equations

Quadratic equations and functions with one known use the concept of variable or an unknown just like in other algebraic expressions for instance indices. The symbolic notation for quadratic equations, which is a meaning familiar from the arithmetic context, has at times different meanings and uses in quadratic equations and functions. Many threshold concepts in quadratics equations are related to difficulties with arithmetic. The following are excerpts from two learners found to have the threshold concept.

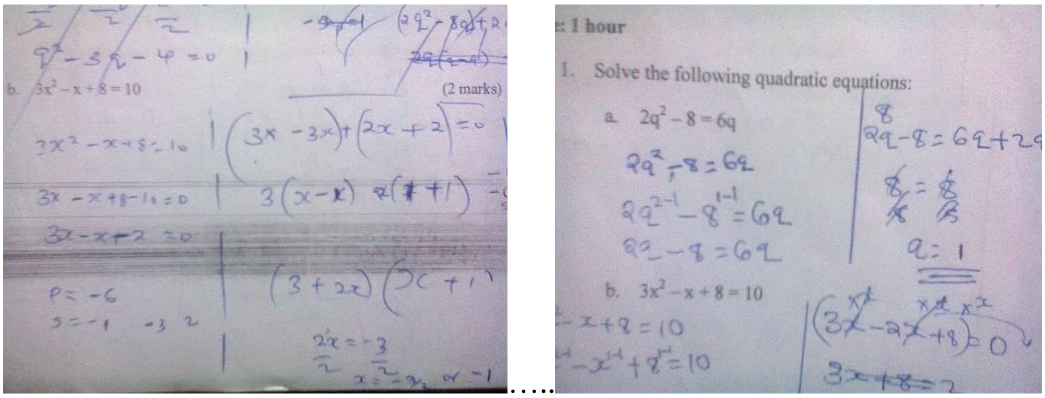


Figure 4.4: Linearizing Quadratic Equations by Learners 47 and 234

In question 1b, the norm was $3x^2 - x - 2 = 0$; $3x^2 - 3x + 2x - 2 = 0$ *to get* $3x(x-1) + 2(x-1) = 0. (x-1)(3x+2) = 0$ *so that either* $(x-1)$ *or* $(3x+2) = 0$ and the correct answer was x = 1 or x = -2/3. Learners 47 dropped the square sign and proceeded to find factors and yet the equation was already linear. The learner ignored the distributive laws when putting the brackets; somehow got $3(x-x)$ from $3x$-$3x$. While learner 234 introduced indices in the second step unnecessarily and reduced $2p^2$ into $2p$ and hence linearized the rest of the steps. This indicates the threshold concept of factorization. The lack of comprehension on threshold concept of learner 234 was the introduction of indices and thereafter linearizing the equation. This affected the performance in quadratic equations and functions. Both learners were found to be in what Vygotsky (1970) regarded as complex stage. In this stage the learners does not understand mathematical logic behind a threshold concept.

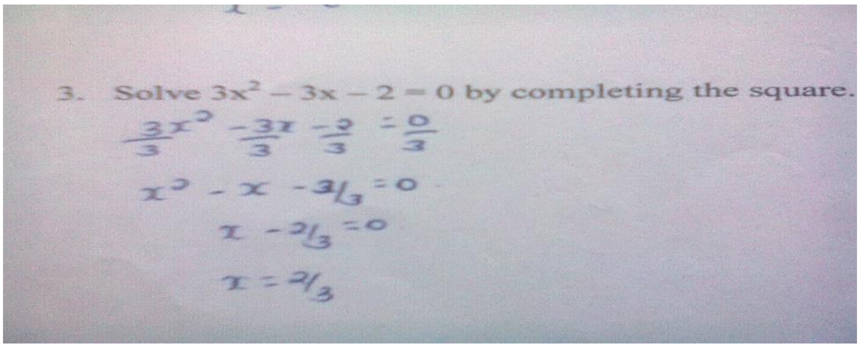


Figure 4.5: Linearizing Quadratic Equations by Learners 256

The first step in this question is to divide through by 3 to get; $x^2 - x + c = \frac{2}{3}$ $\left(c = (\frac{b}{2})^2\right), then\ x^2 - x + (\frac{1}{2})^2 = \frac{2}{3} + (\frac{1}{2})^2$ to get $(x - \frac{1}{2})^2 = \frac{2}{3} + \frac{1}{4}$ hence$(x - \frac{1}{2})^2 = 11/12$. Therefore, $(x - \frac{1}{2}) \pm \sqrt{\left(\frac{11}{12}\right)}$ and$x = 0.96 + 0.5\ or\ x = -0.96 + 0.5\ to\ get\ x = 1.46\ or\ x = -0.46$. But learner 256, as shown in the excerpt above subtracted $x^2 - x$ and got x; the learner interpreted the variables x^2 to be equal to$2x$. In that threshold concept the learner added unlike terms to each other. This implied that lack of the concept of like and unlike terms affected the performance in quadratic equations and functions. Similarly, Lesli 2015, while citing Berger (2005) pointed out that such learners associates the properties of a “new” Mathematical concept with an old one with which the learners were familiar with.

4.3.4 Methods of Solving Quadratic Equations

When a question does not restrict a particular method of solving quadratic equation like the question 1, learners would prefer solving using factorization, formula and completing square methods in that order. In question 1a, 58.07% of the learners used factorization method, 35.68% used the formula and only 9.38% used completing square method. Learner 1 used quadratic formula method in solving quadratic equations and functions with one known because the learner found it to be easier compared to the other methods. The concept the learner lacked in completing square method was making the equation a perfect square. In question 1b, 54.69% of the learners solved using factorization method as 37.24% applied formula method and 10.94% used the completing square method. Learner 3 liked using quadratic formula method in solving quadratics compared to factorization which she reported to have a difficulty in finding factors. She said when their teacher first introduced the formula, then she crammed and used to forget including ‘a’ in the common denominator but divide only by 2. She also reported that getting the determinant proved difficulty especially when c was negative.

Learner 2 had good comprehension in using quadratic formula method compared to other methods though they take more time than the formula method. Similarly, Learner 5 understood quadratic formula method in solving quadratics than any other methods, but still comfortable with completing square method. The learner reported that, when their teacher wrote the formula on the black wall, she first crammed and later mastered the formula after solving more problems. She reported that questioned must be in standard form before writing the formula and making substitution. Therefore, she had no problem as she got all the questions which required the use of quadratic formula. But she said other learners forget the plus and minus sign before the square root which finally would get one value of x. This implied that learner 5 had a very good comprehension on threshold concepts in quadratic equations and had good performance.

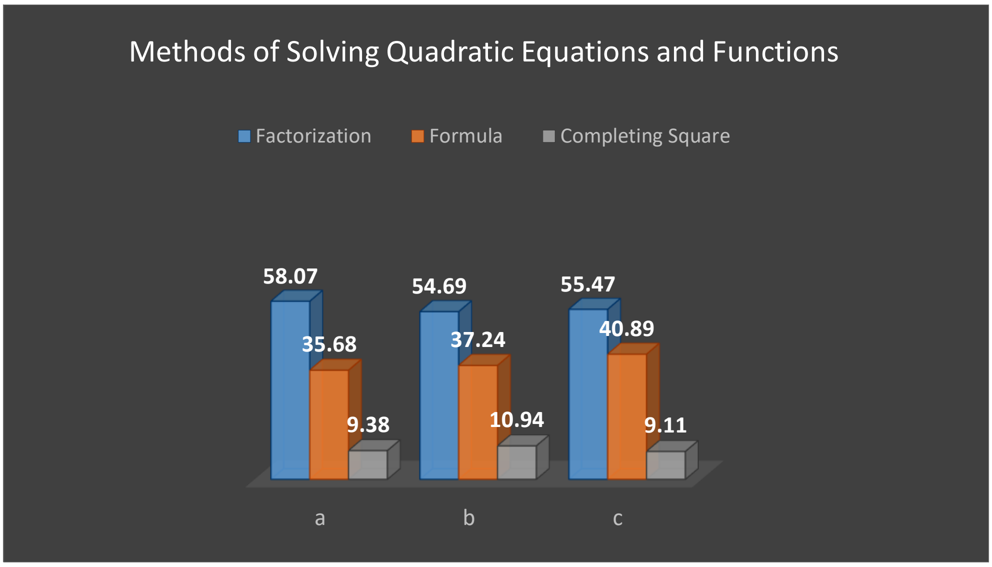


Figure 4.6: Methods used to Solve Question 1.

In question 1c $c^2 - 14 = 5c$, 55.47% of the learners factorized the question as 40.89% used the formula method and 9.11% used completing square method in solving. Learner 9 had the difficulty of making equation perfect square and taking the square root on both side of the equation while using completing square method. But the learner could not find the additive inverse of a fraction easily because he divided the equation which was not in standard form by the coefficient of x^2. Therefore, lack of concepts of completing square method affected the performance in solving quadratic equations.

4.3.5 Threshold Concepts

In order to determined learners' threshold concepts in the performance of quadratic equations and functions with one known, the study analyzed each question and categorized them as symbolic, word and graphical threshold concepts.

4.3.5.1 Threshold Concepts in Solving Symbolic Quadratic Equations

Threshold concepts in solving symbolic quadratic equations were categorized depending on the method of solving; for factorization, for formula and for completing square method.

4.3.5.1.1 Threshold Concepts in Factorization Method

Table 4.3 below indicates the threshold concepts in the performance of symbolic quadratic equations using factorization method in question 1.

Table 4.3: Threshold Concepts in Factorization Q1

Threshold Concepts	Frequency (%)		
	a	b	c
Factorizing Unstandardized Equation	13 (3.4)	`19 (4.9)	23(6.0)
Equation Equated to a Constant	38 (9.9)	9 (2.3)	2 (0.5)
Zero Product Property	11 (2.9)	9 (2.3)	15 (3.9)
Incorrect Factors	6 (61.6)	24 (6.3)	15 (3.9)
Imposing Linear Structure	43 (11.2)	39 (10.2)	42 (10.9)
NA	161 (41.9)	174 (45.3)	116 (30.2)
Correct solution	112 (29.2)	110 (28.6)	171 (44.5)
Total	**384 (100)**	**384(100)**	**384(100)**

The study established why and what makes learners choose a certain method in solving problems in quadratic equations and functions with one known. Consequently, examining learners' answers in choosing the method of solving the problems provided one of the ways to assess learners' threshold concepts in other methods. This was crucial in the view of the fact that provides the reasons can be identified, and then it should be trouble-free to improve the learners' performance to solve a kin quadratic problem in future. In this regard, when learners solved a quadratic equations, some factorized the equations which were not in standard form in the vein of Q1a; $2q^2 - 8 = 6q$ and Q1c; $c^2 - 14 = 5c$ where 3.4% and 6.0% of the learners lacked comprehension on threshold concepts, as indicated in table 4.3 above. Learner 19 guessed the sum and product of the equation to be – 3 and – 2 respectively and maintained the value on the right hand side of the equal sign. Learner 287 as shown in the figure 4.9 below got rid of the square wrongly by squaring the coefficient of x^2 and got 9x. The learner collected like terms to get a wrong value 8. The two learners' lack of comprehension might have affected performance in quadratic equations and functions.

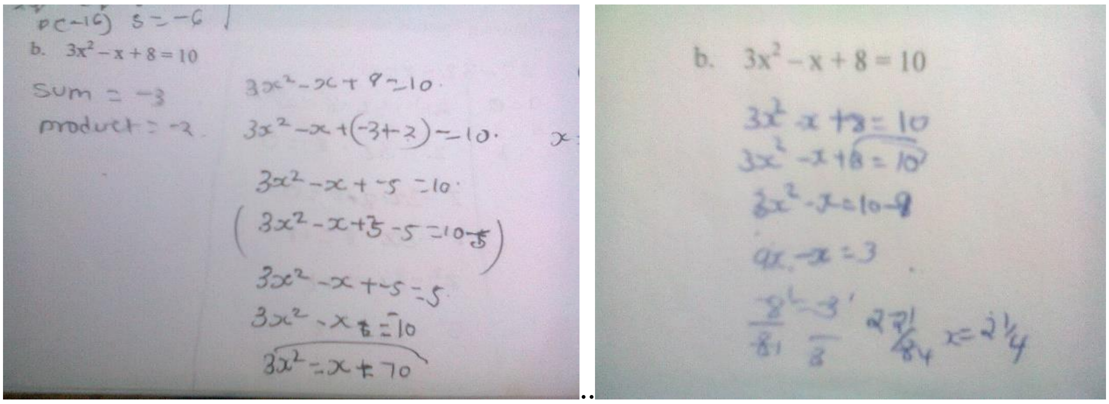


Figure 4.7: Factorizing Unstandardized Equation by Learners 19 and 287

This is a clear indication of lack of fundamental concepts of quadratic equations and functions with one known as designated by 41.9 % 45.3% and 27.3% of the learners in Q1(a, b, c) who considered the method NA. Similarly, Resnick (1982 as cited in Yahya & Shahrill, (2015) stated that threshold concepts in learning are often a result of failure to comprehend the concepts on which procedures are based. Thus, it is important for teachers to develop insights into learner solving in order to identify their threshold concepts.

Other learners did not rewrite their equations in the form of $ax^2 - bx + c = 0$ and therefore, were unable to factorize equation which is equated to a constant as did by 9.9%, 2.3% and 0.5% of the learners. This implied that lack of comprehension on factorization might have affected performance in quadratic equations and functions. The abstract nature of algebraic expressions posed many problems to learners such as understanding or manipulating them according to accepted rules, procedures, or algorithms. Inadequate comprehension of the uses of the equal sign and its properties when it is used in an equation was a major problem that hindered learners from solving equations correctly, Mamba (2012).

The zero-product property is the main procedure in solving quadratic equations by factoring. As shown in table 4.1 above, 2.9%, 2.3% and 3.9% of the learners who applied this method had this threshold concept in solving Q1.

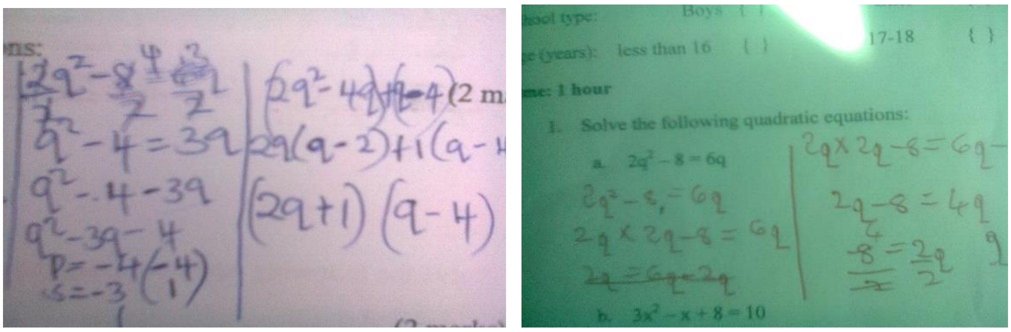


Figure 4.8: Threshold Concept of Zero-Product Property of Learners 74 and 268

The zero-product threshold concept pertains the reasoning that if two factors have a product of zero, then one or both factors must be zero. Therefore, learners would factor one side of an equation to have an equivalent equation such as Q1c; $(c+2)(c-7) = 0$. This results in the two equations$(c+2) = 0$ and $(c-7) = 0$, which can each be solved for x. in figure 4.10 above, learners 74 and 268 did not have that concept. Though the learner could have first rearranged the equation into standard form, there was nowhere which showed that the learner was aware of zero-product property concept. The literature credits a lack of comprehension on the concept of the zero product property as the reason for why learners solve equations such as $x(x+2) = 0$ by dividing both sides by x, thus losing track of the solution x +0, (Kotsopoulos, 2007). Nielsen, (2015), found that learners who were able to factor one side of a quadratic equation were usually able to apply the zero-product property and solve. However, most learners' explanation lacked in completeness, and many provided explanations that appealed to authority regardless of how well they were able to apply the zero-product property.

Even at the post-teaching stage of factorizing a quadratic equation and function with one known, learners still faced lack comprehension of threshold concepts determining the factors. Learners made the wrong use of the third term multiplication factors when doing the splitting method, popularly remembered as product, sum and factors and this might have affected performance in quadratic equations and functions. Matz (1982 cited in Mamba, 2012) explained the persistence of this error that there are two levels of procedures guiding cognitive functioning, namely surface-level procedures, which are the familiar rules of arithmetic and algebra, and deep-level procedures, which create, modify, control and typically guide the surface-level procedures. So, in order to learn quadratic equations and functions a learner should have such a deep-level procedure to overgeneralize numbers; that is, the student must believe that certain procedures work irrespective of the numbers used.

Incorrect factors used in finding the multiplication factors were used to solve the quadratic equations one as indicated by 1.6% of the learners lacked comprehension on the concept in Q1a, 6.3% in Q1b and 3.9% in Q1c and affected performance. Similarly, Lima (2008) and Tall et al. (2014) documented that learners perceive quadratic equations as mere calculations, without paying attention to the unknown as a fundamental characteristic of an equation. Learners mostly focus on the symbolic world to perform operations with symbols. For example, learners used procedural embodiment associated with the exponent of the unknown, and solved the equation by transforming it into $m = 9$ to solve $m^2 = 9$. In this case, learners' use of the procedural embodiments "switching power to roots" resulted in failing to recognize the other root (i.e., *m* = -3). Moreover, they reported that learners attempted to transform quadratic equations into linear equations.

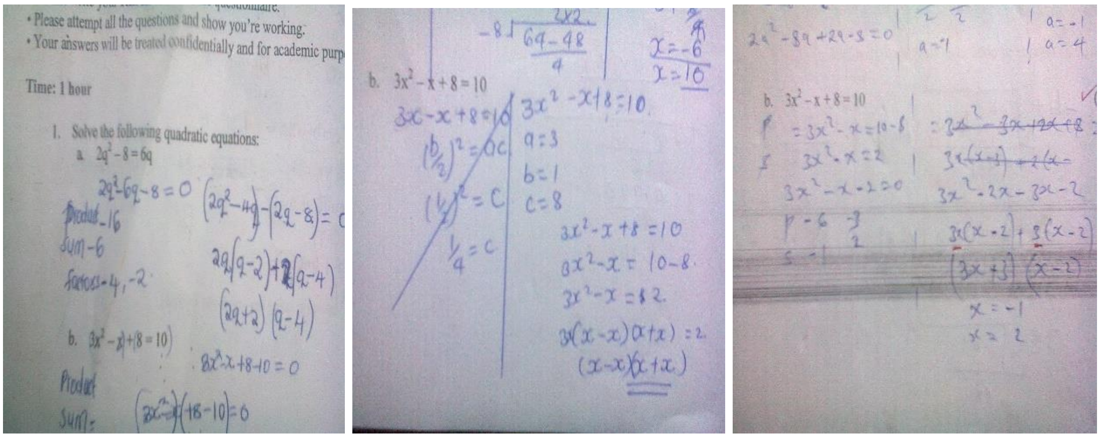


Figure 4.9: Incorrect Factorization by Learners 4, 14 and 45

In question 1a, the factors were 2 and -8, but not only learner 4 had incorrect factors – 4 and – 2 (don't give a product of - 16) but also incorrect distribution of a common factor 2q and 2. This implied that the learner lacked comprehension on threshold concepts of the distribution law of factorization; factorized only the first term inside the bracket and forget the second term. Learner 14 performed additive inverse correctly in question 1b, but equated $3x^2 - x = 2$ and proceeded to perform incorrect factorization. Also learner 45 factorized the question in Figure 4.12 above correctly to obtain -3 and 2 as factors. Hence the learner replaced -x with -2x and -3x which was wrong. Furthermore, -3*x* and -2*x* were not the correct multiplication factors for $-6x^2$. The correct factor multiplication of $-6x^2$to be used here should be 6*x* and – *x*. However, the learner did not fully comprehend the concept of factorizing because after having factorized the incorrect equation, the term before the parenthesis, 3x and 3 are not common to the two terms inside the bracket and so the final solutions were still incorrect. This implied that the learners didn't comprehend the concept well and affected performance in factorizing quadratic equations and functions. These findings suggest that learners have difficulty with basic multiplication table fact retrieval (Kotsopoulos, 2007).

Factorization threshold concepts increase when the parameter a does not equal one (for example in expressions such as $6x^2 - 3x + 2$ and become even more challenging when a and/or c have multiple factors, leading to many possible factor pairs in expressions such as $20x^2 - 63x + 36$. It is worth noting that the research literature on factoring quadratics attends to factoring when $a, b \text{ and } c$ integers are resulting in expressions that can be factored into binomials with integer coefficients.

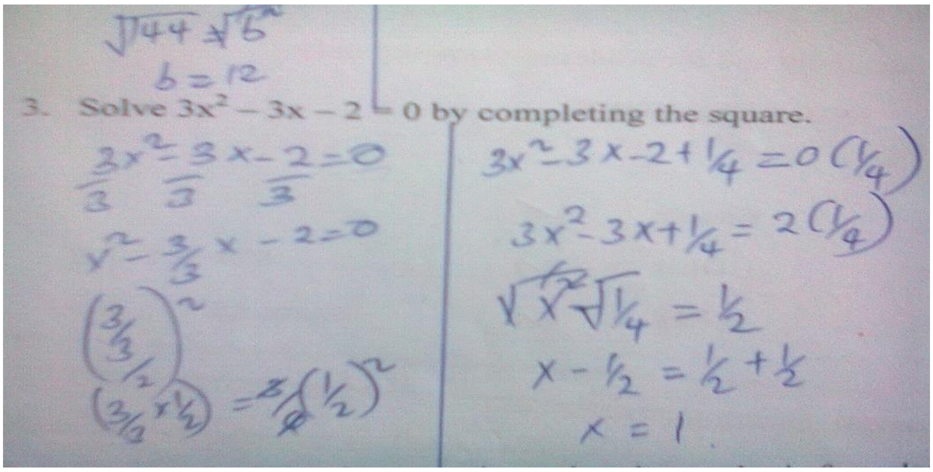


Figure 4.10: Inability to Form Perfect Squares by Learners 138

Learner 138 lacked the concept of dividing through by a common factor. This showed that lack comprehension on the concept of the distributive law which, from a mathematical standpoint, is fundamental not only to the process of factorization in algebra, but also to the reverse process of 'expanding brackets' (Lim, 2000 as cited in Yahya & Shahrill, 2015). That is, learners have a choice of either a rote-learned cross-multiplication method or a rote learned grouping method when factorizing a quadratic equation; however, neither was ever related to the distribution law. The selection of the method really depended on what their teachers preferred their learners to use.

