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NEWTON COTE'S QUADRATURE METHOD VERSUS STIRLING'S QUADRATURE METHOD

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In this paper we make a comparison between the Newton's Cote's quadrature method and Stirling quadrature method. The numerical quadrature rules related to the Stirling interpolation polynomial are developed as opposed to the commonly used Newton's interpolation polynomial. This is done for the case n = 1 *and* n = 2. *The Newton's Cote's and Stirling's quadrature methods are compared by making good use of well known integrals for the two cases* n = 1 *and* n = 2. *It is found that the Newton Cote's formula provides better accuracy than the Stirling's quadrature formula.*

KEYWORDS: *Numerical quadrature, Interpolation, Forward difference operator, Central difference operator.*

1. INTRODUCTION

Numerical integration techniques are indispensable tools for Applied mathematicians, scientist and engineers. They are used where analytical solutions fail. The well known numerical integration quadrature) rules include; the mid-ordinate rule, trapezoidal rule, Simpson's rule and the Weddle's rule. This rules may be obtained as a result of subdivision of the area under the curve $y = f(x)$ and between the ordinates $x = a$ and $x = b$. They may also be derived by using Newton's Cote's interpolation polynomial. For instance when $n = 1$, $n = 2$, $n = 3$ and $n = 6$ we obtain the trapezium, Simpson's one third rule, Simpson's two third rule, and the Weddle's rule respectively. In this paper we shall develop rules related to the Stirling's interpolation polynomial for $n = 1$ and $n = 2$ and compare the results with those of Newton Cote's polynomial case.

2. MATHEMATICAL FORMULATION

We intend to evaluate the integral,

$$I = \int_a^b f(x)\,dx . \qquad [1]$$

Let the range $(b - a)$ be divided into equal parts, each of which is of width h, then $(b - a) = nh$. Also suppose

$$x_0 = a,\ x_1 = a + h,\ x_2 = a + 2h, \ldots, x_n = a + nh = b .$$

It follows that

$$I = \int_a^b f(x)\,dx$$

$$I = \int_{x_0}^{x_0 + nh} f(x)\,dx$$

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$$I = \int_0^n f(x_0 + th).hdt \qquad [2]$$

Where $t = \frac{x - x_0}{h}$ or $dx = hdt$.

Now approximating the function by Newton's forward interpolation polynomial, we have

$$I = h\int_0^n \left[f_0 + t\Delta f_0 + \frac{t(t-1)}{2!}\Delta^2 f_0 + \frac{t(t-1)(t-2)}{3!}\Delta^3 f_0 + \dots \right] dt$$

$$I = h\left[nf_0 + \frac{n^2}{2}\Delta f_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 f_0}{2!} + \left(\frac{n^4}{4} - n^2 + n^4\right)\frac{\Delta^3 f_0}{3!} + \dots upto\,(n+1)\,terms \right] \qquad [3]$$

Δ is the forward difference operator.

Equation [3] is the Newton Cote's quadrature formula.

Shochiro (1991) and Mathur (1992) showed that if $n = 1$, $n = 2$, $n = 3$ and $n = 6$, we get the trapezoidal, Simpson's one third, Simpson's two third rules and the Weddle's rules respectively.

Suppose we now use the Stirling interpolation polynomial instead of the Newton's interpolation in equation [2] we have

$$I = h\int_0^n \left[f_0 + t\mu\delta f_0 + \frac{t^2}{2!}\delta^2 f_0 + \frac{t(t^2-1)}{3!}\mu\delta^3 f_0 + \dots \right] dt\,. \qquad [4]$$

The Stirling's interpolation polynomial is derived in Mathur (1992) where

$$\mu\delta f_i = \frac{1}{2}(f_{i+1} - f_i)\,,$$

$$\delta^2 f_i = f_{i+1} - 2f_i + f_{i-1}\,,$$

$$\delta^3 f_i = f_{i+\frac{3}{2}} - 3f_{i+\frac{1}{2}} + 3f_{i-\frac{1}{2}} - f_{i-\frac{3}{2}}\,.$$

Higher central difference operators are derived in Shochiro (1991), Curtis (1980) and Ward (2004).

Here δ is the central difference operator.

μ is the average operator.

