GECH – WA'S THEOREM OF FINDING THE LENGTH OF SIDES OF A RIGHT ANGLED TRIANGLE

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GECH – WA'S THEOREM OF FINDING THE LENGTH OF SIDES OF A RIGHT ANGLED TRIANGLE

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Roof construction has been a challenge for many years both in developed and developing countries. For instance, right angled triangle application has been common but has been consuming plenty of time due to tedious calculations of the sides using the traditional Pythagora's theorem and trigonometrical ratios. In this paper, we present a solution to this problem through the discovery of the Gech-Wa's theorem of finding the lengths of sides of a right angled triangle (non-issosceles) when the length of only one side is known. This greatly reduces the time and effort spent on finding the lengths of sides of a right angled triangle and its application on roof construction among others.

Keywords: *Gech-Wa's theorem, right angled triangle, roof construction, Pythagora's theorem, trigonometrical ratios.*

1. Introduction

The lengths of the three sides of a right angled triangle have always been known to be related by the traditional Pythagora's theorem which can be written in the equation form as; $A^2 + B^2 = H^2$

Where H is represents the hypotenuse length, A and B represents the lengths of the other two sides of the triangle. The Pythagora's theorem states that "In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides", [1]. From the theorem, the lengths of two sides must be given for one to determine that of the third side.

On the other hand, trigonometrical ratios can be used to find the length of a side of a right angled triangle if one of the other lengths and one of the acute angles is known, [2]. In this case we use the term "SOHCAHTOA" which represents

$$Sin\ \theta = \frac{Opposite}{Hypotenuse}, \quad Cos\ \theta = \frac{Adjacent}{Hypotenuse} \quad \text{and} \quad Tan\ \theta = \frac{Opposite}{Adjacent}.$$

In this paper, we however, present a new formula of determining the lengths of any two unknown sides of a right angled triangle with the knowledge of only one side. Thus, if for instance we know the length of the hypotenuse, then from the results of this discovery, the lengths of the other two sides can be determined. Consequently, if we know the length of the shortest or medium side, then the length of the hypotenuse and one the other side can be determined.

2. Mathematical Formulation

We present various formulae that can be applied to calculate the lengths of any two sides of a right angled triangle if only one of the sides is known. First, we consider a set of integers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and divide them into two groups of five integers to form two classes. The first class has components 1, 2, 3, 4, 5 while the second class has the components 6, 7, 8, 9, 10. The

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largest integer in the first class is 5 and that of the second class is 10. We now present the formula for finding the other two sides when given the hypotenuse only, the medium side only, and the shortest side only, respectively.

a) If only the Hypotenuse is known

Consider the first class that is, 1, 2, 3, 4, 5. If 5 is the length of the hypotenuse of a right angled triangle as shown below, then it is a fact that the length of the medium side will be 4 units while that of the shortest side will be 3 units.

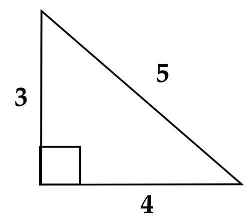

Figure 1

Hence we can find the length of the other two sides using the hypotenuse only, that is,

Medium side,

$$w = \left(H - \frac{H}{5}\right) \tag{1}$$

where H is the hypotenuse and 5 being constant.

Shortest side,

$$g = \left(H - \frac{2H}{5}\right) \tag{2}$$

Thus from the first class, if 5 is the hypotenuse, then the length of the other sides can be found by substituting $H = 5$ into the equations (1) and (2) gives the medium side as $w = 4$ and the shortest side as $g = 3$. This can also be applied to the second class.

b) If only the Medium side is known

From the equation of finding the medium side when given the hypotenuse only, that is,

$$w = \left(H - \frac{H}{5}\right) \Rightarrow H = \frac{5w}{5-1} = \frac{5}{4}w \tag{3}$$

After determining the hypotenuse, we can then find the shortest side using the formula $g = (H - \frac{2H}{5})$ i.e equation (2).

c) If only the Shortest side is known

From the original equation (2) relating the shortest side and the hypotenuse we have;

$$g = \left(H - \frac{2H}{5}\right) \Rightarrow H = \left(\frac{5w}{5-2}\right) = \frac{5}{3}g \tag{4}$$

After finding the hypotenuse, we can use the equation (1) to find the medium side.

d) Relationship between the shortest side, the medium side, and the Hypotenuse

From the equation (3) relating the medium side and the hypotenuse, that is; $H = \frac{5}{4}w$ and equation (4) relating the shortest side to the hypotenuse, that is $H = \frac{5}{3}g$ we can develop a ratio given by

$$H = \frac{5}{4}w = \frac{5}{3}g \;\Rightarrow\; \frac{5}{4}w = \frac{5}{3}g \;\Rightarrow\; \frac{w}{g} = \frac{4}{3} \;\Rightarrow\; w = \frac{4}{3}g \tag{5}$$

Hence if the shortest is g, the medium side will be $\frac{4}{3}g$ and the hypotenuse will be $\frac{5}{3}g$ and this is given in the figure below (Fig. 2).

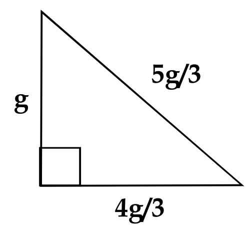

Figure 2

3. RESULTS (SPECIFIC EXAMPLE)

Given any right angled triangle with the hypotenuse as 16 units, we can determine the medium side and the shortest side and verify the result by the Pythagora's theorem. Therefore from Fig. 2 above, we have;

Hypotenuse, $H = \frac{5}{3}g = 16 \Rightarrow$ shortest side, $g = 16 \times \frac{3}{5} = 9.6$ units and

Medium, $w = \frac{4}{3}g = \frac{4}{3} \times 9.6 = 12.8$ units

Hence we have the shortest side as 9.6 units, medium side as 12.8 units.

By Pythagora's theorem,

4. CONCLUSION

From Fig. 2 above, Gech-Wa's theorem states that; "For a right-angled triangle, the Hypotenuse is $\frac{5}{3}$ times the shortest side and the medium side is $\frac{4}{3}$ times the shortest side provided the triangle is NOT issosceles." Thus provided one of the sides of the triangle is known, the remaining two sides can be determined using the theorem. The method has the advantage of being simple and convenient to use.

REFERENCES

[1] *www.mathisfun.com/pythagoras.html*, 22/10/2013.

[2] *www.mathisfun.com/trigonometricalratios.html*, 24/10/2013.