Because the symbols for the parameters of linear and quadratic equations are often the same, 11.2%, 2.3% and 10.9% of the learners imposed linear structure in Q1 (a, b, and c) respectively.

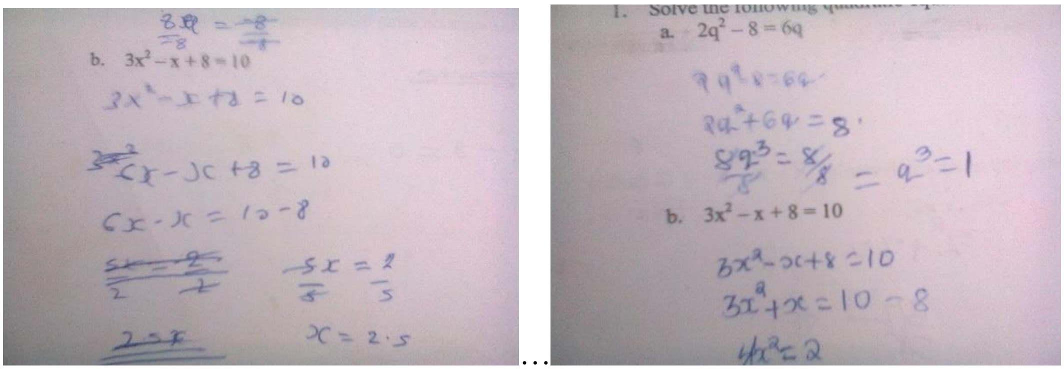


Figure 4.11: Imposing Linear Structure Excerpts from Learners 14 and 18

Previously learnt concepts of linear equations seemed to affect learners 14 and 18 as shown in the excerpts above. Learner 14 interpreted $3x^2$ to be equals to 6x by multiplying 3 which is the coefficient of $x^2$and 2 the power of x^2. Student 18 added $2q^2$and $6q$ to get $8q^3$ forgetting that they were unlike terms in the equations and therefore, lost the properties of quadratic equation.

The figure below shows an excerpt of learner 141 who in the process of dividing the equation by half, wrongfully cancelled $3x^2$to get 3x hence, linearizing the equation. Learner 187 interpreted x^2in $3x^2$to mean $2x$, then wrongly subtracted x to incorrectly remained with$3x$, linearizing it. The results support what Tall et al. (2014) that, while attempting to solve $m^2 = 9$, some students applied the exponent associated with the unknown as if it were the coefficient; that is, m^2 equals to 2m, and learners showed a tendency to use the quadratic formula as the only valid method in solving every quadratic equation. Stacey and MacGregor (1997, as cited in Mamba, 2012) stated that learners may draw on prior learning from other fields to their work with algebraic symbols, e.g., in chemistry, adding oxygen to carbon produces CO_2. Due to similar meanings of 'and' and 'plus' in ordinary language, it is not uncommon for learners to regard 'ab' to mean the same as '$a + b$' because the symbol 'ab' is read as '$a\ and\ b$' and may be interpreted as '$a + b$', so $3 + x$ may be taken as $3x$. Alternate thinking is that leaners often disregard a difficult question and reformulate it to another easier question such as changing $3 - x\ to\ 3$ (Tall & Thomas 1991 as cited in Mamba, 2012).

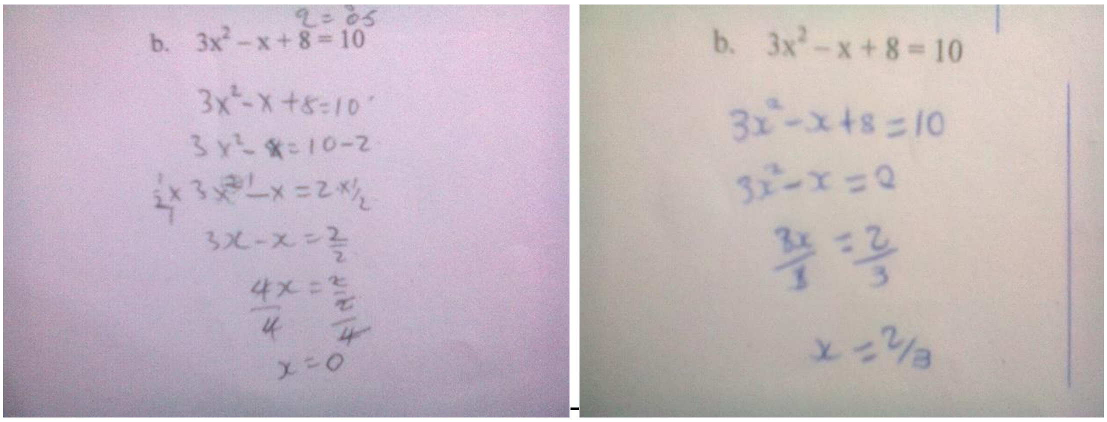


Figure 4.12: Imposed Linear Structure by Learners 141 and 187

This question 1(b) $3x^2 - x + 8 = 10$; was supposed to be solved as $3x^2 - 3x + 2x - 2 = 0$ to get $3x\ (x - 1)$ + $2(x - 1)$ = 0, therefore, $(x - 1)(3x + 2) = 0$ hence either $(x - 1)\ or\ (3x - 2)$ and the answer was $x = 1\ or\ x = -2/3$. This implied that learners could not comprehend the concepts of solving quadratic equations and functions well and affected performance. Learners either erroneously apply concepts of linear equations to quadratics or use them to "linearize" quadratic equations, Lima and Tall (2010). Didis (2010) interprets this as learners knowing the zero-product property but not being able to apply it appropriately when the structure of the equation is changed. This could also be an example of learners imposing linear structure on a quadratic as they work to solve the equation using techniques that have worked to solve linear equations.

4.3.5.1.2 Threshold Concepts in Completing Square Method

Table 4.4 below indicates the threshold concepts in solving symbolic quadratic equations using completing square method.

Table 4.4: Threshold Concepts in Completing Square Method in Q1

Threshold Concepts	**Frequency** (%)		
	a	**b**	**c**
Not Dividing the Equation by Coefficient of x^2	13 (3.5)	7 (9.4)	8 (2.1)
Not Adding Half the Coefficient of x	6 (1.6)	10 (2.6)	9 (2.3)
Unable to Convert the sol. to Squared Form	13 (3.4)	14 (3.6)	9 (%)
NA	348 (90.6)	342 (89.1)	349 (90.9)
Correct Solution	8 (2.1)	4 (1)	9 (2.3)
Total	**384 (100)**	**384(100)**	**384(100)**

Completing square is a method of solving quadratic equations with one known and is helpful if appropriately applied in finding the solution to the equations though learners have difficulties as the study found that 3.5%, 9.4% and 2.1% of those who used this method did not divide the equation by the coefficient of x^2 in Q1a, Q1b and Q1c respectively.

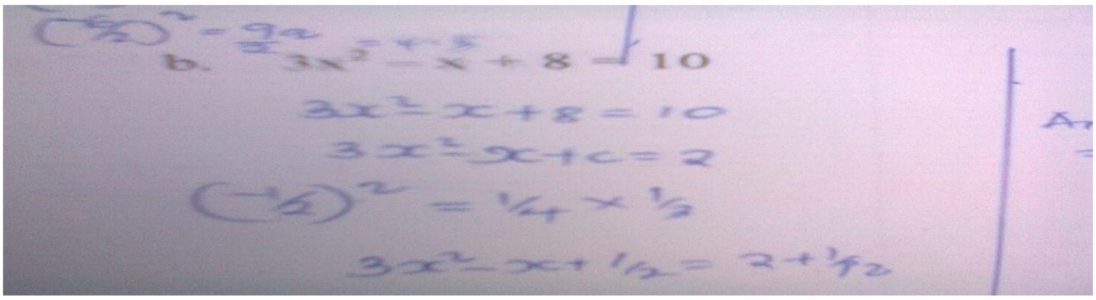


Figure 4.13: Excerpt of Learner 139 did not divide the Eq by the Coefficient of x^2

The correct method of solving question 1b by method of completing square is; get $3x^2 - x + c = 10$, $then\ c = (-\frac{1}{6})^2$. Divide through by 3 to get $x^2 - \frac{1}{3}x + (\frac{1}{6})^2 = \frac{2}{3} +$

$\frac{1}{6}$ .This gives $\left(x-\frac{1}{6}\right)^2 = \frac{2}{3} + \frac{2}{36}$. So $\left(x-\frac{1}{6}\right)^2 = \frac{25}{36}$; $\left(x-\frac{1}{6}\right) \pm \sqrt{\left(\frac{5}{6}\right)}$ and therefore either $x = \frac{5}{6} + \frac{1}{6}\ or\ x = -\frac{5}{6} + \frac{1}{6}\ to\ get\ x = 1\ or\ x = \frac{2}{3}$. But most of the learners didn't use this method as indicated by 90.6%, 89.1% and 90.9% hence was NA for them. Laridon et al. (2010 as cited in Makgakga, 2016) reported that completing square is a method which if students can use appropriately provides correct answers. However, prior to finding the additive inverse of a constant using completing a square, learners should ensure that the coefficient of x^2 is 1 and if it is greater or less than 1, they should divide by that coefficient the equation $ax^2 - bx + c = 0$.

Learners who did not add half the coefficient of x to both side of the quadratic equation were 1.6%, 2.6% and 2.3% in Q1(a, b and c) respectively.

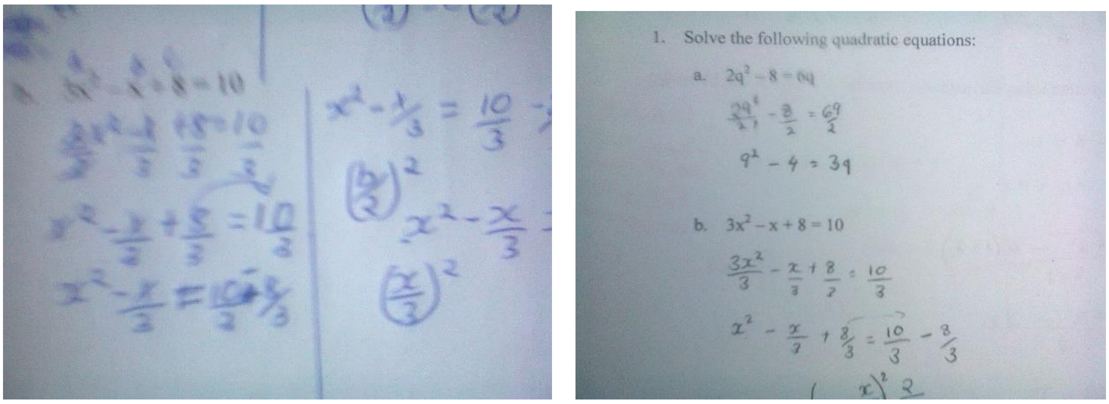


Figure 4.14: Learners 219 and 138 did not add Half the Coefficient of x

In solving question 1b by completing square method should be done as $3x^2 - x + c = 10 - 8$; then $c = \left(\frac{-1}{6}\right)^2$. Divide through by 3 to get $x^2 - \frac{1}{3}x + \left(\frac{1}{6}\right)^2 = \frac{2}{3} + \frac{1}{6}$. So $\left(x-\frac{1}{6}\right)^2 = \frac{25}{36}$; $x - \frac{1}{6} = \pm\frac{5}{6}$ and therefore either $x = \frac{5}{6} + \frac{1}{6}\ or\ x = -\frac{5}{6} + \frac{1}{6}$ to get $x = 1\ or\ x = -\frac{2}{3}$. Learner 219 did not add half the coefficient of x to both side but only wrote $\left(\frac{b}{2}\right)^2$ while learner 138 divided the equation by the only by the coefficient of x^2. It implied that the learners did not comprehend the relation of the concept and affected the performance in quadratic equation. Although the learners correctly converted the left hand side of the equation to its squared form 3.4%, 3.6% and 2.3% of the learners were unable to convert the results to squared form in the same question Didis et al (2011). It is important that students don't view mathematics as a set of strange rules and tricks but as a harmonious system where everything as a reason. Therefore, algebraic ideas of completing squares can be connected with intangible process of working with rectangles and squares using algebraic tiles.

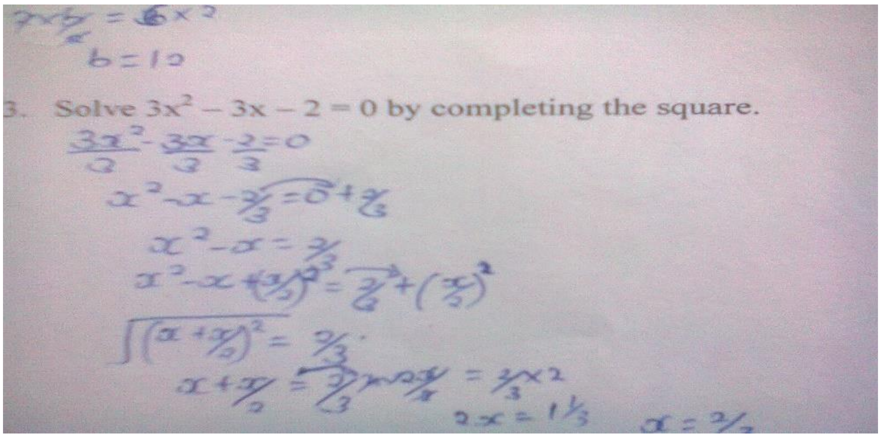


Figure 4.15: Incorrect Completion of Square Excerpts of Learner 93

The study results found four main difficulties in solving question one using quadratic formula. Learners had a difficulty of encoding and carelessness in substituting quadratic equations which were not in standard form using the quadratic formula as indicated by 9.1%, 12.2% and 12.7% of them in question 1 (a, b and c). The standard quadratic formula is given by, x=$\frac{-b\pm\sqrt{b^2}-4ac}{2a}$. So in Q1 (a) $x = \left[6 \pm \sqrt{6^2} - (4x2x8)\right] \div 2x2 = [6 \pm 10] \div 4\ and\ q = 4\ orq = -1$.

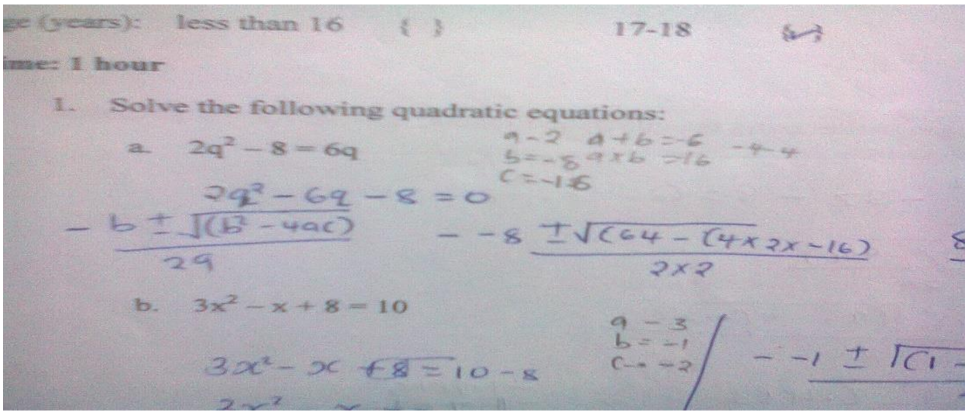


Figure 4.16: Excerpt of Learner 226 Encoding and Carelessness in Substitution

In figure 4.16 above learner 226 rearranged the equation in standard form correctly, but the constant term c was not supposed to be -16 as indicated. The learner got $c = axb$ instead of -8 as in the equation. This implied that the learner did not comprehend the concept of identifying terms in the quadratic equation. The learner also lacked the concept and made incorrect substitution by using the terms from the equation which was not in standard form.

This result clearly shows that learners did not fully have the concept as the solution of the problem is wholly determined by the combined information of the used cues, the content and structure of the retrieved schema, the solution would be wrong if the quadratic formula in the schema was flawed. In other words, the schema mediates the solution (Abdullah, Shahrill & Chong, 2014, as cited in Yahya & Shahrill, 2015).

4.3.5.1.3 Threshold Concepts in Quadratic Formula Square Method

Table 4.5 below indicates the threshold concepts in solving symbolic quadratic equations using completing square method.

Table 4.5: Threshold Concepts on Quadratic Formula Method Q1

Threshold Concepts	Frequency (%) a	b	c
Encoding and Carelessness in Substituting Unstandardized Equation	35 (3.5)	43 (9.4)	49 (2.1)
Incorrect Discriminant when equated to a Negative Constant	11 (1.6)	19 (2.6)	2.1 (2.3)
Dividing Only Discriminant by 2a	4 (1.0)	6 (1.6)	6 (1.6)
NA	247 (64.3)	241 (62.8)	227 (59.1)
Correct Solution	87 (22.7)	75 (19.5)	91 (23.7)
Total	**384 (100)**	**384 (100)**	**384 (100)**

From table 4.5 above, when equation is equated to a negative constant in the quadratic formula, 1.6 %, 2.6% and 2.3% of the learners found incorrect discriminant in question 1 (a, b and c). This implied that the learners did not comprehend the threshold concepts in quadratic equations and led to incorrect solutions.

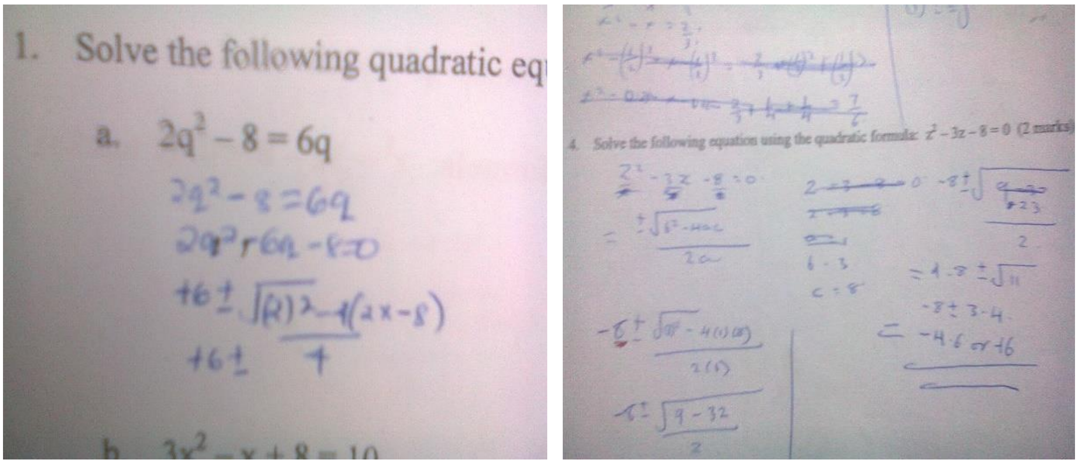


Figure 4.17: Excerpt of Incorrect Discriminant by Learners 86 and 35

Solving Q4 using quadratic formula is given by, $z = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $z = \frac{3\pm\sqrt{3^2-4x1x-8}}{2x1}$ and $z = 4.7$ or $z = -1.7$. Learner 35 used formula method in solving question 4 in which the learner used -8 to substitute for b as shown in the excerpt instead of -3, interchanging the second term and the constant term. The second difficulty was the incorrect discriminant. The learners found $3^2 - 4x1x8 = 9 - 32$ instead of $3^2 - 4x1x(-8) = 9 + 32$, because since the constant term c was negative (-8) the operation would no longer be subtraction but addition. Learner 86 interchanged the coefficient b to be a in the formula, hence getting wrong discriminant. Majority of the learners liked to use this method of solving as shown in the table 4.5 above, but 1.0%, 1.6% and1.6% of the learners lacked comprehension of the concept of the formula and divided only the discriminant by $2a$ like the case of learner 99 who divided the quadratic formula by $2c$ instead of $2a$. Didis et al. (2011) reported that learners' incorrect solutions were mainly based on either the incorrect calculation of the discriminant or incorrect use of the quadratic formula, because they made calculation errors while they were finding the discriminant of the quadratic equation.

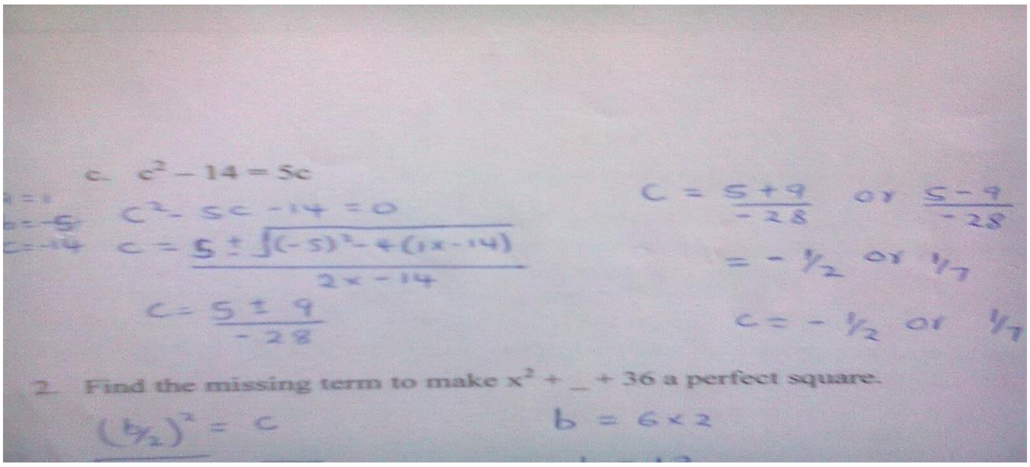


Figure 4.18: Incorrect Denominator of the Formula by Learner 99

Solving Q1c using quadratic formula is given by, c=$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, hence $c = \frac{5\pm\sqrt{5^2-(4x1x-14)}}{2\text{x}1}$ and$c = 7\ or\ c = -2$. In this difficulty, many of the learners misremembered the quadratic formula and applied the formula as shown in figure 4.18 above to solve the equation. The learner instead of substituting 1 for letter a in the denominator, the learner used -14 to get an incorrect denominator -28. From the interview it came out clearly that teachers do not derive the formula from completing square method and which could the best way of introducing it to the learners for the first time. Due to lack of practice, learner 7 performed a wrong substitution as she wrote b^2 instead of $-b$. From her interview she said:

> *Quadratic formula method of solving quadratics was just written on the black wall by the teacher and told us that the formula is found at the back of the mathematical tables.*

Majority of the learners did not comprehend the concept of how to use this method and consequently reported not applicable (NA) as shown by 64.3%, 62.8% and 59.1% they were expected maybe to learn by rote the quadratic formula and to be able to apply it to solve quadratic equations despite not being taught how this formula could be derived, (Lim, 2000 as cited in Yahya & Shahrill, 2015). Thus learners developed a perception that their main task was only to gain knowledge and to be able to solve quadratic equations using the quadratic formula; there was no real need to really develop the concept of understanding why the method works.