Integrating equation [4] with respect to n we get

$$I = h\left[nf_0 + \frac{n^2}{2}\mu\delta f_0 + \frac{n^3}{3}\frac{\delta^2 f_0}{2!} + \left(\frac{n^4}{4} - \frac{n^2}{2}\right)\frac{\mu\delta^3 f_0}{3!} + \dots upto\,(n+1)\,terms \right]. \qquad [5]$$

Which is the general case.

Let $n = 1$ in equation [5] and neglect second and higher differences. Then

$$\int_{x_0}^{x_0+h} f(x)\,dx = h\left[f_0 + \frac{1}{2}\mu\delta f_0 \right] = h\left[f_0 + \frac{1}{4}f_1 - \frac{1}{4}f_0 \right] = h\left[\frac{3}{4}f_0 + \frac{1}{4}f_1 \right].$$

Similarly;

$$\int_{x_0+h}^{x_0+2h} f(x)\,dx = h\left[\frac{3}{4}f_1 + \frac{1}{4}f_2\right]$$

$$\int_{x_0+(n-1)h}^{x_0+nh} f(x)\,dx = h\left[\frac{3}{4}f_{n-1} + \frac{1}{4}f_n\right]$$

Adding these integrals, we have

$$\int_{x_0}^{x_0+nh} f(x)\,dx = h\left[\left(\frac{3}{4}f_0 + \frac{1}{4}f_n\right) + f_2 + f_3 + \ldots + f_{n-1}\right]. \qquad [6]$$

This rule corresponds to the trapezium rule when Newton Cote's quadrature formula is used.

Let $n = 2$ in equation [5] and neglect third and higher differences. Then

$$\int_{x_0}^{x_0+2h} f(x)\,dx = h\left[2f_1 + 2(f_2 - f_1) + \frac{4}{3}(f_2 - 2f_1 + f_0)\right]$$

$$= h\left[\frac{4}{3}f_0 - \frac{5}{3}f_1 + \frac{7}{3}f_2\right].$$

Similarly we have

$$\int_{x_0+2h}^{x_0+4h} f(x)\,dx = h\left[\frac{4}{3}f_2 - \frac{5}{3}f_3 + \frac{7}{3}f_4\right]$$

$$\int_{x_0+(n-1)h}^{x_0+nh} f(x)\,dx = h\left[\frac{4}{3}f_{n-2} - \frac{5}{3}f_{n-1} + \frac{7}{3}f_n\right]$$

Adding these n integrals we have

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}[(4f_0 + 7f_n) + 11(f_2 + f_4 + \ldots + f_{n-2}) - 5(f_1 + f_3 + \ldots f_{n-1})]. \qquad [7]$$

This rule corresponds to the Simpson's one third rule when Newton Cote's quadrature formula is used.

3. RESULTS

Table 1
Comparison of Results for the Different Methods and Integrands $n = 1, h = 0.02, a = 1, b = 2$

	Stirling quadrature formula		Newton's quadrature formula		Analytical formula
$f(x)$ Value	Approximatedvalue	% error	Approximated value	% error	value
$\frac{1}{1+x}$	0.397136	2.0954110	0.397136	2.054110	0.405465
$\frac{1}{1+x^2}$	0.314762	2.172080	0.314762	2.172080	0.321751
$\frac{1}{2+2x+x^2}$	0.138900	2.111860	0.138900	2.111860	0.141897
ln x	0.397136	2.806680	0.397136	2.806680	0.386294
sin x	0.938991	1.83891	0.938991	1.83891	0.956449

Table 2
Comparison of Results for the Different Methods and Integrands $n = 2, h = 0.02, a = 1, b = 2$

	Stirling quadrature formula		Newton's quadrature formula		Analytical formula
$f(x)$ Value	Approximatedvalue	% error	Approximated value	% error	value
$\frac{1}{1+x}$	0.403840	0.400775	0.405465	$6.59532e-08$	0.405465
$\frac{1}{1+x^2}$	0.318853	0.900697	0.321751	$6.34500e-08$	0.321751
$\frac{1}{2+2x+x^2}$	0.140927	0.683595	0.141897	$1.08241e-07$	0.141897
ln x	0.393076	1.755520	0.386294	$4.02000e-07$	0.386294
sin x	0.956840	0.040913	0.956449	$9.88931e-08$	0.956449

4. CONCLUSION

The Newton Cote's quadrature formula and Stirling's quadrature method are equally accurate for $n = 1$. However, the Newton Cote's quadrature formula is slightly more accurate than the Stirling's quadrature method for $n = 2$. The newly developed Stirling's quadrature rules may be used as integration techniques.

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