Table 4.6: Threshold Concepts in Calculating Constant Term using Completing Square Method in Q2

	Frequency Percentage (%)			
Threshold Concepts	**Boys**	**Girls**	**Mixed**	**Total**
Incorrect $\left(\frac{b}{2}\right)^2 = ac$ Relation	16 (36.4)	6 (13.6)	22 (50)	44(100)
Gives only Positive Value of x	74 (43)	36 (20.9)	62 (36)	172(100)
Inserts root of C^2 and Factorize	3 (8.3)	4 (11.1)	29 (80.6)	36(100)
Put Zero and Attempt to Solve	16 (36.4)	6 (13.6)	22 (50)	44(100)
Correct Solution	12 (38.7)	15 (48.4)	4 (12.9)	31(100)
Blank	6 (36.4)	12 (13.6)	70 (79.5)	88(100)
Total	**115 (29.9)**	**77 (20.1)**	**192 (50)**	**384 (100)**

Learners should divide by a throughout before finding the additive inverse of c both sides to have$x^2 + \frac{b}{a} = -\frac{c}{a}$. Suppose half the coefficient of x is C, then $x^2 + \frac{b}{a} + C = -\frac{c}{a} + C$ gives the relation$\left(\frac{b}{2a}\right)^2 = C$, hence $b^2 = 4aC$ but 36.4%, 13.6% and 50% of the learners from boys', girls' and mixed schools respectively who had this relation incorrect.

Learners can then add half the coefficient of x to both sides, before the equation could be factorized Laridon et al. (2010 cited in Makgakga, 2016). There seems to be a serious difficulty in the use of square roots as majority of the learners gave only positive value of x as done by 43.0%, 20.9% 36% of the from boys', girls' and mixed schools respectively. The value of $b = \pm\sqrt{4ac}$ and a positive and negative value was supposed to be given and in this case +12 and -12 were correct values. Most of the learners could not recall clearly they inserted square root of C^2 and factorize as were indicated by 8.3%, 11.1% and 80.6% of the learners from boys', girls' and mixed schools respectively.

Learners showed lack of comprehension of threshold concept putting zero and attempting to solve as shown by 36.4%, 13.6% and 50% of them from boys', girls' and mixed schools respectively. In addition, learners could not find the additive inverse of the equation. Learners complete a square on the left hand side and were unable to do it on the right hand side. This clearly indicated lack of comprehension of threshold concepts to be able to solve quadratic equations correctly using this method. Similarly, teachers have inevitably been asked why they add $\left(\frac{b}{2}\right)^2$ and not any other number, to both sides of the equation (Vonogradova & Wiest, 2007).

Table 4.7: Threshold Concepts in Using Completing Square Method in Q3

	Frequency (%)			
Threshold Concepts	**Boys**	**Girls**	**Mixed**	**Total**
Not Dividing the Equation by the Coefficient of x^2	9 (20.9)	11 (25.6)	23 (53.5)	43 (100)
Not Adding Half the Coefficient of x	10 (17.9)	15 (26.8)	31 (55.4)	56 (100)
Unable to Convert the results to Squared Form	16 (32.7)	13 (26.5)	20 (40.8)	49 (100)
Gives positive root only	16 (36.4)	6 (13.6)	22 (50)	44 (100)
Wrong Method	15 (19.7)	10 (13.6)	51 (50)	76 (100)
Blank	16 (32.7)	3 (6.1)	30 (61.2)	49 (100)
Correct Solution	40 (42.1)	23 (24.2)	32 (33.7)	95 (100)
Total	**115 (29.9)**	**77 (20.1)**	**192 (50)**	**384 (100)**

Solving quadratic equation using completing square method requires division of the equation by the coefficient of x^2 when it is greater than 1 or less than zero. This threshold concept was reported by 17.9%, 26.8% and 55.4% of the learners from boys', girls' and mixed schools respectively in Q3. Lack of comprehension of this concept led to poor performance in quadratic equation. Laridon et al., (2011) advise that when adding half the coefficient of *x*, learners should ensure that the coefficient of x^2 is 1. Further, half the square of the coefficient of *x* should be added both on the left hand side and the right hand side. This view was supported by Zemelman et al., (1998) that learners without true understanding of the underlying concepts guarantee serious problems in learning other concepts. This is what happened to these learners as this reveal that most of them were unable to make connection of factorization to the concept of completing a square result in using procedures of solving problems inappropriately.

More difficulties were found when 32.7%, 26.5% and 40.8% of the learners from boys', girls' and mixed schools respectively were unable to convert the results to squared form which revealed that they lacked the concept of completing square. In solving Q3 the first step is to divide through by 3 to get; $x^2 - x + c = \frac{2}{3}$; then $c = \left(\frac{1}{2}\right)^2$. Then $x^2 - x + \left(\frac{1}{2}\right)^2 = \frac{2}{3} + \frac{1}{4}\ hence\ \left(x - \frac{1}{2}\right)^2 = \frac{11}{12}; x - \frac{1}{2} = \pm\sqrt{\frac{11}{12}}$ and therefore either $x = 0.96 + 0.5\ or\ x = -0.96 + 0.5$ to get $x = 1.46\ or\ x = -0.46$. Learner 14 in the figure below divided the equation through by 3 in the first step but still retained 3x in the second step instead of adding half the coefficient of b squared.

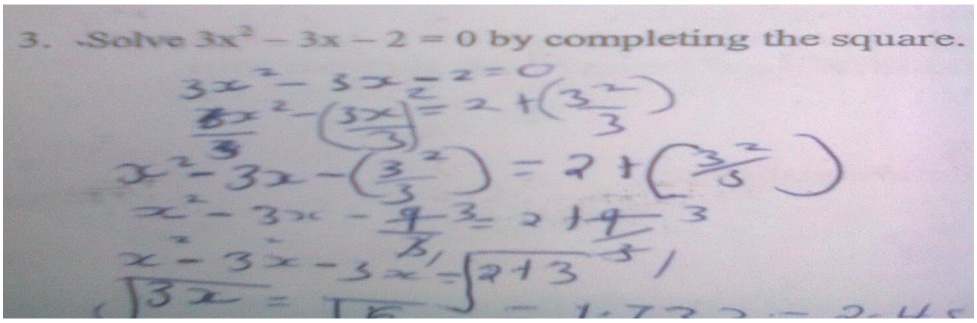


Figure 4.19: Inability to make a Perfect Square by Learner 14

Table 4.7 indicated that learners applied the wrong method in getting the solution to this question as shown by 19.7%, 13.6 and 50% of them from boys', girls' and mixed schools respectively. While 32.7%, 6.1% and 61.2% of the learners from boys', girls' and mixed schools respectively left this question blank. This implied that learners did not comprehend the threshold concept of solving quadratic equations and functions. Similarly, threshold concept is specific to a particular task and if a learner does not understand the concept, it would be difficult for learners to articulate the procedures to solve equations based on that particular concept (Battista, 2001 as cited in Makgakga, 2016a).

Table 4.8: Threshold Concepts in Using Quadratic Formula Q3

	Frequency (%)			
Threshold Concepts	**Boys**	**Girls**	**Mixed**	**Total**
Encoding and Carelessness in Substitution	18 (37.5)	9 (18.8)	21 (43.8)	48 (12.5)
Incorrect Discrimant	17 (25.4)	10 (14.9)	40 (59.7)	67 (17.4)
Dividing Discrimant by 2a	3 (18.8)	1 (6.3)	12 (75.0)	16 (4.2)
Wrong method	6 (13.6)	6 (13.6)	33 (73.3)	45 (11.7)
Correct Solution	58 (13.8)	48 (29.6)	56 (34.6)	162 (42.2)
Blank	13 (28.3)	3 (6.5)	3 (6.5)	19 (4.9)
Total	**115 (29.9)**	**77 (20.1)**	**192 (50.0)**	**384 (100)**

A number of learners experienced difficulty in solving Q4 as indicated by 37.5%, 18.8% and 43.8% learners from boys', girls' and mixed schools who had encoding and carelessness in substitution. This demonstrates that they were unable to deal with equations involving the substitution, using ± signs and finding square root. They tend to ignore the square root or unintentionally forget about it. Learners did not use the quadratic formula correctly, and were prone to make careless mistakes in the substitution especially when the constant term, C is a negative integer a difficulty which 25.4%, 14.9% and 59.7% of the learners from boys', girls' and mixed schools respectively. Learner 35 forgot to put negative 8 in bracket which could have found the discriminant to be $9 + 32$ instead of $9 - 32$.

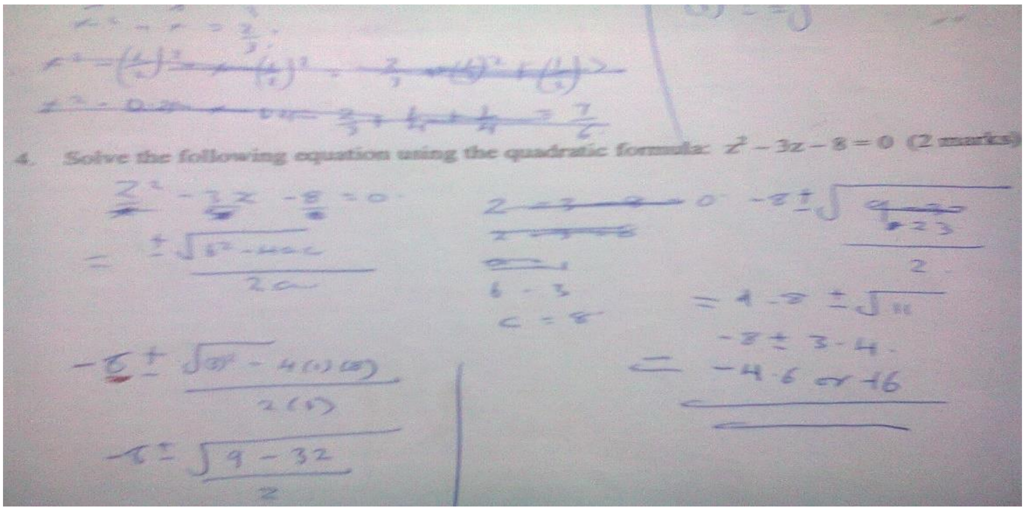


Figure 4.20: Excerpt from Learner 35 with a Difficulty of Incorrect Determinant

During the class observations and interviews conducted, all the learners ascertain to the fact that their teacher expected them to memorize the quadratic formula and apply it to solve quadratic equations, and were not expected to understand where the formula was derived. This concurred with what had been found by Lim (2000) in his study. Literally all the learners absolutely could not write the quadratic formula in absence of the mathematical table. The question was left blank as indicated by 28.3%, 6.5% and 6.5% of the learners from boys', girls' and mixed schools who said that he could not recall the quadratic formula, whereas 13.3%, 13.6% and 73.3% from the same categories used the wrong method to solve the question to compensate for their failure to apply the quadratic formula during the examination. Learner 14 as shown in the excerpt below used the wrong method instead of quadratic formula.

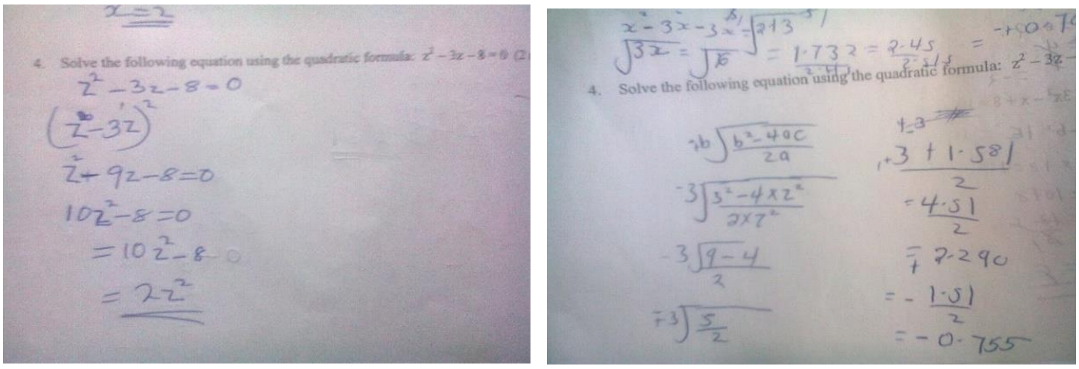


Figure 4.21: Wrong Method and Formula from Learners 14 and 304

Based on the researchers' observations, learners who relied on memorizing the quadratic formula without really understanding it were prone to make a lot of errors. Learner 304 in the figure above forgot plus or minus before the square root and also substituted 4ac as 4 x z^2 and got 4. In certain situations which required solving quadratic equations using quadratic formula only, learners without the correct concept get incorrect solution. The results above indicates that 18.8%, 6.3% and 75.0% of the learners divided only the discriminant by $2a$ arriving at the wrong answers in all the questions. The observations made and interview conducted showed that teachers just write the formula on the board and learners are just told that it is found at the back of the mathematical table. Indisputably, having not been taught to comprehend the quadratic formula increased the possibility of learners deriving the incorrect formula or retrieved schema that was flawed or incomplete, which resulted to confuse learners that the term $2a$ is only divided by $b^2 - 4ac$ like in the formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. Learners also forgot to put a negative sign in front of the first letter b in the formula. It is implied from this findings that learners lacked the concept of using the quadratic formula method. The was similar to what Oliver (1992 as cited in Yahya & Shahrill, 2015) who explained in her article about learners' need to possess the schema that is needed in order to answer questions correctly.

4.3.5.1.4 Threshold Concepts in Solving Word Problems

The learners' performance in solving quadratic word problems resulted from threshold concepts in understanding, representing and translating word problem.

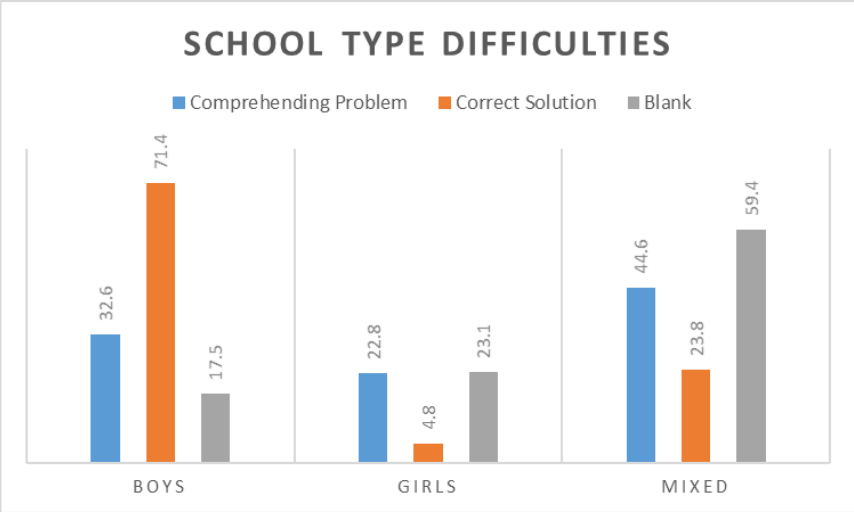


Figure 4.22: Learners' Threshold Concepts in Solving Word Problem

In questions 5 and 6, learners were expected to understand the relationships among the variables, form equation(s) and solve them. The performance indicated that learners used arithmetic methods, working backwards or guessing to find solutions rather than quadratic methods. Unlike symbolic questions, most learners had no simple ways to use arithmetic methods in this question other than guessing or using trial and error.

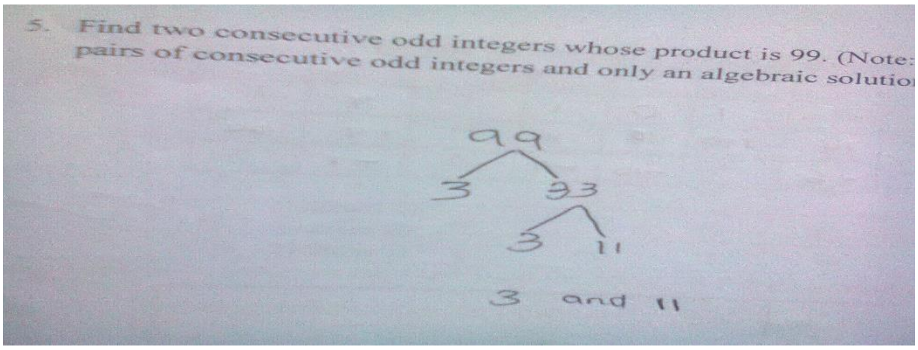


Figure 4.23: Threshold Concept of symbolic representation of word problem from Learner 256

In Q5 of the diagnostic test, learners were asked to, "Find two consecutive odd integers whose product is 99"; learners would take the square root to get two numbers, a positive and a negative. The quadratic expression could have looked like, $n(n-2) = 99$, to get $n^2 + 2n - 99 = 0$. So using factorization method to solve, $n^2 + 11n - 9n - 99 = 0$giving$(n+11)(n-9) = 0$, then either $(n+11) = 0 \; or \; (n-9) = 0$ and the answer is $n = -11$ or $n = 9$. But the learner instead prime factorize the product just to satisfied the condition of a quadratic solution by just getting two answers of the problem, probably thinking the learner got the question correctly as shown in figure 4.23 above. This implied that lack of comprehension of threshold concepts might have led to learners' inability to solve the question. Solving word problems involves a triple process: assigning variables, noting constants, and representing relationships among variables. Among these processes, relational aspects of the word problem are particularly difficult to translate into symbols. Thus, learners' lack of comprehension in the threshold concept in translating from natural language to quadratic and vice versa are one of the three situations that generally arise when learners are in secondary education (Mayer, 1982; Bishop, Filloy & Puig, 2008 as cited in Egodawatte, 2011). Hinsley et al. (1977, as cited in Egodawatte, 2011) showed that the translation of quadratic word problems is guided by schemas. These schemas are mental representations of the similarities among categories of problems. Translation errors frequently occur during the processing of relational statements.

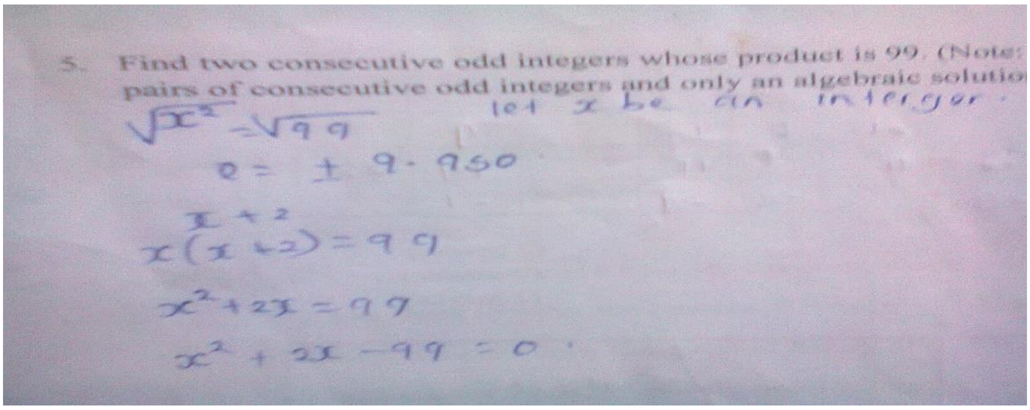


Figure 4.24: Excerpt from Learner 279 of Inability to Solve Quadratic Equation

Learner 279 had translated the word problem well but the solving difficulty occurred when the student got confused trying to formulate a solution for the problem. Learner 279 set up the quadratic equation correctly as $n^2 + 2n - 99 = 0$, but could not solve the equation using any of the methods and as such, and therefore the solution was incomplete. One noticeable feature in the answers was that 32.6%, 22.8% and 44.6% of learners from boys, girls and mixed schools respectively had difficulties in comprehending the relationship among two varying quantities. In the comprehension phase, a problem-solver comprehends and then forms the text base of the problem, utilizing words as an internal representation in his or her memory. In the solution phase, she or he expresses this internal representation externally and applies the rules of algebra to reach a conclusion (Koedinger & Nathan, 2004; Mayer, 1982 cited in Didiş et al., 2011).

Learners have poor deductive reasoning abilities as 17.5%, 23.1% and 59.4% of them from boys', girls' and mixed schools respectively left this question blank. This gives 59.6% of learners who left the question blank compared to a total of 16.4% of the learners who got the correct solution. Lack of this concepts led to poor performance in quadratic equation. Moreover, Briars and Larkin (1984 in Didis & Erbas, 2015) attributed the word problem-solving difficulty to the learners' psychological processes. They also emphasized factors relating to the problem's features, such as the number of words in the problem, the presence of cue words and the size of the numbers.

Table 4.9: Threshold Concepts in Solving Word Problem Q6

	Frequency (%)			
Threshold Concepts	**Boys**	**Girls**	**Mixed**	**Total**
Make Drawing and Examining in a Different Point of View	15 (27.3)	15 (27.3)	25 (45.5)	55 (14.3)
Difficulty in Grasping Relation between two Varying Quantities	43 (30.7)	24 (17.1)	73 (52.1)	140 (36.5)
Correct Solution	12 (75.0)	0 (0.0)	4 (25.0)	16 (4.2)
Blank	45 (26.0)	38 (22.0)	90 (52.0)	173 (45.1)
Total	**115 (29.9)**	**77 (20.1)**	**192 (50.0)**	**384 (100)**

The major source of learner's threshold concepts in solving Q6 was translating the narrative into appropriate quadratic expressions as 27.3%, 27.3% and 45.5% of students from boys, girls and mixed schools respectively made drawing and examining it in a different point of view as shown in the figure below.

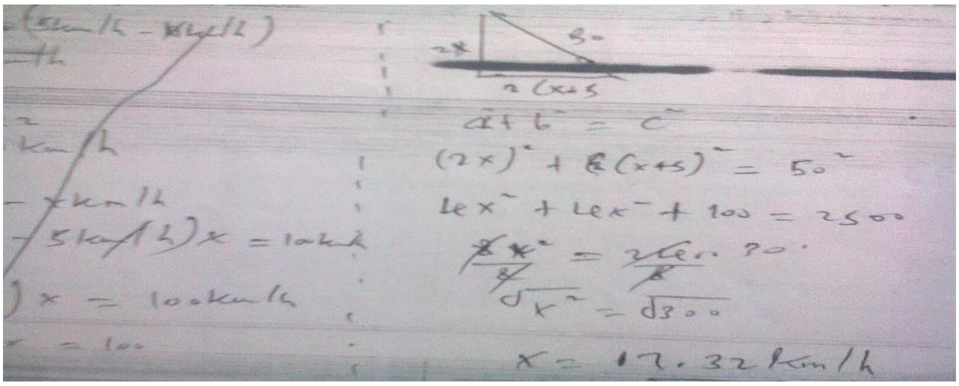


Figure 4.25: Excerpt of Incorrect Expansion from Learner 30

Learner 30 had a very good understanding of the problem which was expressed with a drawing, but the only difficulty the learner had was the incorrect expansion of $2(x+5)^2$to arrive at $4x^2+100$ instead of $4x^2+20x+10=0$as shown in the figure below. Building up a single quadratic relationship to satisfy the conditions was so hard for them showing that most students lack the fundamental concept of distinguishing terms used mathematically, Egodawatte, (2011). While citing Clement (1982), Didis, (2015) indicated that learners' lack of comprehension in solving quadratic word problems stem from the difficulties they have in symbolizing meaningful relationships within quadratic equations.

Difficulty in grasping relation between two varying quantities was found with 30.7%, 17.1% and 52.1% of learners from boys, girls and mixed schools respectively and this result from the semantic structure and memory demands of the problem. Consequently, these learners might have translated the syntax of the relational statement into quadratic expression without considering the magnitude of the relationship. Although the linguistic form of the problem's text conveys the significant factors that affect the comprehension process, Stacey and MacGregor (2000 cited in Didis, 2015) claimed that a major reason for lack of comprehension of threshold concept with word problems arises from logic of a problem. They argue that because of their prior experiences with arithmetic word problems, learners perceive the problem-solving process as a series of calculations and shift their thought process from quadratic thinking to arithmetic thinking when solving quadratic word problems.

Operative reasoning is a concept learners used to perform hypothetical operations on two quantities to match the symbols with the words. But 26.0%, 22.0% and 52.0% of learners from boys, girls and mixed schools respectively left the question blank expressing lack of comprehension of threshold concepts of mathematical relationship, Weinberg (2007 as cited in Egodawatte, 2011).

4.3.5.1.5 Threshold Concepts in Solving Graphical Problems

Parent, (2015) while citing Burger et al. (), described a function as, a relation in which the first coordinate is never repeated. There is only one output for each input, so each element of the domain is mapped to exactly one element in the range. In this study, the concept of function was narrowed down to quadratics and learners were first required to use the given range of values of x to get the corresponding values of y. the following figure shows learners' performance in Q7.

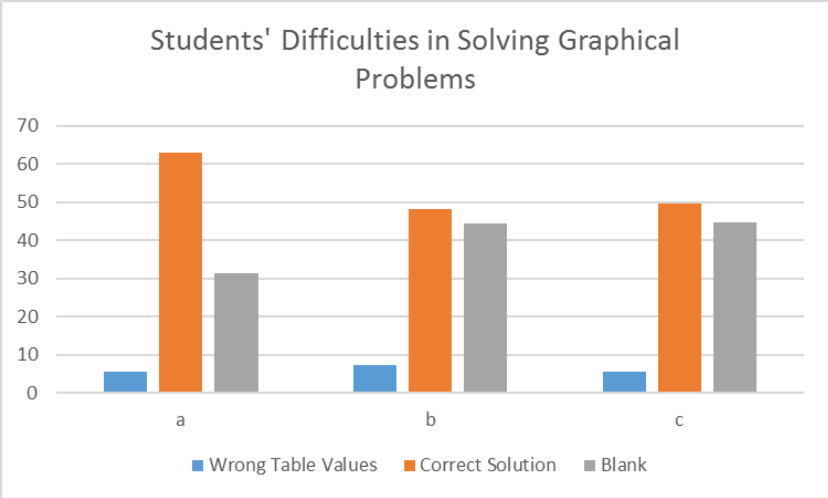


Figure 4.26: Learners' Performance in Q7

In plotting a graph learners showed a difficulty of not able to find the corresponding values of y within the given range of the values of x as 5.5%, 7.3% and 5.5% of the learners had wrong table values in Q 7 a, b and c respectively. It was also reported by Ellis and Grinstead (2008, as cited in Parent, 2015) that when working with quadratic functions, learners' lack of comprehension on threshold concepts mainly appear with connections between algebraic, tabular, and graphical representations, a view of graphs as whole objects, struggles to correctly interpret the role of parameters, and a tendency to incorrectly generalize from linear functions. Learners with correct solutions question a, b and c were 63%, 48.2% and 49.7% respectively. But some of the learners left the question blank as indicated by 31.5%, 44.5% and 44.8% in question a, b and c respectively.

Table 4.10: Threshold Concepts in Graphical Solution Q.7

	Frequency (%)	
Threshold Concepts	**I**	**II**
Other Transformation	20 (5.2)	15 (3.9)
Confusing Intercepts and Coordinates	7 (1.8)	7 (1.8)
Finds Equation of a Line	51 (13.3)	50 (13)
Correct Solution	12 (3.1)	15 (3.9)
Blank	294 (76.6)	312 (81.3)
Total	**384 (100)**	**384 (100)**

Table 4.11: Threshold Concepts in Finding Roots of a Problem Q.8

	Frequency (%)				
Threshold Concepts	**a**	**bi**	**b ii**	**b iii**	**b iv**
Wrong table Values	19(4.9)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0(0.0)
Does not Know roots of a function	0.0(0.0)	36 (9.4)	0.0(0.0)	0.0(0.0)	0(0.0)
Incorrect Graph	0.0(0.0)	0.0(0.0)	9.0(2.3)	0(0.0)	0(0.0)
Gives Coordinates	0.0(0.0)	0.0(0.0)	0.0(0.0)	20(5.2)	0(0.0)
Use Roots find Equation	0.0(0.0)	0.0 (0.0)	0.0(0.0)	0.0(0.0)	8(2.1)
Correct solution	337(87.8)	74(19.3)	222(57.8)	137(35.7	18(4.7

Question 8 was a simultaneous quadratic equation and function and in part a 4.9% of the learners had wrong table values, 87.8% got the question correct and 7.3% left the question blank.

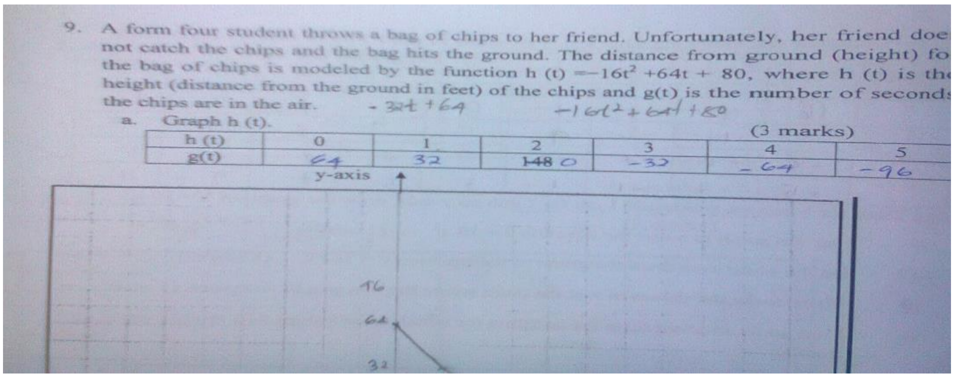


Figure 4.27: Wrong Table Values for Learner 86

In question 8 the values of Y within the given range of x were $-6, 1, 6, 9, 10, 9, 6, 1$ *and* -6 which the learners were supposed to draw the graph. Question 8 bi, 9.4% of the learners gave the x-intercept of the function as the solution of the equation$0 = 10 - x^2$. There were 19.3% of the learners who got the question correct but majority, 71.4% left the question blank. Question 8 b (ii) required the learners to draw the graph of the equation$y = 2x + 3$; equation of a straight line, 2.3% of them had incorrect graphs, 57.8% got the question correct while 39% left the question blank. The excerpt of learner 33 below indicates an incorrect graph with a y intercept being 0 instead of 80. The learner used the wrong y values within the range of x coordinates.

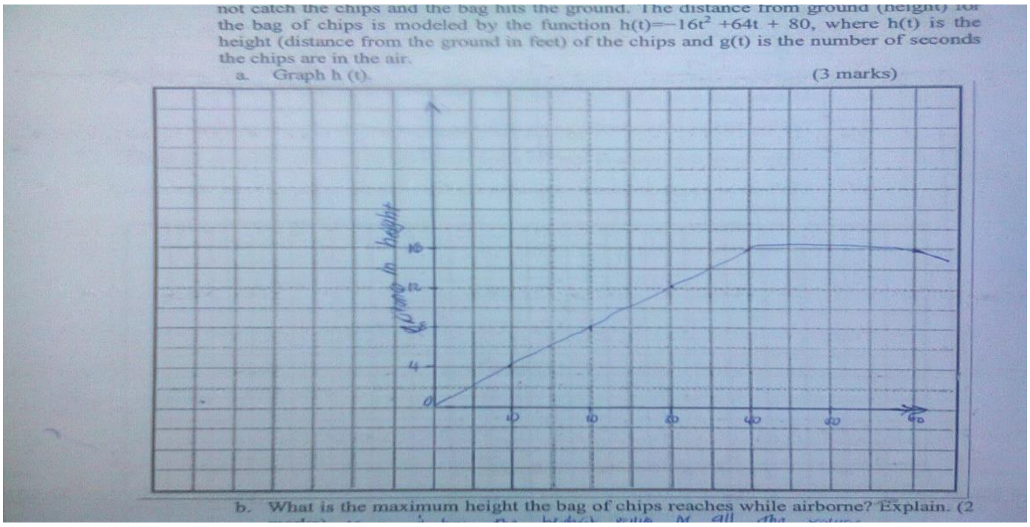


Figure 4.28: Excerpt of Incorrect Graph of Learner 33

8While in question 8 b (iii), learners were required to give the values of x at the point where the two graphs intersected, hence$x = -3.83\ or\ x = 1.83$. But 5.2% gave the coordinates of the points instead meaning they didn't differentiate between coordinates and values of x. 35.7% of the learners got the correct solution but 59.1% left the question blank. Parent, (2015) reported that when looking at various graphs, though, and indicating the location roots of x from the graphs, learners appeared to not understand the task and what was being asked of them. Instead of simply looking at the quadratic graphs and observing the point of the functions that touched or crossed the x-axis.

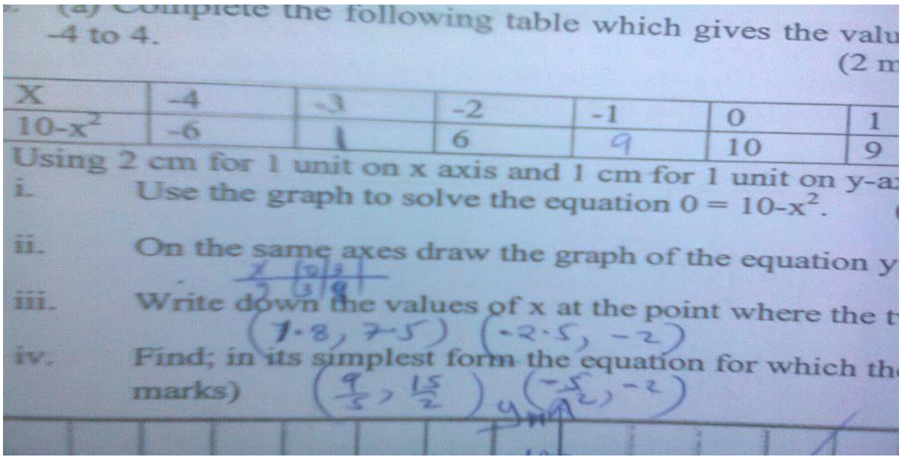


Figure 4.29: Learner 55 gave Coordinates of the Points of Intersection

In the last question; 8 b (iv) the equation from the roots of x was $x^2 + 2x - 7$. The results showed that majority of the learners didn't know how to form the equation using values of x as roots as 2.1% used roots of x to find linear equation instead of a quadratic equation. The excerpt of learner 55 above gave the coordinates of the point of intersection and was unable to form an equation just using the x-coordinates of this point. A few got the question correct as 4.7% had correct solution while 93.2% didn't attempt the question and left it blank.

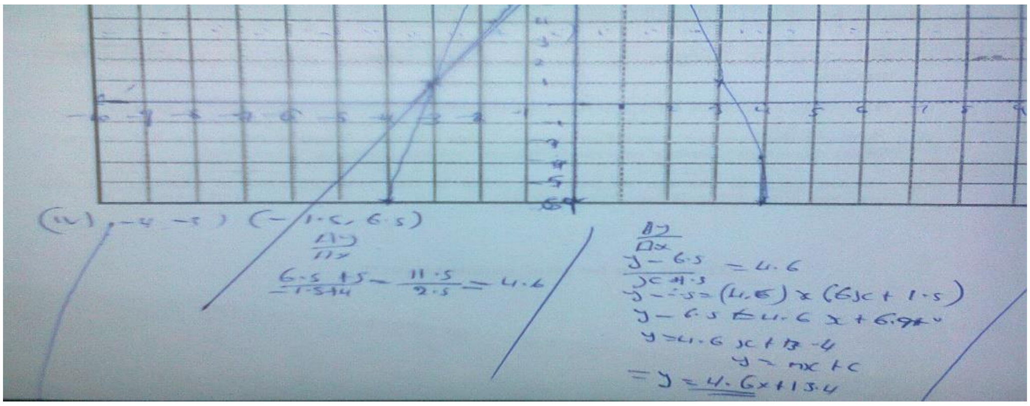


Figure 4.30: Learner 33 formed Linear Equation

Learner 33 formed linear equation instead of forming a quadratic equation. This shows that the student lack the idea of formation of quadratic equations using the given roots of x.

Table 4.12: Threshold Concepts in Solving Real Life Problem

	Frequency (%)				
Q9: Threshold Concepts	**a**	**b**	**c**	**d**	**e**
Wrong tables	15.0 (3.9)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Wrong scale	7.0 (1.8)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Gives range only	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	18.0(4.7)	0.0 (0.0)
Gives height of the form fours	0.0 (0.0)	10.0 (2.6)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Gives the max.	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	16.0 (4.2)
Correct solution	124 (32.3)	319 (83.1)	335 (87.2)	30 (7.8)	21 (5.5)
Blank	238 (62.0)	55 (14.3)	49 (12.8)	336 (87.5)	347 (90.4)
Total	**384 (100)**	**384 (100)**	**384 (100)**	**384 (100)**	**384 (100)**

The emphasis on functions as a unifying Mathematical concept, as a representation of real-world phenomena, and as an important Mathematical structure was central to this study. Question 9 was a real-life situation where the learners were required to apply the knowledge of quadratic equation and functions to solve. The value of $h\ (t)$ with respect to the corresponding values of $g(t)$ was $80, 128, 144, 128, 80, 0, -112$. In question 9a 3.9% of the learners had incorrect table values, 1.8% used the wrong scale, and 32.3% got the correct solution while 62% left the question blank.

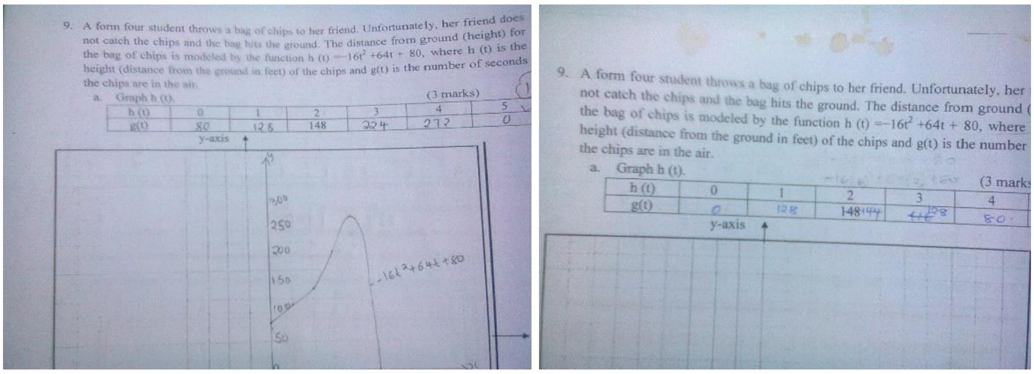


Figure 4.31: Wrong Graphs for Learners 120 and 125

In question 9b, learners were asked to find the maximum height the bag reaches while airborne which was 144ft, but 2.6% gave the height of the form fours in the bus, 83.1% got the correct solution and only 14.3% left the question blank. The excerpt of learner 120 and 125 in the figure above indicated that the learner had the difficulty of not plotting a good graph due to incorrect y-coordinates for the given range of x.

Majority of the learners, 87.2% were able to find the time taken after the bag was thrown did it hit the ground and only 12.8% left the question blank. Learner 115 in figure 4.33 below differentiated the quadratic equation and solved for t instead of reading it direct from the graph.

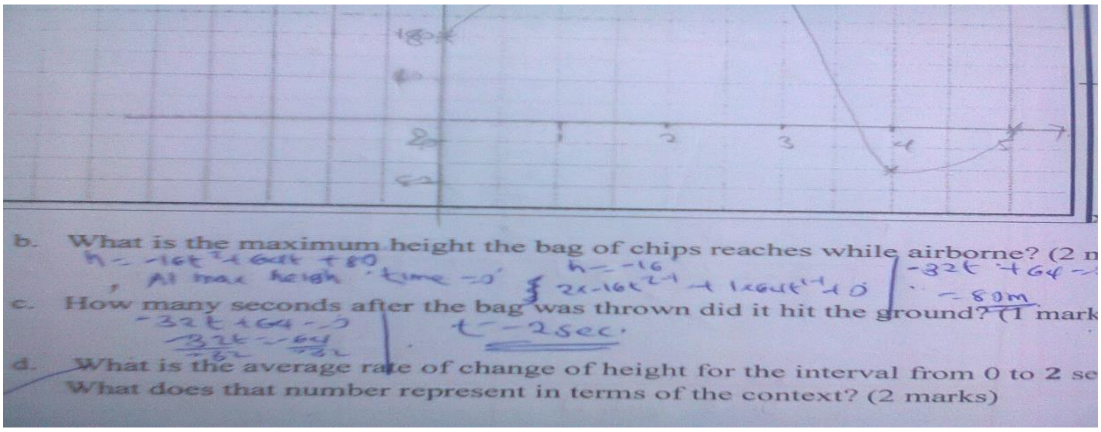


Figure 4.32: Excerpt of Incorrect Time taken from Learner 115

When learners were asked to find the range of change of height for the interval from 0-2 seconds most of them were not able to understand and 4.7% gave the range only without taking the average, 7.8% had correct solution and the rest 87.5% left the question blank. In question 9e, 4.2% of the learners gave the maximum height of the function when they were asked to give the height of the form four learners while 5.5% had correct solution and 90.4% left the question blank. From table 4.12 results, only questions 9b and 9c in which slightly over 300 learners got the correct solution but in questions 9 a, d and e less than half of the learners got the correct solution. This implies that learners cannot comprehend threshold concepts in real life situation problems. This would pose a great challenge to the learners and affect their performance in quadratic equations and functions in the national examinations. The difficulty in comprehension of the concepts was also reported by Ellis and Grinstead (2008, as cited in Parent, 2015) that in quadratic functions, learners' difficulties in tabular, and graphical representations and tend to incorrectly generalize from linear functions.

4.4 Teaching Strategy

The following hypothesis was tested:

H_{O1}: There is no statistical significant difference in teaching strategy on students' performance in solving quadratic equations and functions with one known.

In order to respond to the above hypothesis, the study employed the one-way analysis of variance (ANOVA) in determining whether learners' performance on the diagnostic test was a function to each of the teaching strategies (problem solving, use of examples and lecture method). The table below represents the output of the findings for the means and standard deviations for each of the three teaching strategies.

Table 4.13: Descriptives

		N	Mean	Std. Dev.	Std. Error
Performance	Problem Solving	115	34.94	17.43	1.63
	Lecture Method	192	28.94	17.38	1.25
	Use of Example	77	32.94	15.94	1.81
	Total	384	31.54	17.28	0.88

The first SPSS output gave descriptive statistics for each strategy and the students' performance. However, the problem solving strategy ($M = 34.94, SD = 17.43$) produced the highest performance compared to lecture and use of example teaching strategies with $(M = 28.94, SD = 17.38$ and $(M = 32.96, SD = 15.94)$ respectively. This implies that teacher's strategy employed during the lesson was reflected in the final learners' performance. Mamba, (2012) while citing Stepans (1994) reported that sometimes even the demonstrations used by teachers usually do not involve active participation of the learners but they sit back and merely observe with no opportunity to have a hands-on manipulation of materials or experiencing the phenomenon individually or in small groups.

The second table of the output was the ANOVA source table. This was where the various components of the variance were listed, along with their relative sizes. There were two components to the variance: between groups (which represents the difference due to the teaching strategy used) and within groups (which represents individual difference in learners within each level of the strategy). In the ANOVA source table, the primary answer was a ratio of explained variance (F ratio). The one-way ANOVA in table 4.14 revealed a statistically significant main effect, $F\ (2, 381) = 4.73, p = .009$, indicating that not all the three teaching strategies resulted in the same students' performance.

Table 4.14: ANOVA Source Table

Performance	Sum of squares	df	Mean of squares	F	Sig.
Between groups	2769.59	2	1384.80	4.73	.009
Within groups	111575.83	381	292.85		
Total	114345.41	383			

Since a significant difference was found among the teaching strategies used, then post hoc comparisons using Turkey's procedures were used to determine which pairs of the three teaching strategy means differed. These results are given in Table 4.13 and indicate that learners who had received the problem solving teaching strategy ($Mean = 6.00, p = .009$) scored significantly higher on the diagnostic test than did learners who had received the lecture teaching strategy. Also, learners who received the problem solving teaching strategy scored insignificantly higher than those who received use of example teaching strategy($Mean = 1.98, p = .712$). Specifically, the results suggested that when a teacher guides the learners in solving a quadratic problem, learners performs better. However, it was noted that when learners were shown an example first and allowed to solve after the teacher yielded medium performance. Lecture teaching strategy produced the least results. The approach a teacher takes when teaching a threshold concept in mathematics is influenced by their own conception of those concepts, as well as what the teacher wants the learners to be able to do with those concepts (Swan, 2006). For example, if the teacher has a conception that Mathematics is just about doing procedures correctly, then would teach a mathematical formula and show the learners how to use it and then would expect the learners to be able to apply it. On the other hand, if the teacher has a conception that Mathematics is a reasoning science, the teacher expects the learners to be able to analyze the Mathematical formula, decide whether it has a solution and apply that formula only if necessary.

Table 4.15: Multiple Comparisons

Turkey HSD	(I) Strategy	(J) Strategy	Mean difference (I-J)	Std. error	Sig.
	Problem Solving	Lecture Method	6.00	2.02	.009
	Lecture Method	Problem Solving	1.98	2.51	.712
		Use Of Example	-4.02	2.30	.188
	Use of Example	Problem Solving	-2.00	2.51	.712
		Lecture Method	4.02	2.30	.188

The mean difference is sig at the .50 level

Learners' threshold concepts in factorization of quadratic equations are; the zero product property and forming meaningless connections with quadratic roots. If these threshold concepts are experienced by learners it is possible that their teachers experience the same threshold concepts as a result of their educational background and experiences. This would imply they need to participate in strategies to keep them abreast and aware of the latest ideas and approaches geared towards assisting learners in facing the threshold concepts mentioned above and gaining a greater understanding of the quadratic concepts (Mamba, 2012).

4.4.1 Gender Performance

4.4.1.1 Gender performances for the cognitive levels

The following research hypothesis was tested:

H_{O2}: There is no statistical significant difference in the performance of quadratic equations and functions with one known based on gender.

The conditions to be tested here was identified by looking at all possible combinations of the different levels of each independent variable which produced a factorial matrix with 4 conditions as shown in table 3.5 below:

Table 4.16: Factorial Matrix for Gender and Performance

GENDER	COGNITIVE PERFORMANCE		
	Low	High	Grand Mean
Male	21.71	11.56	16.64
Female	19.64	9.90	14.77
Grand Mean	20.68	10.73	15.71

The mean gender performances for the cognitive levels were represented in the figure below.

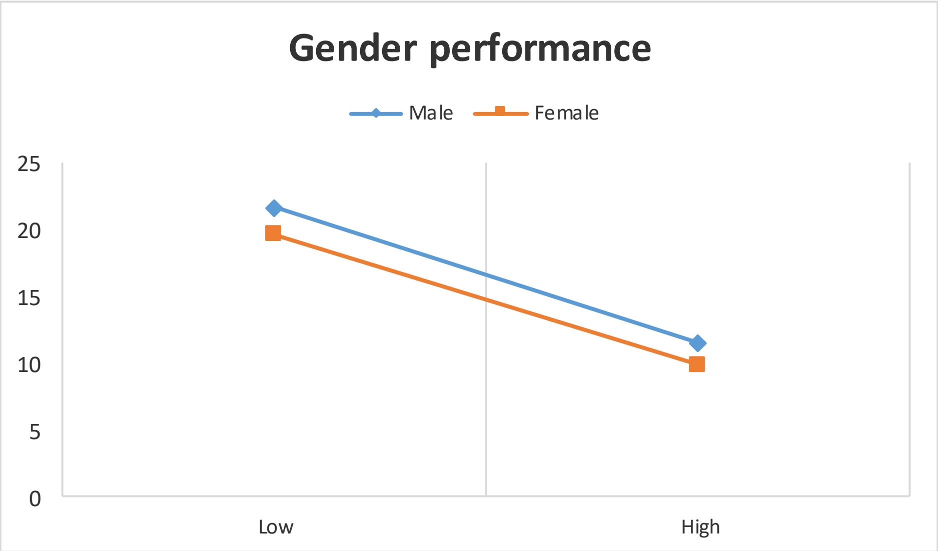


Figure 4.33: Gender Performance for the Cognitive Levels

In this hypothesis, 2 kinds of results occurred; main effects and interactions. The main effects referred to the overall influence of the independent variables. Therefore, determining the main effects for gender involved using data for all levels of the cognitive performance. Similarly, the main effect for cognitive performance was determined by combining the data for both genders. Therefore, one way MANOVA was conducted to test for the main effect and the results are shown in the following output.

Table 4.17: Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.
Intercept	Wilk' Lambda	.214	701.317[b]	2.000	381.000	.000
Gender	Wilk' Lambda	.988	2.256[b]	2.000	381.000	.106

a. Design: Intercept + Gender
b. Exact statistic

Since the results in the first section were not significant, the study could not perform univariate tests. Therefore, the results of the one-way MANOVA examined that gender (male or female) does not affect low and high order performance as no significant influence was found, $\text{Lambda}(2, 381) = .988, p = .106$. Neither low nor high order performances were significantly influenced by gender as illustrated in the table below. Boaler and Staples, (2008 cited in Tutkun et al., 2012) also reported that some teachers implement tasks that promote higher-order thinking with specific populations of learners because it's frustrating for low performing students to solve.

Table 4.18: Tests of Between-Subjects Effects

Source	DV	Type III Sum of Squares	df	Mean Square	F	Sig.
Correct Model	Low	405.177[a]	1	405.177	3.449	.064
	High	262.986[b]	1	262.986	3.534	.061
Intercept	Low	162518.094	1	162518.094	1383.30	.000
	High	43758.486	1	43758.486	588.041	.000
Gender	Low	405.177	1	405.177	3.449	.064
	High	262.986	1	393.802	3.534	.061
Error	Low	44879.562	382	117.486		
	High	28426.137	382	74.414		

a. R Squared = .009 (Adjusted R Squared = .006)
b. R Squared = .009 (Adjusted R Squared = .007)

4.4.1.2 Gender and School Type Performance

A statistical test between cognitive performance and school type was performed using one way MANOVA. Cognitive performance had 2 levels (low and high) and school type had 3 levels (boys, girls and mixed) and the design produced 6 conditions as shown in table 4.19 below.

Table 4.19: Factorial Matrix for Cognitive Performance and School Type

Cognitive Performance	**School Type**			
	Boys	Girls	Mixed	**Grand Mean**
LOW	22.04	21.73	19.60	21.12
HIGH	13.00	9.91	9.86	10.92
Grand Mean	17.52	15.82	14.73	16.02

From the table above a line graph was drawn and the lines are non-parallel. However, an interaction probably existed, and lead to a statistical decision to determine by the analysis of variance. Generally, since the lines were not parallel the relationship which was drawn from the graph was that boys', girls' and mixed schools performance were higher in low order questions than high order questions. Secondly, boys' performances were higher in both low and high order questions than girls' and a declined noted in the mixed schools.

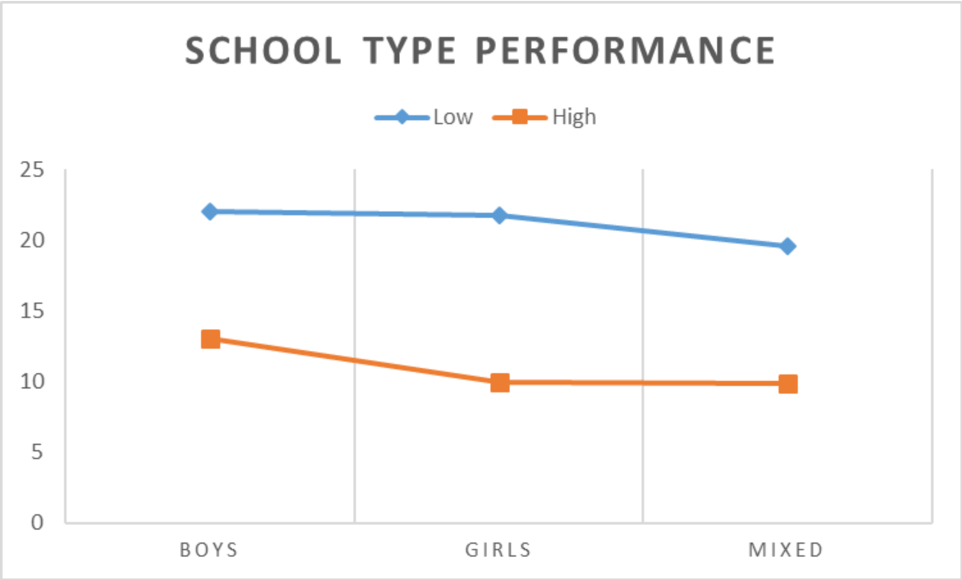


Figure 4.34: Cognitive Performance and School Type

The other results tested using the data for all levels of the school type was the main effect for cognitive performance. The main effect for school type was determined by combining the data for both cognitive performances. The F-ratio was calculated using a one way Multivariate Analysis of Variance (MANOVA) since there were more than one dependent variable (low and high). Therefore, in order to examine the possibility of a main effect for cognitive performance, another for school type and for potential interactions between the two it was possible to run these tests at once.

Table 4.20: Multivariate Tests[a]

Source		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared (η^2)
Intercept	Wilks' Lambda	.224	657.347[b]	2.0	380.00	.000	0.81
School Type	Wilk, Lambda	.965	3.453[b]	4.000	760.00	.008	.340

a. Design: Intercept + School Type
b. Exact statistic
c. The statistic is an upper bound on F that yields a lower bound on the significance level.

The table above represents the MANOVA using the Wilk's λ test. Using an alpha level of .05, the test was significant, $Wilk's\ \lambda\ (4, 760) = .965, p = .008$, multivariate $\eta 2 = .19$. The significant *F* indicated that there are significant differences among the school type groups on a linear combination of the two dependent variables (low and high). The multivariate $\eta 2 = .340$ indicates that approximately 34% of multivariate variance of the dependent variables is associated with the group factor. The results of the pairwise comparisons are shown below. Type I error across the two univariate ANOVAs was controlled for by testing each at the .025 alpha levels. To be consistent with this decision, there was also need to control the probability of committing one or more Type I errors across the multiple pairwise comparisons for the dependent variable at the .025 alpha levels. In order to be able to maintain this family wise error rate across comparisons for a dependent variable the study selected .025 for the significance level in the multivariate.

Table 4.21: Tests of Between-Subjects Effects

Source	DV	Type III Sum of Squares	df	Mean Square	F	Sig.
Correct Model	Low	517.577[a]	2	258.789	2.202	.112
	High	787.605[b]	2	393.802	5.377	.005
Intercept	Low	152286.795	1	152286.795	1296.068	.000
	High	40737.896	1	40737.896	556.283	.000
School Type	Low	517.577	2	258.789	2.202	.112
	High	787.605	2	393.802	5.377	.005

a. R Squared = .011 (Adjusted R Squared = .006)
b. R Squared = .027 (Adjusted R Squared = .022)

Since the results in the first section were significant, the study interpreted the univariate tests. Follow–up univariate ANOVAs indicated that low order performance were not significantly influenced by school type, $F\ (2, 380)\ =\ 2.202, P\ =\ .112$. High order performance, however, were significantly influenced by school type $F\ (2, 380)\ =\ 5.377, P\ =\ .005$ as illustrated in the table below. Stein and Kaufman, (2010) reported some factors associated with maintenance of high-level cognitive demands. They included sustained press for justifications, correctness or meaning through teacher questioning, comments, and/or feedback, tasks build on students' prior task behavior and teachers should draw frequent conceptual sustained engagement in high-level cognitive connections.

4.4.2 Gender, Teacher Strategy and School Type versus Performance

The study tested the following hypothesis:

HO3: There is no significant relationship between gender, school type and teaching strategy on one hand and performance in quadratic equations and functions with one known on the other hand.

Moderated multiple regression analysis was used to test the hypothesis that gender, school type and teaching strategy was a function of performance.

Table 4.22: Model Summary

Model	R	R^2	Adjusted R^2	R^2 Change	Std. Error of the Estimate	Sig.
1.	.169[a]	.029	.021	.029	17.09631	.011[a]
2.	.187[b]	.035	.022	17.08	.006	.285[b]

a. **Dependent Variable: PERFORMANCE**
b. **Predictors: (Constant), GENDER, SCHOOL TYPE, TEACHING STRATEGY**

The study results indicate that in model 1, unadjusted value of R^2 meant that all subsets of predictor variables had a value of multiple R that was smaller than .029 (2.9%). Remarkably, these variables in combination significantly $(Sig. F\ Change = .011)$ predict student's performance. In model 2, with interaction terms between school type and teaching strategy accounted for insignificantly $F\ (3, 380) = 3.738, p < .05$. The model also indicate more variance than just school type and teaching strategy by themselves, $R2\ change = 17.08, p = .285$. Since the relationships were not significant, the study implied that there was a complete moderation between the predictor and the moderators.

Table 4.23: ANOVA [a]

Model		Sum of squares	df	Mean square	F	Sig.
1.	Regression	3277.582	3	1092.527	3.738	.011b
	Residual	111067.832	380	292.284		
	Total	114345.414	383			
2.	Regression	4012.888	5	292.284	2.738	.19[c]
	Residual	110332.526	378	291.885		
	Total	114345.414	383			

a. Dependent Variable: PERFORMANCE
b. Predictors: (Constant), GENDER
c. Predictors: (Constant), GENDER, SCHOOL TYPE, TEACHING STRATEGY

The first model used only Gender (X_1) to predict Performance (Y) indicating that gender was a predictor, with $F\ (3, 380)\ =\ 3.738, p\ =\ .011$. The second model was used to predict Performance (Y) from the additive effects of School Type (X_2) and Teaching Strategy (X_3), assuming no moderation. The results were significant, $F\ (5, 378)\ =\ 2.738, p\ =\ .019$. From unstandardized coefficients the following regression equations was formed:

Predicted $\bar{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ (1)

$\hat{Y}\ =\ 39.49\ -\ 4.35x_1 - 2.95x_2\ +\ 2.52x_3;$ (2)

From the study results the coefficient $\beta_1\ =\ -4.35$ indicated that for either male or female student, one additional unit of X_1 (either male or female) and controlling for other variables was associated with -4.35 less units of predicted Y (-4.35 on the students' performance). However, since there were gender*school type and gender*teaching strategy interactions in the model, the effects of performance for males and females, would lead to incorrect interpretation of the results. Therefore, the study tested for the moderation effects as shown below.

Table 4.24: Coefficients [a]

Model	Unstandardized β	Coefficients Std. Error	Standardized Beta	Coefficients t	Sig.
1. (Constant)	39.40	3.035		13.680	.000
Gender	-4.350	2.444	.125	1.780	.076
Teaching	-2.947	1.165	-.148	-2.529	.012
School Type	2.524	1.868	.103	1.352	.177
2. (Constant)	136.63	61.29		2.23	.026
Gender	-100.73	60.80	-2.90	-1.66	0.98
School type	-21.32	14.04	-1.07	-1.52	.130
Teaching Strategy	-18.71	18.23	-.760	-1.03	.305
Gender* School Type	20.20	13.98	2.19	1.45	.149
Gender* Teaching Strategy	18.77	12.69	2.04	1.48	.140

a. **Dependent Variable: Performance**

The model 2 in table 4 includes the interaction term, resulting in the following equation:

$$\bar{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 \quad (3)$$

$$\hat{Y} = 136.63 - 100.73\, x_1 - 21.32 x_2 - 18.71 x_3 + 20.20 x_1 x_2 + 18.77 x_1 x_3; \quad (4)$$

The study tested for the statistical significance of the interaction terms Gender* School Type which yielded $t\,(5, 378) = 1.45, p = .149$ and $t\,(5, 378) = 1.48, p = .140$ for Gender* Teaching Strategy. Since the predictors and the moderators were not significant with the interaction terms added, then the study found that a complete moderation occurred. Therefore, there was a relationship between gender and school type and also gender and teaching strategy differed for males and females and computed the regression equations separately for males and females learners. In the data set, $=$

$\hat{Y} = Performance, X1 = Gender\ (Male = 1, Female = 2), X2 =$
$School\ Type\ (Boys = 1, Girls = 2\ and\ Mixed = 3) and\ X3 =$
$Teaching\ Strategy\ (Lecture = 3, Example = 2\ and\ Problem\ Solving = 1)$. Thus, for males the regression equation reduced to;

$$\hat{Y} = 136.63 - 100.73\,(1) - 21.32\,(x_2) - 18.71\,(x_3) + 20.20(1 * x_2) + 18.77(1 * x_2) \quad (5)$$

$\hat{Y} = 136.63 - 100.73 - 21.32\,(x_2) - 18.71 x_3 + 20.20(x_2) + 18.77 x_3$, which was written as

$$\hat{Y}_{\boldsymbol{M}} = 35.90 - 1.12(x_2) - 0.06 x_3 \quad (6)$$

For females the equation is $\hat{Y} = 136.63 - 100.73\,(2) - 21.32\,(x_2) - 18.71\,(x_3) + 20.20(2 * x_2) + 18.77(2 * x_3)$, was written as $\hat{Y}_F = -64.83 + 19.08\, x_2 + 18.83 x_3$. (7)

The weight on the x_1x_2 interaction term ($\beta_4 = 20.20\, in\, equation\, 3$) was the difference in the regression weight on x_1 for males and females (- 1.12 vs 19.08 in equation 6 and 7). Thus, a test of β_3 was a test of the sex difference in the regression weight on school type when predicting student's performance. Also, the weight on the x_1x_3 interaction term (β_5 =18.77 in equation 3) was the difference in the regression weight on x_1 for males and females (-0.06 vs 18.83 in equation 6 and 7). As a consequence, a test of β_5 was a test of the sex difference in the regression weight on teaching strategy when predicting learner's performance. The study then concluded that, on average, gender had a statistically significantly stronger relationship with student's performance for females than for males. In the model without the interaction term, the regression weight of – 4.35 (equation 2) on gender overestimated the relationship for males and underestimates the relationship for females.

In order to find if linear relationships exist between gender (X) and performance (Y), school type (M) was taken as a moderator, and teaching strategy (N) as moderator of the relationship between X and M. The table below shows the coefficients of the regression terms.

Table 4.25: Coefficients

Model	Variable	coeff	se	t	P
1	Constant	-1.9613	.1208	-16.2336	.0000
	Gender	3.4257	.0788	43.4819	.0000
2	Constant	-1.2858	.1566	-8.2114	.0000
	Gender	3.1862	.1021	31.2041	.0000
3	Constant	36.1303	3.8851	9.2997	.0000
	Gender* School	.8421	1.1709	.7192	.4725
	Gender* Strategy	-1.9654	.9035	-2.1754	.0302
	Gender	-3.914	5.720	-0.684	.9455

The following conceptual model illustrates the moderation effects used in the PROCESS macro, using model 8 which instantly tested the model.

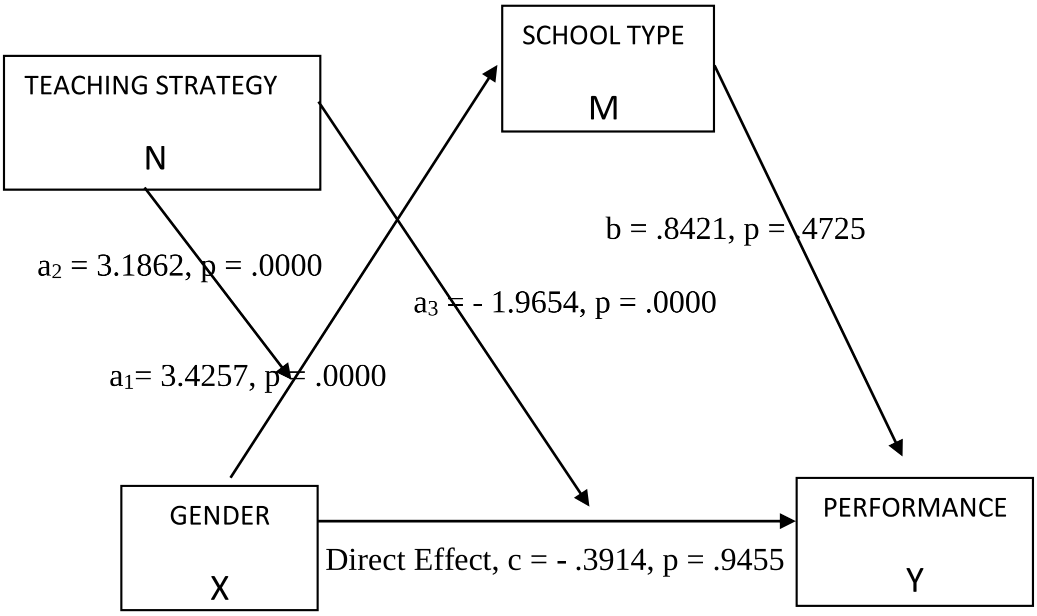


Figure 4.35: Conceptual Model for the Moderated Macro Process

The moderated regression indicated that teaching strategy moderated the relationship between gender and performance due to the significant interaction $a_3 = -1.9654, p = .0000$. A significant direct effect of gender on school type ($a1 = 3.4257, p = .0000$) was also noted, but not on performance (c = - 0.3914, p = .9455). Teaching strategy moderated the relationship between gender, and performance, due to the significant interaction ($a_2 = 3.1862, p = .0000$).

On the conditional direct effect(s) of X on Y at values of the moderator(s), teaching strategies in which the teacher used his/her pedagogical knowledge well where the learners were actively involved indicated an impact on learners' performance. So a teaching strategy like problem solving showed a positive relationship between gender, and good performance was noted. The school type generally did not significantly moderate the relationship between gender and learners' performance. In this regard, it does not matter which type of school a learner attended but perform well if and only if a teacher employ the right strategy in the lesson. For the conditional indirect effect(s) of X on Y at values of the moderator(s), only teaching strategy was significant, and its effect increases as the teacher engages the students actively in the lesson. Thus the school type had no effect on learners' performance. The study results are supported by Moeti, (2016) who reported that teaching strategy in mathematics teacher preparation and professional development activities should address two crucial themes regarding teaching mathematical routines. First, is the features and applicability of the strategies for carrying out a fundamental procedure in school mathematics: why, when, and how these strategies work, in what cases they would work the best, when they would not work well, and how to compare them and choose the optimal one both in general and in a specific problem context.

Since the relationship between the independent variable (gender) and the dependent variable (performance) in the study was hypothesized to be an indirect effect that existed due to the influence of a third variable (teaching strategy), the effect of the independent variable was reduced and the effect of the mediator remained significant. in order to determine whether the reduction in the effect of the independent variable, after including the mediator in the model, was a significant reduction and therefore whether the mediation effect is statistically significant, a Sobel test was conducted and a full mediation in the model was found ($z = -2.169, p = .0301$). It was found that teaching strategy mediated the relationship between learners' gender and performance. School type did not mediate the relationship in the model ($z = .7189, p = .4722$) as indicated in the table below.

Table 4.26: Normal Theory Tests for Specific Indirect Effects

	Effect	se	Z	p
Gender*school	2.8849	4.0130	.7189	.4722
Gender*Strategy	-6.2622	2.8871	-2.1690	.0301

The results therefore, shows that as much as the school type of the learner matters, students' performance in quadratic equations and functions with one known depended on the teaching strategy employed during the lesson. Eisenkopf et al., (2015) also found that girls' performance in mathematics improved in single sex classes and that this improvement was greater when taught by a male teacher. This could be an indication that apart from the single sex setting girls' also thrives if taught the subject by male teachers.

4.5 Chapter Summary

The research findings found out that Mathematics teachers mostly assume that learners have understood the concept as expected without employing relevant teaching strategies which identify learners' threshold concepts in solving quadratic equations and functions. The methods of solving quadratic equation and functions noted in the study were; factorization, quadratic formula method, completing square method and the graphical method. The study found that Mathematics teachers focused on symbols and the apparently in-comprehensible rules that show procedures using them in the abstract domain.

In the study, it was found that when a question does not restrict a particular method of solving quadratic equation like the question 1, learners would prefer solving using factorization, formula and completing square methods in that order. The learners' difficulties when solving quadratic equations and function using factorization included;

Factorizing equation which is not in standard form, being unable to factorize equation which is equated to a constant, zero product property, incorrect factors and imposing linear structure. In completing square method, learners were had a difficulty of not dividing the equation by the coefficient of x^2, not adding half the coefficient of x to both sides of the equation and being unable to convert the results to squared form. Formula method posed so many difficulties to the learners, including; encoding and carelessness in substituting equations which are not in standard form, incorrect discriminant when equation is equated to a negative constant and dividing only discriminant by 2a. Some of the difficulties found in solving word problems included; make drawing and examining in a different point of view and difficulty in grasping relation between two varying quantities

On the relationship between gender, school type and teaching strategy on one hand and performance in quadratic equations and functions with one known on the other hand, the study tested for the statistical significance of the interaction terms Gender* School Type. Since the predictors and the moderators were not significant with the interaction terms added, then the study found that a complete moderation occurred. Therefore, there was a relationship between gender and school type and also gender and teaching strategy differed for males and females and computed the regression equations separately for males and females learners. The moderated regression indicated that teaching strategy moderated the relationship between gender and performance due to the significant interaction. Teaching strategy moderated the relationship between gender, and performance, due to the significant interaction. On the conditional direct effect(s) of X on Y at values of the moderator(s), teaching strategies in which the teacher used pedagogical knowledge well where the learners were actively involved indicated an impact on learners' performance.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.0 Introduction

This chapter presents a summary of the findings from which it draws conclusions and recommendations. Finally, makes recommendations based on research objectives.

5.1 Summary of the Findings

Since the purpose of the study was to establish learners' comprehension and performance on threshold topics in Mathematics, the findings was summarized in the light of the following research objectives: To discuss the influence of teaching strategy on learners' performance in quadratic equations and functions with one known, to describe learners' scores in solving quadratic equations and functions with one known, to analyze learners' threshold concepts in solving quadratic equations and functions with one known that may attribute to gender, to determine gender difference if any, that may exist in the performance of quadratic equations and functions with one known and to determine relationships if any among teaching strategy, gender and school type on one hand, and performance in quadratic equations and functions with one known on the other hand.

The study was able to identify three majorly used teaching strategies; problem solving, lecture method and use of example. The average performance of the learners taught using problem solving strategy got a mean score of 36%. The second teaching strategy used by most of the teachers in this topic was the use of examples in which students scored on average 34%. Learners who were taught using lecture method scored 30%. Consequently, problem solving was the best strategy used due to its higher learners' performance.

When a question does not restrict a particular method of solving quadratic equation like the question 1, students would prefer to solve using factorization, formula and completing square methods in that order. Factorizing equation which is not in standard form, unable to factorize equation which is equated to a constant, zero product property, incorrect factors and imposing linear structure. In completing square method, the students had the following threshold concepts not dividing the equation by the coefficient of x^2, not adding half the coefficient of x to both sides of the equation and inability to convert the results to squared form. While in using the formula students difficulties included encoding and carelessness in substitution, incorrect discriminant when c is negative, dividing only discriminant by 2a and wrong method.

Consequently, their performance also depended on how effectively they used factorization, completing the square, and quadratic formula for solving quadratic equations. As shown in table 4.1, the structural properties of the quadratic equation in Q1 (the question with the highest percentage of correct solutions) and Q3 and Q4 (the one with the lowest percentage of correct solutions) differ from one another. Although the quadratic equation in Q1 was factorable, has integer coefficients, and its roots were all rational numbers, the quadratic equations in Q3 and Q4 were not factorable; their roots were giving decimal numbers.

Interference from previously learned arithmetical procedures like prime factorization was witnessed to hinder the development of subsequent quadratic concepts. A learner whose questionnaire number was 256, prime factorized 99 and wrote 3 and 11 as pairs of consecutive odd numbers. Therefore, apart from the difficulties encountered by students when translating word problems into quadratic language, there were other difficulties such as interferences from other systems, applying the square root unnecessarily like questionnaire number 279, not understanding the equal sign as a relationship, and other misconceptions in simplifying quadratic expressions.

The study findings regarding word problems in quadratic equations and functions equations was that they were quite difficult for students almost all the questions were left blank. In this regard the study found comprehension of the problem statement to be the central reason for learners' threshold concepts with the word problems, rather than cognitive challenges in the solution phase of the symbolic equations. Learners experienced lack of comprehension and interpreting the word problem, as well as in representing the relationships symbolically due to complex syntactic structures. The learners interviewed honestly said those questions did not even look quadratic, like in Q5 most of the learners linearized the problem.

Moreover, even though some of the learners did comprehend the problem statement fully, they experienced failure because they did not know which mathematical procedures to conduct or how to formulate the correct relationships. In fact the Pythagoras relations for example in Q5 could not be remembered by the learners despite having related the three quantities in a right angled triangle. Some learners' incorrect solutions indicated that they knew which quantity in the problems could be symbolized as the unknown; however, they could not construct certain meaningful relationships in terms of a quadratic equation. Therefore, learners were required to recognize the underlying structure of relationships between quantities, in order to use quadratic methods of solving quadratic equation and functions.

The one-way MANOVA results examined that gender (male or female) does not affect learners' low and high order performance as no significant influence was found. The hypothesis to test for the significant gender difference with school type was tested using one way MANOVA. Using α=.05, the test was significant. The significant F indicated that there are significant differences among the school type groups on a linear combination of the two dependent variables (low and high). Hence, approximately 34% of multivariate variance of the DVs were associated with the group factor.

Teaching strategy moderated the relationship between gender and performance due to the significant interaction, $a3 = -1.9654, p = .0000.$ significant direct effect of gender on school type ($a1 = 3.4257, p = .0000$), but not on performance ($c = -0.3914, p = .9455$) . Teaching strategy moderated the relationship between gender, and performance, due to the significant interaction ($a2 = 3.1862, p = .0000$).

On the conditional direct effects of X on Y at values of the moderators, for teaching strategies in which the teacher has used his/her pedagogical knowledge well in which the students are actively involved there is an impact on learners' performance. So a teaching strategy like problem solving showed a positive relationship between gender, and good performance. The generally school type was not significantly moderating the relationship between gender and learners' performance. In this regard, it does matter which type of school a learner attended but perform well if and only if a teacher employ the right teaching strategy during the lesson presentation. For the conditional indirect effects of X on Y at values of the moderators, only teaching strategy was significant, and its effect increased as the teacher engages the students actively in the lesson. Thus the school type had no effect on learners' performance.

5.2 Conclusion

The results suggest that learners displayed high performance in solving the quadratic equation in Q1 especially using factorization method because it was easy to find factors and learners have had more practice and greater procedural abilities in solving these types of equations. However, when it came to the non-factorable structure of the quadratic equation in Q3 and Q4 which were to be solved using completing square and formula methods respectively, learners were unsuccessful, due to their lack of comprehension on threshold concepts in the formation of quadratic equation concepts. These questions required more quadratic symbol manipulation and arithmetic operations, while applying either a quadratic formula or complete square. The data also revealed that the method chosen to solve each quadratic equation affected the learners' performance. On the other hand, the number of learners who applied the completing the square method was quite low.

Although the nature of the quadratic equation in Q3 explicitly allowed learners to use the completing square method, a few of them attempted to use it. One of the fundamental reasons which learners interviewed gave was that they lack sufficient quadratic and arithmetic concepts to divide the equation through by the coefficient if x^2, additive inverse and complete the squares. Also, how to get factors, taking square root on both sides and zero product property to efficiently use the complete the square technique. Moreover, learners found it easier and faster using a quadratic formula and factorization methods. Therefore, since students memorize the rules, formulas, and procedures to solve quadratic equations without understanding the meaning, they could not transfer these rules, formulas, and procedures to solve the quadratic equations with non-standard structured properties. They also have a tendency to forget the formula after some time has passed since they learnt it. In addition, learners usually do not think about alternative techniques for solving quadratic equations in terms of their effectiveness and usefulness but think of one with least procedural concepts.

The learners interviewed reported that they were memorizing quadratic formula without really understanding how the formula was derived suggesting that teachers traditional teaching strategies in informing learners about the quadratic formula. Since completing square method precedes the teaching of quadratic formula method, a better alternative should be to assign learners into groups to explore how the quadratic formula is derived using completing square method and present their findings to the class. These way learners better remember knowledge they acquired through self-discovery.

The study found that school type did not affect learners' performance as such so long as the Mathematics teacher employs a good teaching strategy. In addition, if the teacher employs both higher-level and lower-level questions in the classrooms and the performance on both genders will be high as learners are able to apply their abstract knowledge in solving problems of higher order in which this study will establish based on satisfactory or dismal performance.

5.3 Recommendations

The study came up with three research recommendations:

5.3.1 Policy

The following policy recommendation was made by the study with respect to objectives i and iii

i. The Ministry of Education should frequently organize workshops, seminars and in-service training for Mathematics teachers specifically on the modern and research based teaching strategies.

ii. The Kenya Institute of Curriculum Development should document learners' threshold concepts encountered in their performances to enable teachers take precautionary measures during their lesson preparations.

5.3.2 Practice

With respect to objectives iv, and v, this study has the potential to offer many practice recommendations.

i. Threshold concepts were associated to the lack of conceptual knowledge and a real-life situation like using area model while teaching completing square method and deriving quadratic formula by completing squares and this would make the Mathematical threshold concepts not to look so abstract.

5.4 Future Research

This study has the potential to add to the mathematical community's knowledge of threshold concepts in solving of the entire spectrum of Mathematics.

i. Learner comprehension on threshold concepts regarding other branches of Mathematics could be of interest for further research.

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APPENDICES

Appendix A: Teacher's Pre-Lesson Interview Schedule

Name (Pseudonym): Date of Interview:

Elements Of Teaching Strategy	Question Related To Teaching Strategy	Responses
Knowledge of the subject matter	1. What are the key concepts in the lesson that you are about to teach? 2. Draw a concept map illustrating the sequence you will follow to teach these key concepts. 3. Does the lesson involve any procedural knowledge that the learners must know? If so, what does the procedure involve?	
Knowledge of teaching methods	➢ Which teaching method will you employ to ensure successful delivery of this lesson? ➢ Why did you choose such a method? ➢ In your selection of the method to be used in this lesson, have you selected real-life situation? Eg area model for factorization, completing square and projection or cup in graphical method.	
Knowledge of students' conceptions	i. What is the goal/aim of your lesson? ii. Which students' prior knowledge is regarded as important before the above key concepts can be successfully taught to students? iii. What possible student difficulties do you anticipate regarding this lesson? iv. How will you assist students who experience difficulties with this lesson? v. Have you prepared an assessment instrument to evaluate whether the goal of the lesson have been achieved?	

Teacher Pre-Lesson Interview Schedule

Teacher A

Teacher A was prepared to teach graphical method as a method of solving quadratic equation and functions with one known.

The following were the elements of teaching strategy observed.

a. Knowledge of the subject matter

   The key concepts in the lesson of Teacher A were the concept of graph, plotting, drawing and solution reading. When asked to draw a concept map illustrating the sequence Teacher A would follow to teach these key concepts, she mentioned that she would give an equation & use x values within the given range to find corresponding y values. After that she would draw the curve of the function and solve it by drawing the equation of a straight line. Teacher A said that the procedural knowledge included the reading of the solutions from the graph.

b. Knowledge of the teaching strategy

   In order to ensure successful delivery of the lesson, Teacher A reported that she would use illustration by using a given example. She chose such a method because it involves the learner and also the class size. She never selected real-life situation since it was only a graphical method of solving.

c. Knowledge of student's conceptions

   Teacher A aimed that by the end of the lesson, the students should be able to use graphical method in solving quadratic equations and functions. She reported that Cartesian plane, integers, x-y axis, scale drawing and co-ordinates were students' prior knowledge which were regarded as important before the above key concepts could be successfully taught to students. When asked about the possible students' difficulties she anticipated regarding the lesson, Teacher A mentioned subsisting x values to get the corresponding y values in the function especially x values are negative.

   Choice of current scale, plotting of a smooth curve and getting the linear equation from the quadratic function in order to solve for this simultaneous functions were also reported.

Teacher A had prepared an assessment instrument to evaluate whether the goal of the lesson had been achieved. They included the previous work on graphs and coordinates.

Teacher B

Teacher B was going to teach completing square method of solving quadratic equation and functions with one known.

Teacher B's content element of teaching strategy observed were as follows:

a. Knowledge of the subject matter

   The key concepts Teacher B mentioned were the making of perfect squares and taking square roots on both sides of the quadratic equations. The concept map Teacher B drew illustrating the sequence she followed to teach these key concepts included: adding or subtracting the constant term on both sides of the equations and making the coefficient of x^2 to be 1. Also mentioned were additive inverse of half the square of the coefficient of x & factorization the perfect square root on both sides and finally collecting like terms in order to get the values of the unknown.

   Teacher B agreed that the lesson involved some procedural knowledge that learners must know. There were making a perfect square and taking the square root on both sides.

b. Knowledge of teaching methods

   Teacher B employs discussion method to ensure successful delivery of the lesson. The choice of the method was because it had a defined procedure to follow and students would understand better because they were involved Teacher B reported that a rectangular surface like a wall was a real-life situation she selected.

c. Knowledge of the student's conceptions

   By the end of the lesson Teacher B aimed that the students would have been able to solve a quadratic equation using completing method.

d. The student's prior knowledge which was regarded by Teacher B as important before the above key concepts could be successfully taught to students were squares and square roots, integers, factorization and linear algebra.

The possible student difficulties Teacher B anticipated regarding that lesson included; addictive inverse of the content term especially when the constant term is a negative number. Also mentioned included the difficulties in factorization of the perfect squares and giving one value when taking the square roots.

When asked how Teacher B would assist students who experienced difficulties with that lesson: she said she would use other students and more examples.

Teacher B had prepared an assessment instrument to evaluate whether the goal of the lesson had been achieved. They included; giving supervised work in class, giving assignments after the lesson and giving random assessment test (RAT) after completing the topic.

Teacher C

Teacher C prepared a lesson to teach factorization as a method of solving quadratic equation and functions. The following were the elements of teaching strategy found.

a. Knowledge of the subject matter

   Factorization of expressions and using factors to solve the given equations were the key concepts in the lesson that Teacher C was about to teach. The concept map drawn by Teacher C illustrating the sequence he would follow to teach the key concepts were factorizing expression – using the factors to solve equations.

   Teacher C involved a procedural knowledge that the students must know in the lesson. Students was supposed to know how to factorize quadratic expressions.

b. Knowledge of teaching methods.

   Teacher C would employ lecture method to ensure successful delivery of the lesson. The choice of the method was to allow the learners to actually see how it is done and then do the same through factorization.

   Teacher C had not selected real-life situation to use.

c. Knowledge of students' conceptions

   The students of Teacher C should know how to factorize quadratic equations by the end of the lesson.

   Factorizing quadratic expressions, done in form 2, was the students prior knowledge TC regarded as important before the above key concepts could be successfully taught to students.

Finding factors involving fractions was mentioned by Teacher C as the possible student's difficulties he anticipated regarding that lesson. He would assist students who experienced difficulties with that lesson by taking time to go through more exercises in order to reinforce the lesson.

Teacher C had prepared an assessment instrument to evaluate whether the good of the lesson have been achieved.

Teacher D

Teacher D was prepared to teach quadratic formulate method of solving quadratic equations and functions with one known. The following were the pre-lesson interview observed relating to teaching strategy.

a. Knowledge of the subject matter

   Subsisting the constants a, b and c in the given formula were the key concepts in the lesson that Teacher D was about to teach quadratic equation – arranging the quadratic equation in standard form ($ax^2+bx+c=0$) – identifying a, b & c – write the formula – making correct substitution, were the concept map illustrating the sequence Teacher D would follow to teach these key concepts. The lesson did not involve any procedural knowledge that the students must knew.

b. Knowledge of teaching methods

   Teacher D would employ lecture method to ensure successful delivery of that lesson because it involved more substitutions. No real-life situation had been selected by Teacher D to be used in that lesson.

c. Knowledge of the students conceptions

   By the end of Teacher D's lesson the students should know how to make correct substitution in the quadratic formula. Completing square method was student's prior knowledge regarded by Teacher D as important before the above key concepts could be successfully taught to students.

   Teacher D mentioned that substitution involving negative integers was the possible student difficulties he anticipated regarding that lesson he would assist students who experienced difficulties with that lesson by ensuring that they do a lot of exercises.

Teacher D had prepared an assessment instrument to evaluate whether the goal of the lesson had been achieved.

Teacher E

Quadratic formula method of solving quadratic equation and function was a lesson Teacher E had prepared to teach. He pre-lesson interview on the elements of teaching strategy were as indicated as follows:

a. Knowledge of the subject matter

   Coefficients of a standard quadratic equation, squares and square roots and operations of integers were the key concepts in the lesson that he was about to teach.

   The key concepts Teacher E drew illustrating the sequence he would follow to teach were: having a standard quadratic equation - substituting terms in the formula and solving for x.

   The lesson involved procedural knowledge that the students must know. It involved students knowing the quadratic formula before making substitution and solving for x.

b. Knowledge of the teaching methods

   Teacher E would employ problem solving method to ensure successful delivery of the lesson. He chose such a method because it would actively engaged the students during lesson.

   Teacher E had not selected real-life situation to be used in the method selected.

c. Knowledge of student's conceptions

   By the end of the lesson, Teacher E expects the students to be able to solve quadratic equations and functions using quadratic formula method.

   Completing square method was the students' prior knowledge. Teacher E regarded as important before the above key concepts could be successful taught to students. He anticipated the making of correct substitution, taking the square root and dividing through by 2a as the possible students difficulties. To assist students who experience difficulties with that lesson, he would give more practical problems to solve.

   Revision exercises at the end of the topic were prepared as an assessments instrument to evaluate whether the goal of the lesson had been achieved.

Teacher F

Teacher F was ready to teach graphical method of solving quadratic equations and functions with one known. Illustrated below were the pre-lesson interview on elements of teaching strategy.

a. Knowledge of the subject matter

   The key concept of Teacher F was the drawing of a quadratic function. The concept map illustrating the Sequence Teacher would follow to teach that concept included X- coordinates, Y-coordinates and the plotting of smooth curve. Teacher F's lesson involved procedural knowledge that the students must knew he mentioned the finding of the corresponding y values from the given x-values and x-y Cartesian plane as the procedure involved.

b. Knowledge of teaching method

   Teacher F would employ lecture method and examples to ensure successful delivery of the lesson. It was chosen because it fitted the explanation of graphical method of solving simultaneous function. No real-life situation had been selected by Teacher F in that lesson.

c. Knowledge of students' conceptions

   By the end of the lesson, the students should be able to use a graphical method to solve quadratic equations and functions. A graph of a straight line was regarded as students' prior knowledge Teacher F and was important before the above key concepts could be successfully taught to students. Mentioned that finding the corresponding values of y within the range of values of x and the wrong scale were the students' difficulties he anticipate regarding that lesson.

In order to assist students who experienced difficulties with that lesson, Teacher F would address the problem as it arouse in class. Teacher F had prepared an assessment instrument to evaluate whether the goal of the lesson had been achieved. The question $y=2x^2+3$ was to be given to students at the end of the lesson.

Appendix B: Classroom Observation Protocol

Lesson number:-____ Topic___ Duration of period: ___ Date of Interview: _____

Which strategies do teachers use to teach quadratic equations and functions with one known?

Content element to be observed	Evident when the Teacher….	Observed Practice Displayed
a. Knowledge of the subject matter	1. Exhibits deep and thorough conceptual understanding of identified aspects of equations and functions. 2. Identifies critical mathematical components within the concept of equations and functions that are fundamental for understanding and applying that concept. 3. Displays skills for solving problems in the area of equations and functions.	
b. Knowledge of Teaching strategies	1. Use Examples 2. Use Group Work 3. Use Problem Solving 4. Use Real-Life Situation	
c. Knowledge of students' understanding	1. Addresses students' difficulties 2. Displays expectations of possible difficulties students may face during learning and address such. 3. Discusses students' ways of thinking about a concept. 4. Shows an awareness of the instruments to measure student learning and how to use them	

DESCRIPTION OF THE LESSON OBSERVATION
Teacher A

CONDITION OF THE CLASSROOM

There were 49 girls in the classroom. All the hand a locker and sited in pairs to comfortably share mathematics textbook. Teacher A had a full view of the class. The classroom had a white board, teacher's table and duster in front of the class. She had carried a marker pen and KLB mathematics course book.

LESSON: SOLVING QUADRATIC FUNCTION USING GRAPHICAL METHOD

Teacher A introduced the lesson by asking the students to get their graph books. "I want us to draw the graph of the quadratic function." She wrote the quadratic function on the white board using her blue marker pen.

a) $Y=2x^2-x-3$, $-3\leq x\leq 3$

b) Using a suitable line solve the quadratic function $y=2x^2-3x-5$

Teacher A then asked "what is the first step?" All the students answered in a chorus to the question, "Draw a table." She continued, "is it clear to everybody?" "Yes" the class once again responded in a chorus.

Teacher A then guided the student in drawing the table below:

x	-3	-2	-1	0	1	2	3
y	18	7	0	-3	-2	3	12

THEMES

The classroom presented a highly conducive learning environment despite a large class size

Teacher A used topical example as a method of teaching. She didn't recap the lesson by reminding the students of the graph of a straight line as indicated in pre-lesson interviews. The students had just learned the parent function of the form $y=x^2$ and this could have helped in emphasizing the key terms in drawing quadratic function. Teacher A's closed question of asking the students the first step elicit chorus answers without an effort of explaining their thinking.

Teacher A used a topic specific example to demonstrate how to draw the graph of a quadratic function. She displayed poor questioning (pedagogical knowledge) with regards to assessing whether the students have understood her explanation of choosing her a suitable scale.

Teacher A used content specific –procedural knowledge to get the equation of a straight line. She didn't ask any individual directed question to ensure that a common understanding of the concept.

Teacher A repeated content specific-procedural knowledge in finding the equation of a straight line and also in reading the values of x points of intersection which must be very accurate ($x=-1$ or $x=2^1/_2$)

She went on to say "you now need to come up with a scale by considering the highest value (18) and the lowest value (-3)." He explained further to the students, "Draw now a smooth curve by plotting ordered pairs of points in x-y plane." She went on to say "you realize the curve is turning upwards!"

Teacher A proceeded to guide the students to solve the quadratic function by eliminating the constant term. "Let us now subtract the two equations to get $y=2x+2$; which kind of equation is this? "The whole class responded in a chorus, "An equation of a straight line".

Teacher A proceeded to explain how to get an equation of a straight line by emphasizing that we do not only subtract but sometimes add the two functions. She went on to explain that to solve the quadratic function, any two points of the straight line equation are jointed. She concluded that the values of x are found at the points of intersection of the two curves. "..is it understood?" she asked. "Yes" the students responded in a chorus she ended the lesson with an exercise at the end of the topic.

Teacher B

Condition of the classroom

Teacher B had 51 male students each provided with a locker and a chair. Mathematics textbooks were shared by students seated in pairs and arranged in rows. She had a full view of all the students in the class. In the classroom there were a teachers table, a duster and a black wall. The classroom provides a conducing learning environment not only in terms of class organization but also the availability of teaching /learning aids.

Square Method

Teacher B introduced the lesson using content specific knowledge. She reviewed about making perfect squares and emphasized that completing square method is used to solve quadratic equations.

Teacher B used the lesson specific example to demonstrate how to solve quadratic equation using completing square method. Teacher B used content knowledge to explain to the student how $(x-{}^{3}/_{2})$ was found i.e. taking the square root on both sides. The students actively involved in the lesson. She explained using her content knowledge of the subject matter, though she could have used her pedagogical knowledge well if she could have used a student to explain.

Teacher B displayed good content knowledge on how to get values of x but displayed poor pedagogic knowledge by asking the students any problem? She should instead probe with regard to assessing whether her students have understood or not.

Teacher B's choice of real life situation problem displayed a good content knowledge. She further displayed a very good pedagogic knowledge when she helped the student with difficulty by asking another student to respond to the question.

Geometrical models, charts and a white grid was also available.

Lesson: Solving Quadratic Equations Using Completing

Teacher B, standing in front of the class, introduced the lesson "yesterday we learned how to make perfect squares using the three quadratic identities. In today's lesson we are going to learn how to solve quadratic equations using completing square method". She said and wrote the following quadratic equation on the black wall;

$x^2-3x-4=0$

Teacher B explains that the first step is to add 4 to both sides of the equation (additive inverse) to get $x^2-3x+c=4+c$ and in order to get C we use the relation $c = (^b/_2)^2$.

Teacher B moved on to the next step to get $(x-{}^3/_2)^2=2{}^5/_4$ which gave $x-{}^3/_2=\pm{}^5/_2$ and a student asked "way $(x-{}^3/_2)$? "she responded that you take the square root on both sides of the equation. Then she went on to get x=4 or $x={}^-1$, and ended with the question any problem?" There was silence.

Teacher B then read the second question to the class; "Using completing square method, calculate the side of a square whose area is $30cm^3$ if its length is $(3x+1)$ cm and a width of $(3x-2)$ cm".

Teacher B guided the students in getting the quadratic equation $9x^2-3x+2=0$. Once again a student asked why $(x-1/6)=\pm{}^{11}/_6$. She asked Korir to explain before making a conclusion and

The classroom presented a good learning environment.

Teacher C presented the content using lecture method. He didn't review about expansion of quadratic expressions for the students to quickly understand that factorization is the reverse process. He's question allowed chorus answers.

Teacher C used a topic specific example to demonstrate how to solve quadratic equation using factorization. He dominated the lesson hence didn't give room to the student to ask questions. He had good content knowledge but failed to show good pedagogic knowledge by not giving room to address student's difficulties especially when the coefficients of x or constant term is negative. Although the class size was very large, there was a conducive learning environment created because the students were arranged in rows which facilitated movement.

The D had a good content knowledge to present using lecture method. He didn't review completing square method by deriving the quadratic formula before informing the students that it is at the back of the mathematical tables. He emphasize substitution in solving for x and this makes students reluctant in learning. His question of asking if it's question of asking if it's clear didn't allow the students to think and express their views but only to give the easiest answer, yes.

Teacher D used a lesson specific example to demonstrate how to solve quadratic formula giving the values of x=2 or $x=-{}^{5}/_{3}$. She left a class assignment.

Teacher C

Condition of the classroom

Teacher C had a class population of 53 girls. Every student was provided with a locker and a chair and shared mathematics textbook in pairs. There was a black wall, a teacher's table and a duster in front of the classroom.

Lesson Topic: Solving Quadratic Equation using factorization method.

Teacher C introduced the lesson by writing a question on the black wall. He then asked the students, "What type of equation is x2+8x +12=0? The whole class responded in a chorus, "quadratic equation".

Teacher C went on to explain that when a quadratic expression is equated to zero, it is called a quadratic equation. He went on to solve by finding 2 factors whose sum is equal to 8 and whose product is equal to 12 (Product, Sum and factors). He then wrote the values of X=$^{-}$2 or -6.

Teacher D

Condition of the classroom

There were 32 male and 21 female students present during the lesson but 3 male and a female student were absent. Each student has a locker and a chair. In order to facilitate sharing of mathematics textbooks, students were seated in pairs which were arranged in rows. Teacher D had his checkbox and KLB mathematics copies. A black wall, a teacher's table, a chair and a duster were available in the classroom.

method. He engages the students in making correct substation into the formula.

Teacher D once again used content knowledge to provide an explanation on how to make correct substation. He further gave a conceptual reason why

$\sqrt{9+40}$ and not $\sqrt{9-40}$.

Teacher D, though he demonstrate content knowledge predominantly used procedural knowledge to provide explanation on how to get $\sqrt{49}$ =$\pm$7. He used inefficient questioning strategy by asking if it is clear. He could have probed from another student (pedagogical knowledge) which could have helped exposed the students' difficulties with the use of quadratic formula.

Despite that it was a big class size in the whole school, there was a good classroom organization which provided a conducive environment for learning.

Quadratic Formula.

Teacher E introduced the day's lesson by recapping it with the previous lesson. When these was silence in the class he assigned and proceeded. In fact the best was to revise the previous lesson was to derive the quadratic formula using completing square method. He used his content knowledge to present quadratic formula. His topic specific example helped him to demonstrate how to make substitution.

Teacher E demonstrated his good content knowledge in forming the students that a,b & c are coefficients. His procedural knowledge

Lesson observation

Lesson topic: solving quadratic equation using quadratic formula

Teacher D greeted the class and wrote a quadratic formula on the black wall; x=$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. He explained that, "this quadratic formula is found at the back of the mathematical tables, so your only required to know how to substitute terms into the formula and solve for x, is that clear?" "Yes" the whole class responded in a chorus.

Teacher D showed the students how to solve x^2-3x-10=0 using quadratic formula method He mentioned that the quadratic equation must be in a standard form $ax^2+bx+c=0$ before using the formula. He then asked the class, "in our case, what is a? b? and c? "a-1, b=-3 and c=-10". He proceeded to show the students how to make correct substitution into the formula.

Teacher D proceeded with the lesson, "Now, x= $\frac{-3\pm\sqrt{9-40}}{2}$

And therefore x=$\frac{-3+7}{2}$ of x=$\frac{-3+7}{2}$ which gives x= 2 or x=-5. Then he asked, "Have you understood?" A student responded "No teacher, 'why not $\sqrt{9-40}$ =$\sqrt{-31}$?".

Teacher D reminded the students that since c=-10, the minus (-) before 4ac and the negative 10 gives a positive number. "is it now clear?". He asked a question which the class responded "Yes".

helped him to explain clearly how to make accurate substitution of terms into the formula.

Teacher E discouraged chorus answers and his efficient pedagogical knowledge enabled him probe the student which gave them an opportunity to express their comprehension of the use of quadratic for formula. This was a diagnostic strategy which enable the students to expose his difficulty regarding the division of the formula by 2a. In fact in order to eliminate other difficulties he could have asked another student with different answers to explain how they got them.

Teacher E displayed a good content knowledge by using topic specific example in order to deal with student's difficulties because they solved using the newly acquired knowledge. He also used his pedagogical knowledge well by giving another question as he goes round marking thus being able to assist students with difficulties in solving equations using quadratic formula.

Teacher classroom looked over grounded but there was a conducive environment for learning presented.

Teacher F used lecture method to introduce the lesson. He didn't recap the lesson by reminding the students that if a=o a straight line equation of the form y= mx + c is formed something they learned in form two.

Teacher F had a good content knowledge but did not use his pedagogical knowledge well when he asked a closed questions. Teacher F used his quadratic function to demonstrate

Teacher E

Condition of the classroom

Teacher E taught in a boy's school with a class population of 45 students. The students sat in pairs and the rows provided limited movement. He had his chalks in a box and carried mathematics teacher's copies. A chair and a table with a duster were also available in the classroom.

Lesson Topic: Solving Quadratic Equations Using the

Teacher E standing in front of the class said, "Good morning class?" "Good morning teacher". The whole class responded. He asked the students if they have any question regarding the provisions assignment on solving quadratic equation using completing square method. There was silence in the classroom but he proceeded to introduced the lesson of the day by writing the question on the black wall, solve the quadratic equation, $4x^2+7x+3=0$ using quadratic formula. He then wrote quadratic formula on the black wall white informing the students that the formula is found at the back of the mathematical tables.

Teacher E proceeded to explain that letters a,b and c in the formular represented the coefficients of x^2, x and the constant term in the standard quadratic equations. He then made substitution to have $\frac{-7\pm\sqrt{7^2-4(4)(3)}}{2(4)}$

Teacher E then asked. "What is the answer? "It is negative ……………..'' students tried to respond in a chorus but he pointed at Kipkirui who gave -$5^7/_8$ of -$5^5/_8$ as the answer. The student's explanation showed that he had not divided 7 the how to get the corresponding values of y for every value of x within the range.

Teacher F predominantly used his procedural knowledge in making his explanation on how to draw a graph of a parabola. Teacher F's pedagogical knowledge is a good diagnostic strategy in identifying students' difficulties in drawing a graph of a quadratic function. The assignment he gave would enable the students to use the newly acquired knowledge to eliminate any difficulty.

Teacher F

Condition of the class

Teacher F taught in a mixed school with a class population of 29 male and 22 female students. He had a KLB mathematics course book and carried a chalk box. The classroom was arranged in rows and the students sat in pairs.

Lesson Topic: Drawing Graph of a Quadratic Function

Teacher F introduced the lesson to the class by writing on the black wall "Graph of a parabola. "He went on to say, "Standard quadratic equation where a≠0, b=0 and c=0 yields ax^2; is that understood? The whole class responded in a chorus "yes". He remarked to say, "A graph of $y=ax^2$ is called a parabola".

term in the numerator by 8, the common denominator(29)

Teacher E wrote another question on the black wall; $x^2+tx=12$ which the students solved using quadratic formular methods as he moves round to mark. He concluded the lesson by giving a class assignment.

Teacher F proceeded and wrote the following question on the black wall,

1. Draw a graph of $y=x^2$, $-3\leq x \leq 3$. He went on to explain to the students how to get the corresponding values of y by substituting the given values of x as follows

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

Teacher F drew the table as shown above and went onto explain, “using an ordered pairs of x and y, plot a graph of which should be ¾ of a page and jointed with a smooth curve.

Teacher F inquired if there was anyone with a difficulty as he moved around the class. He concluded the lesson by giving a class assignment of the graph.

$Y=2x^2+3$ $-3\leq x\leq 3$.

Appendix C: Diagnostic Test Instrument

My name is Gilbert Tonui, a student at Moi University, School of Education and I am carrying out a research study entitled, *"Students' Difficulties in Solving Quadratic Equations and Functions with One Known*". Your response to the questions and interview will be treated confidentially. Anonymity will be the highest priority to all information recorded during the interview and it will **ONLY** be used for the purpose of the study.

INDICATE YOUR:

Name (Pseudonym):

Gender: Male { } Female { }

School type: Boys { } Girls { } Mixed { }

Age (years): less than 16 { } 17-18 { } above 19 { }

INSTRUCTIONS

- Do not write your **REAL** name on the questionnaire.
- Please attempt all the questions and show you're working.
- Your answers will be treated confidentially and for academic purposes only.

Time: 1 hour

1. Solve the following quadratic equations:

a. $2q^2 - 8 = 6q$ (2 marks)

b. $3x^2 - x + 8 = 10$ (2 marks)

c. $c^2 - 14 = 5c$ (2 marks)

2. Find the missing term to make $x^2 + \_ + 36$ a perfect square. (2 marks)

3. Solve $3x^2 - 3x - 2 = 0$ by completing the square. (3 marks)

4. Solve the following equation using the quadratic formula: $z^2 - 3z - 8 = 0$

(2 marks)

5. Find two consecutive odd integers whose product is 99. (Note: There are two different pairs of consecutive odd integers and only an algebraic solution will be accepted).

(3 marks)

6. The buses A and B leave a bus station at the same time, one traveling to the north and the other travels to the east. The speed of the bus B is 5 km/h is more than that of the bus A. Determine the (average) speeds of the buses if they are 50 km apart after 2 hours they left the station.

(4 marks)

7. Using the same axis, draw the graph of:

a) $y = x^2$, (2 marks)

b) $y = x^2 + 10$, (2 marks)

c) $y = x^2 - 10$ for $-2 \leq x \leq 4$ (2 marks)

i. What maps $y = x^2 + 10$ onto each of the other functions? (2 marks)

ii. What maps $y = x^2 + 10$ onto $y = x^2 - 10$? (2 marks)

8. (a) Complete the following table which gives the value of $10-x^2$ where x ranges from -4 to 4. (2 marks)

X	-4	-3	-2	-1	0	1	2	3	4
$10-x^2$	-6		6		10	9			-6

(b) Using 2 cm for 1 unit on x axis and 1 cm for 1 unit on y-axis, draw the graph of $Y= 10-x^2$.

i. Use the graph to solve the equation $0 = 10-x^2$. (2 marks)

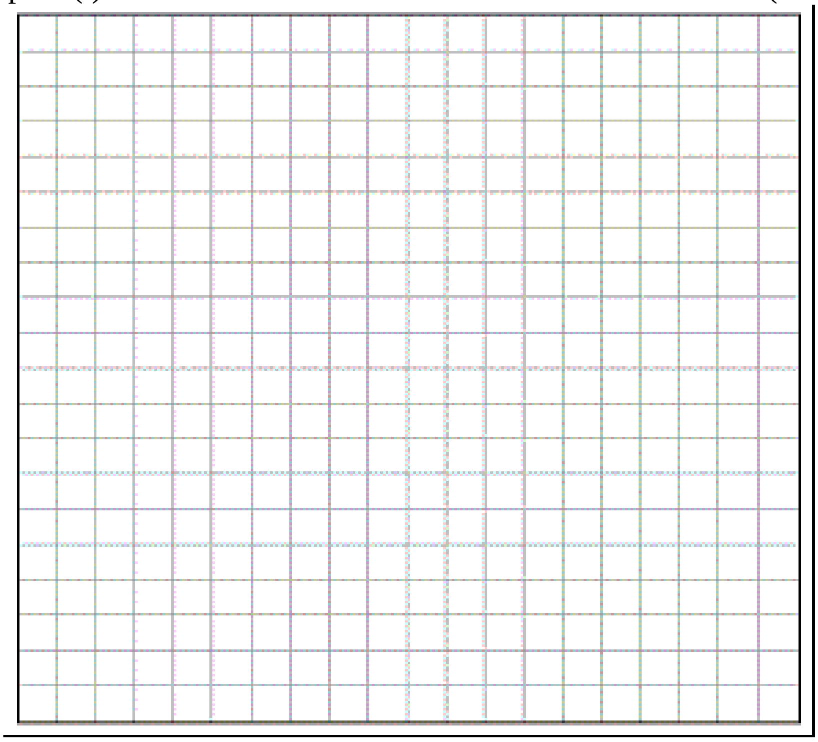
ii. On the same axes draw the graph of the equation $y = 2x+3$.
(3 marks)

iii. Write down the values of x at the point where the two graphs intersect.
(1 mark)

iv. Find; in its simplest form the equation for which these values of x are the roots. (2 marks)

9. A form four student throws a bag of chips to her friend. Unfortunately, her friend does not catch the chips and the bag hits the ground. The distance from ground (height) for the bag of chips is modeled by the function $h(t)=-16t^2 +64t + 80$, where h(t) is the height (distance from the ground in feet) of the chips and g(t) is the number of seconds the chips are in the air.

a. Graph h (t). (3 marks)



b. What is the maximum height the bag of chips reaches while airborne? Explain. (2 marks)

c. How many seconds after the bag was thrown did it hit the ground? (1 mark)

d. What is the average rate of change of height for the interval from 0 to **12** second? What does that number represent in terms of the context? (2 marks)

e. Based on your answer to part (e), what is the average rate of change for the interval from **15** to **2** sec.? (2 marks)

Thank you for your cooperation!!!

Appendix D: Student Interview Protocol

Preamble: Thank you so much for attempting the quadratic problems today. I am wondering about students' difficulties in solving quadratic equations and functions with one known and would like to ask you some questions.

Is it okay that we talk about them now?
May I take images of your work?
Thanks!

Name (Pseudonym): **Gender** **Date of Interview**

STUDENT RESPONSE FORM

Question	**Follow up Questions**	**Student Response that indicates Developed Concept/ Difficulty**
I have your answer script for the quadratic equations and functions here. Starting from question 1 to 9, which ones were easy, difficult and why?	As the student has solved these problems, press for how s/he has solved and the quality of the explanation. For each question ask about: ✓ Which method do student used in solving quadratic equations and functions and why? ✓ What does it mean for an equation to have two solutions? ✓ What does it mean to have solved a quadratic function?	

Thank you so much for doing all of these problems with me! I really appreciate it.

STUDENTS INTERVIEW RESPONSES

STUDENT 1 **Threshold Concepts**

QUESTION 1. Student 1 used quadratic formula method in solving quadratic equations and functions with one known because he found to be easier compared to the other methods. He said the difficulty of *completing square method is making the equation a perfect square*.

3 Student 1 got the question 3 correct, though he mentioned that he used to have a difficult in *adding half the square of the coefficient of x (the relation* $(b/2)^2 = c$*)*. He also mentioned *additive inverse of the constant term* as another difficult student faced.

5. Student 1 left the question 5 blank because he had the difficulty of *getting the two consecutive odd integers.*

6. Student 1 didn't comprehend question 6, making it hard for him to *interpret it symbolically*.

7. Student 1 got part of question 7 but part i and ii were left blank because he had the difficulty of *understanding the question*.

8. Student 1 got part of question 8 but in part iii, he had the difficulty of *locating the point of intersection.*

9. Student 1 had a difficulty of *understanding the* question as it was very unfamiliar.

STUDENT 2

QUESTION	Threshold Concepts
1.	Student 2 used quadratic formula method in solving because she had good knowledge about the method. With the other methods she said she would try her level best although *they take more time than the* formula method.
2.	Regarding question 2, student 2 gave *a positive value* only as she didn't know that two numbers were required when from a square root of a number.
3.	Student 2 got question 3 correctly, though she remembered having a difficulty of *taking the square root of all the terms on the left hand side of the quadratic equation*. Therefore, she was not able to *make perfect squares*.
5.	Student 2 was unable to symbolically represent the word problem quadratically. She associated this to *her negative attitude towards long sentences and lack of practice*.
6.	Expressing it methodically, student 2 repeated that she didn't *comprehend the question*.
7.	Student 2 had no difficulty with question 7. She reported that other students couldn't *find the corresponding y values within the range of x values*. She stressed that if brackets were not used in squaring negative values of x, *the negative signs would end up giving wrong value of y.*
	Student 2 got part of question 8 but was not able to find quadratic equation in which x are roots because she couldn't *understand how to find values of x after getting the equation of a straight line.*

STUDENT 3	**Threshold Concepts**
QUESTION 1.	Student 3 liked using quadratic formula method in solving quadratics compared to factorization which she reported to have a difficulty in *finding factors*. She said when their teacher first introduce the formula, she crammed and she used to forget to include a in the common denominator but divide only by 2. She
1.c	also reported that *getting the determinant* proved difficulty especially when c was negative.
2.	When quadratic equation was not in standard form student 3 had the difficulty of bringing into the standard form. In this question instead of -14 he wrote 14, interchanging the sign. Since the sign of the constant term changed, he got *an incorrect determinant during substitution.*
3. 5.	Student 3 had a difficulty of getting the square root of a number as she said *she forgot to give -6.*
6. 7.	Student 3 had no difficulty in question 3 but she said that the difficulties associated to this question were *forgetting to divide through by the coefficient of* x^2 *if it's greater than 1*. She also said bringing the equation *in standard form is another* difficulty.
8. 9.	Student 3 had a difficulty in *symbolically representing the word problem*, otherwise she understood the terms used in the question.
	Student 3 had the difficulty of *comprehending* the question.
	Student 3 got the part of the graph correctly but reported that the most common difficulty to most of the students is *the use of the wrong scale and poor curves*.
	She got part of the question but she could not *formulate a quadratic equation using the values of x as roots.* Student 3 complained *about time* in question 9.

STUDENT 4 QUESTION	Threshold Concepts
1.	Student 4 used quadratic formula method in solving because she was formula with and found it easier than factorization method. She said the difficulty with factorization is *getting factors of the* equation.
1 b.	Student 4 had the difficulty *of substitution using the formula method in question* 1b because she interchanged b^2 for a^2 thus getting the *wrong determinant*.
1 c.	Student 4 could use quadratic formula method competently, but her big difficulty is *bringing a quadratic equation to standard form*. She associated this to her lack of practice.
2.	Student 4 laughed when she realized that question 2 was a quadratic equation which needed two values of the missing term because she had *given only one value*.
3.	Student 4 had the difficulty *of additive inverse of the negative constant*. Her difficulty was also using the *relation* $b^2 = 4ac$.
5.	Student 4 had a difficulty *in understanding question* 5 due to her negative attitude to word problems since they require critical thinking.
6.	The difficulty student 4 had in question 6 was inability to *comprehend the relationships between the variables*.
7.	In question 7i student 4 said it was very difficult as she *could not understand the term function*.
8.	With joy in her face, student 4 admitted not to have any difficulty in the quadratic function. She did not hesitate to mention other students' difficulties which included *wrong values of y especially when x values in the given range are negative*. She said they *retain negative sign after squaring x values*.
9.	Student 4 claimed to have ran short of time and so did not attempted question 9.

STUDENT 5 QUESTIONS	**Threshold Concepts**
1.	Student 5 said she understood quadratic formula method in solving quadratics than any other methods, though she is still comfortable with completing square method. She said when their teacher wrote the formula on the black wall, she first crammed and later mastered after solving more problems. She mentioned questioned must be in standard form as they were told by their teacher before writing the formula and making substitution. Therefore, she had no problem as she got all the questions which required the use of quadratic formula. But she said other students forget the *plus and minus sign* before the square root which finally would get one value of x.
2.	Student 5 gave only a *positive value* of b after using the relation $b^2 = 4ac$, as she forgot and had only understood up to that level.
3.	Before mustering completing square method of solving quadratics, student 5 *additive inverse of the constant term* as she could add the term only to one side.
5.	Student 5 represented the equation with x and y implying that she *didn't comprehend* the question.
6.	Student 5 reported that the statement was so confusing and therefore could not understand the relationships between distance and speed in the question.
7.	Student 5 substituted the second equation with the first getting unrealistic results which made it difficult to *interpret the transformation*.
8.	After getting the equation of the line y = 0, student 5 could not get the values of x.
9.	Student 5 said real-life problems were unfamiliar to her and had the difficulty in *understanding the question*.

STUDENT 6 QUESTIONS	**Threshold Concepts**
1.	Student 6 preferred factorization to any other because he found it to be easier. He said the difficulty *then is when the equation does not look factorable like when the constant term, c is negative leading to negative factor(s).*
1 b.	Student 6 chose quadratic formula method in solving question 1b because he could not *get factors easily*. He reported that their teacher just wrote the formula on the wall and told them to do a lot of practice in order to understand.
2.	Student 6 had the difficulty of applying the relation $b^2 = 4ac$, in order to make a perfect square.
4.	Student 6 proved not to have any difficulty in solving using quadratic formula. He did mentioned that the difficulties faced included *giving only positive value of x, getting wrong determinant and dividing the determinant only by 2a*.
5.	Student 6 had the difficulty of *not comprehending* the word statement hence could not represent it symbolically to form quadratic equation.
8.	Student 6 got *confused on how to use* the graph of $y = 10 - x^2$ in solving the equation $0 = 10 - x^2$.
9.	Student 6 could not get accurate values of x due to a *small scale* used.

STUDENT 7

QUESTIONS	**Threshold Concepts**
1.	Although she understand other methods of solving quadratics student 7 said she is very comfortable using completing square method. She emphasized that in completing square method *rearranging the equation into standard form and additive inverse*
2.	*of the constant term* are some of the difficulties other students face. Student 7 illustrated that a *perfect square* could using +12 and -12,
4.	she only forgot otherwise she had no difficulty. Due to lack of practice, student 7 performed a *wrong substitution* as she wrote b^2 instead of –b. she reported that quadratic formula method of solving quadratics was just written by their teacher on
5.	the black wall and were told the formula is found at the back of the mathematical tables.
6.	Student 7 acknowledged to have a difficulty in *understanding* question 5 and therefore could not form quadratic equations.
7.	Student 7 said that since she could not *understand* question 6, she decided to do other questions and was finally got up with time.
8.	Student 7 got part of question 7 as she could not understand the *transformation terms* used in the functions in 7i.
9.	Student 7 had the difficulty of *forming quadratic equations* using x values from the graphs in question 8. In question 9, student 7 had no difficulty. She reported that other students had the difficulties of being unable *to draw a smooth curve, unable to find the corresponding y values from within the range of x values, using the wrong scale and inverting the axis*.

STUDENT 8

QUESTIONS **Threshold Concepts**

1. Student 8 had a good mastery of the quadratic formula method of solving quadratics as she was aware that equations must be in standard form before any calculations are done. Her only difficulty was in the *substitution* and got the wrong answer because she never notice that the constant term c was negative. It is a bit difficulty for

2. her sometimes to get factors using factorization method as she reported.

3. Student 8 used the relation $b^2 = 4ac$ correctly but could not remembered to *give the negative value of b.*

The difficulties student 8 has with completing square method is

5. that sometimes she *forgets to make the coefficient of x^2 to be one and also making a quadratic equation perfect square.*

6. Student 8 had a difficulty of understanding question 5 thus *could not form a quadratic equations.*

7. Since question 6 did not look like a quadratic, student 8 *could not comprehend the question.*

8. Student 8 drew good graphs but did not know how to use it to answer the questions as she *could not understand some of the terms used like a function*.

9 Getting the corresponding values of y within the given range of x using the equation $0 = 10 - x^2$ was a difficulty student 8 had and therefore *was not able to solve the quadratic equation.* Otherwise she plotted a very good curve.

Student 8 *drew a very good curve but was only unable* to use it in answering some the questions.

STUDENT 9

QUESTIONS **Threshold Concepts**

1. Student 9 had the difficulty of making *equation perfect square and taking the square root on both side of the equation* while using completing square method.

2.

Student 9 used the method of completing square in 1b. but he could not find the additive inverse of a fraction easily because he *divided the equation which was not in standard form by the coefficient of* x^2.

4.

Student 9 just *ignored -6* in his answer, otherwise he was aware that the value of be must be a plus b and a minus b.

Student 9 had a difficulty of *making correct substitution* when he used quadratic formula in solving question 4. This led him in getting the *wrong determinant* and when asked how they were taught about

5.

the quadratic formula, he said their teacher wrote it on the wall and

6.

told that it was in the mathematical tables.

7.

Understanding question 5 was difficulty for student 9 and was unable to formulate quadratic equation.

Student 9 tried to apply the knowledge of *linear equations in solving* question 6 as it did not look quadratic at all.

8.

Student 9 drew a graphs but did not know how to use it to answer the questions as she could not understand some of the terms used like a function.

9

Student 9 got most of the question correct except only part iv which he said was difficulty. He reported that other students had difficulties in getting the corresponding values of y within the negative values of x. plotting points and drawing of a smooth curve were also the difficulties he cited.

Student 9 had a difficulty in understanding question 9 as it did not look like a quadratic equation.

STUDENT 10

QUESTIONS **Threshold Concepts**

1. Student 10 reported that he had a difficulty in using quadratic formula method in solving and best understand factorization. He said that factorization could only be difficulty if the *quadratic equation is not*

2. *in standard form.*

Student 10 gave a positive answer only because of the *positive signs*

3. in the quadratic expressions.

Student 10 in a hurry forgot the negative value of the solution. He mentioned that some of the difficulties associated with completing square method were *forgetting to divide through by the coefficient of*

5. *x2, taking perfect squares and square roots on both sides of the*

7. *equation.*

Student 10 had a difficulty in *comprehending word problems*.

8. Student 10 didn't have any difficulty in question 7 but he reported that other students were *unable to draw a smooth curve and wrong scale*. Student 10 still didn't have any difficulty in answering question 8. He mentioned that *incorrect subtraction of the equation of a line from that of the function and inaccurate reading of the x values* are some

9. of the difficulties students' faces in quadratic functions. In part iv of this question, he couldn't formulate a quadratic equation using the values of x as roots because he did not understand how to use a graph. In question 9, student 10 *couldn't understand* the question because it was too long.

Appendix E: Codes

A) Codes for Conversations: Quadratic Equations and Function Characteristics

- Conversations about whether quadratics is functions.
- Conversations about the Zero Product Property
- Standard Form
- y-intercept
- x-intercepts
- Line of Symmetry

B) Solving Techniques and Graphing Approaches

Solving Techniques

✓ Factoring
✓ Connects to area model
✓ Completing the square
✓ Quadratic Formula
✓ Tries to use linear techniques
✓ Solves by undoing (appropriate linear techniques)
✓ Uses a table of values
✓ Tries but not able to solve
✓ Says not possible
✓ Supported by the graph
✓ Invalid method (linear and/or interesting approaches)

Intercepts

- Plots *x*-intercepts first
- Plots *x*-intercepts only
- Plots *x*-intercepts, but no idea of how to find y-intercept or other points

Y-intercept

❖ Knows the *y*-intercept occurs where x=0
❖ Knows the *x*-intercepts are solutions to the equation

C) Codes for Connections

▪ Cannot connect between function and graph
▪ Makes connections between ideas

D) Codes for Difficulties

➢ **Factorization**

- Missing constants
- Conceptual and procedural
- Multiplication of factors – incorrect terms used and false guess
- Imposing linear structure (zero product property) - lack of understanding.
- Working to isolate the variable by adding or subtracting terms from both sides or diving both sides by x in the expression.
- Factoring common term in front of parenthesis correctly and resulting expression inside parenthesis incorrectly.

➢ **Quadratic Formula**

- Ignoring the square root sign or unintentionally forget about it.
- Only finds positive root
- Comprehension
- Transformation
- Encoding and carelessness especially in substitution
- Present formula in isolation, not defining the terms in the formula
- Incorrect calculation of the discriminant

➢ **Completing Square**

- Only finds positive root
- Comprehension
- Transformation
- Encoding and carelessness especially in substitution
- Dividing by the coefficient of x^2
- Lack of knowledge of factorization
- Additive inverse of a constant term
- Addition of half the square of x^2 to both sides
- Converting the results to squared form

➢ **Word Problems**

- Make drawing and examining in from a different point of view without formulating quadratic equation to represent relationships.
- Guess - and – test strategy- incorrect reasoning with a familiar context.
- Don’t really comprehend problem.
- Understand the problem and represent information as a quadratic equation but have difficulty in solving and interpretation.
- Difficulty in grasping the relationship between the 2 varying quantities
- Calculation errors using cross multiplication methods and zero-product property.

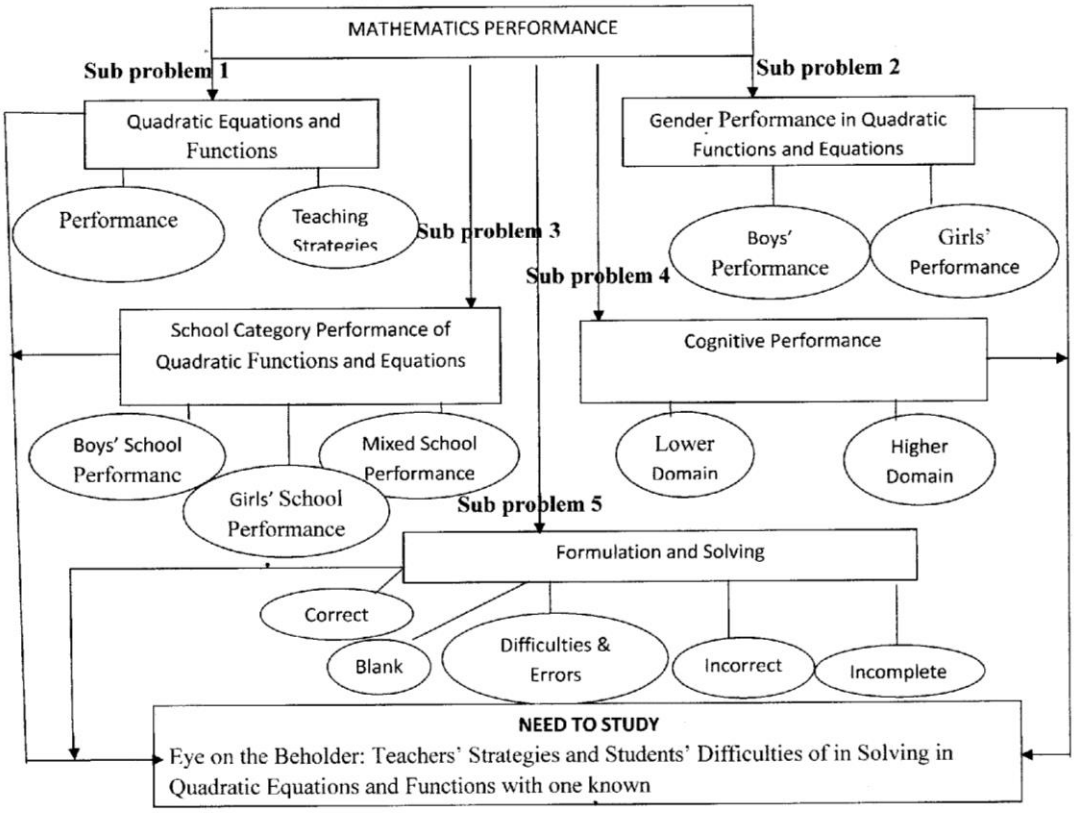
➢ **Graphical Method**

- Plots points incorrectly
- Roots of a function- x intercept
- Confusion between y-intercept and y-coordinates
- Other interesting errors

E) Codes for Justifications

✓ Explains thinking – why something works, or how things are connected
✓ Appeal to authority – says the teacher or book said so
✓ Through example
✓ Deductive argument
✓ Says they can't really explain

Appendix F: Research Literature Map



Appendix G: SPSS Macro Process Output

Run MATRIX procedure:

************* PROCESS Procedure for SPSS Release 2.16.3 ****************

Written by Andrew F. Hayes, Ph.D.   www.afhayes.com

****
Model = 4
Y = PERFORM
X = GENDER
M1 = Gender*school Type
M2 = Gender*Teaching Strategy

Sample size
384

****
Outcome: Gender*school Type

Model Summary

R	R-sq	MSE	F	df1	df2	p
.9121	.8319	.5900	1890.6732	1	382	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	-1.9613	.1208	-16.2336	.0000	-2.1988	-1.7237
GENDER	3.4257	.0788	43.4819	.0000	3.2708	3.5806

****
*****
Outcome: Gender*Teaching Strategy

Model Summary

R	R-sq	MSE	F	df1	df2	p
.8475	.7182	.9911	973.6972	1.0000	382.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	-1.2858	.1566	-8.2114	.0000	-1.5936	-.9779
GENDER	3.1862	.1021	31.2041	.0000	2.9855	3.3870

****

Outcome: PERFORM

Model Summary

R	R-sq	MSE	F	df1	df2	p
.1672	.0280	292.4977	3.6425	3.0000	380.0000	.0129

Model

	coeff	se	t	p	LLCI	ULCI
constant	36.1303	3.8851	9.2997	.0000	28.4914	43.7693
Gender*school	.8421	1.1709	.7192	.4725	-1.4602	3.1445
Gender*Strategy	-1.9654	.9035	-2.1754	.0302	-3.7418	-.1890
GENDER	-.3914	5.7270	-.0684	.9455	-11.6521	10.8692

******************** TOTAL EFFECT MODEL ***********************
Outcome: PERFORM
Model Summary

R	R-sq	MSE	F	df1	df2	p
.1087	.0118	295.7990	4.5645	1.0000	382.0000	.0333

Model

	coeff	se	t	p	LLCI	ULCI
constant	37.0057	2.7051	13.6802	.0000	31.6870	42.3244
GENDER	-3.7688	1.7640	-2.1365	.0333	-7.2371	-.3004

************* TOTAL, DIRECT, AND INDIRECT EFFECTS ****************

Total effect of X on Y

Effect	SE	t	p	LLCI	ULCI
-3.7688	1.7640	-2.1365	.0333	-7.2371	-.3004

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI
-.3914	5.7270	-.0684	.9455	-11.6521	10.8692

Indirect effect of X on Y

	Effect	Boot SE	BootLLCI	BootULCI
TOTAL	-3.3773	5.3360	-13.9680	7.0045
Gender*school	2.8849	3.9295	-4.4896	10.5431
Gender*Strategy	-6.2622	2.8130	-11.6760	-.8922
(C1)	9.1471	4.2702	.8951	18.5317

Partially standardized indirect effect of X on Y

	Effect	Boot SE	BootLLCI	BootULCI
TOTAL	-.1955	.3096	-.8002	.4040
Gender*school	.1670	.2267	-.2562	.6155
Gender*Strategy	-.3624	.1633	-.6762	-.0506

Completely standardized indirect effect of X on Y

	Effect	Boot SE	BootLLCI	BootULCI
TOTAL	-.0974	.1540	-.4094	.1991
VAR1VAR2	.0832	.1128	-.1276	.3062
VAR1VAR3	-.1806	.0812	-.3349	-.0257

Ratio of indirect to total effect of X on Y

	Effect	Boot SE	BootLLCI	BootULCI
TOTAL	.8961	7.7747	-2.3067	9.4124
VAR1VAR2	-.7655	8.9835	-5.9297	2.3039
VAR1VAR3	1.6616	13.5719	.0457	9.6740

Ratio of indirect to direct effect of X on Y

	Effect	Boot SE	BootLLCI	BootULCI
TOTAL	8.6278	8.9745	6.9499	147.5945
VAR1VAR2	-7.3699	9.5884	-200.3003	-5.4630
VAR1VAR3	15.9977	16.9822	11.3512	347.8948

Normal theory tests for specific indirect effects

	Effect	se	Z	p
Gender*school	2.8849	4.0130	.7189	.4722
Gender*Strategy	-6.2622	2.8871	-2.1690	.0301

Specific indirect effect contrast definitions
(C1) VAR1VAR2 minus VAR1VAR3

*************** ANALYSIS NOTES AND WARNINGS *****************

Number of bootstrap samples for bias corrected bootstrap confidence intervals:
1000

WARNING: Bootstrap CI endpoints below not trustworthy. Decrease confidence or increase bootstraps
-200.3003

Level of confidence for all confidence intervals in output:
95.00

------ END MATRIX -----

Appendix H: Map of Kericho County

Appendix I: Research Permit

CONDITIONS

1. The License is valid for the proposed research, research site specified period.
2. Both the Licence and any rights thereunder are non-transferable.
3. Upon request of the Commission, the Licensee shall submit a progress report.
4. The Licensee shall report to the County Director of Education and County Governor in the area of research before commencement of the research.
5. Excavation, filming and collection of specimens are subject to further permissions from relevant Government agencies.
6. This Licence does not give authority to transfer research materials.
7. The Licensee shall submit two (2) hard copies and upload a soft copy of their final report.
8. The Commission reserves the right to modify the conditions of this Licence including its cancellation without prior notice.

REPUBLIC OF KENYA

NACOSTI

National Commission for Science, Technology and Innovation

RESEARCH CLEARANCE PERMIT

Serial No.A 16631

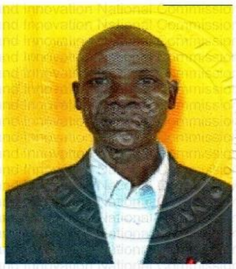
CONDITIONS: see back page

THIS IS TO CERTIFY THAT:
MR. GILBERT KIPKOECH TONUI
of MOI UNIVERSITY, 0-30100 eldoret,has been permitted to conduct research in
Kericho County

on the topic: ***PERFORMANCE AND DIFFICULTIES OF STUDENTS IN FORMULATING AND SOLVING QUADRATIC EQUATIONS AND FUNCTIONS WITH ONE KNOWN***

for the period ending:
20th November,2018

Permit No : NACOSTI/P/17/80946/19229
Date Of Issue : 22nd November,2017
Fee Recieved :Ksh 2000



....
Applicant's Signature

....
Director General
National Commission for Science, Technology & Innovation