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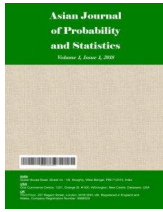
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Statistical Measure of Second Order Response Surface Rotatability Using an Infinite Class of Supplementary Difference Sets

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Rotatability is a desirable feature of a response surface experimental design. In case a design is non rotatable or exhibit surface of constant prediction variances that are nearly spherical then an attempt is made to make the design rotatable. In this paper, a measure of rotatability of five level second order rotatable designs using an infinite class of supplementary difference sets is suggested. The variance function of a second-order response design and an infinite class of supplementary difference sets is used in coming up with the design.

Keywords: Response surface methodology; rotatable designs; second order designs; five level; supplementary difference sets.

1 Introduction

The property of rotatability as a desirable quality of an experimental design was first advanced by [1] and requires that the variances of the estimated response are constant on circles or spheres about the center of the design.

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Koukouvinos et al. [2] gave a general construction method for five level second order rotatable designs. Further research was done by Mutiso, Kerich and Ng'eno [3,4] where they constructed five level rotatable designs using an infinite class of supplementary difference sets. Park et al. [5,6] developed a measure of rotatability that is invariant under rotation and Ng'eno [7] developed a measure of modified rotatability using an infinite class of supplementary difference sets by fixing $c=5$. This article presents a measure of Box- Hunter [1] rotatability by fixing $c = 3$. The measure will be investigated using an infinite class of supplementary difference sets.

The following symmetry conditions (Moments conditions) needs to be satisfied for a design to form a Second order rotatable arrangement [1]

$$\begin{aligned}
 & \text{i) } \sum_{u=i}^N \prod_{i=1}^v x_{iu}^{\alpha_i} = 0 \text{ if any } \alpha_i \text{ is odd for } \sum \alpha_i \leq 4 \\
 & \text{ii) } \sum_{u=1}^N x_{iu}^2 = \text{Constant} = N\lambda_2 \text{ for } i = 1, 2, \dots, v \\
 & \text{iii) } \sum_{u=1}^N x_{iu}^4 = \text{Constant} = cN\lambda_4 \text{ for } i = 1, 2, \dots, v \\
 & \text{iv) } \sum_{u=1}^N x_{iu}^2 x_{ju}^2 = \text{Constant} = N\lambda_4 \\
 & \text{v) } \frac{\sum_{u=1}^N x_{iu}^4}{\sum_{u=1}^N x_{iu}^2 x_{ju}^2} = c \text{ for } i \neq j
 \end{aligned} \tag{1.1}$$

where c , λ_2 and λ_4 are constants.

Using these symmetry conditions the variance and the covariance of the estimates are obtained and are shown below

$$\begin{aligned}
 & \text{i) } V(b_0) = \frac{(c+v-1) \lambda_4 \delta^2}{N[(c+v-1) \lambda_4 - v\lambda_2^2]} \\
 & \text{ii) } V(b_i) = \frac{\delta^2}{N\lambda_2} \\
 & \text{iii) } V(b_{ij}) = \frac{\delta^2}{N\lambda_4} \\
 & \text{iv) } V(b_{ii}) = \frac{\delta^2}{N(c-1)\lambda_4} \left[\frac{(c+v-2)\lambda_4 - \lambda_2^2(v-1)}{(c+v-1)\lambda_4 - v\lambda_2^2} \right] \\
 & \text{v) } \text{Cov}(b_0, b_{ii}) = \frac{-\lambda_2 \delta^2}{N[(c+v-1)\lambda_4 - v\lambda_2^2]} \\
 & \text{vi) } \text{Cov}(b_{ii}, b_{ij}) = \frac{\delta^2}{N\lambda_4(c-1)} \left[\frac{\lambda_2^2 - \lambda_4}{(c+v-1)\lambda_4 - v\lambda_2^2} \right]
 \end{aligned} \tag{1.2}$$

and all other covariances are zero.

An inspection of the variances shows that a necessary condition for the existence of a non-singular second order design is $(c + v - 1)\lambda_4 - v\lambda_2^2 > 0$ which leads to the following non-singularity condition first developed by Box and Hunter [1]

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{c+v-1} \tag{1.3}$$

Harder and Park [8] studied estimates in response at two different points in the factor space. They introduced the analogous form of Box Hunter rotatability and termed it as slope rotatability. Harder and Park [4] and Park [9] stated that the necessary and sufficient condition for slope rotatability is $4V(b_{ii}) = V(b_{ij})$. The condition was simplified by Victorbabu and Narasimham [10] and they developed equation (1.4) below which is the necessary and sufficient condition for a 2nd order design to be slope rotatable

$$\lambda_4[v(5 - c) - (c - 3)^2] + \lambda_2^2[v(c - 5) + 4] = 0 \tag{1.4}$$

2 Methodology

Seberry Wallis [11] defined supplementary difference sets and stated that the parameters of e - $[v; k_1, k_2, \dots, k_e, \lambda]$ SDS satisfies $\lambda(v-1) = ek(k-1)$. Koukouvinos et al. [2] came up with the following useful relationships which we will utilize in this study.

- i) $\sum_{u=1}^N x_{1u} = \sum_{u=1}^N x_{2u} = \sum_{u=1}^N x_{3u} = \sum_{u=1}^N x_{4u} = 0$
- ii) $\sum_{u=1}^N x_{iu}^2 = 2^{t(m)}(e-1) + 2b^2 = N\lambda_2$
- iii) $\sum_{u=1}^N x_{iu}^4 = 2^{t(m)}(e-1) + 2b^4 = cN\lambda_4$
- iv) $\sum_{u=1}^N x_{iu}^2 x_{ju}^2 = 2^{t(m)}(e-2) = N\lambda_4$

Mutiso et al. [4] stated in a theorem that the Supplementary difference sets with parameters e - $[v:2:1]$ gives a five level second order rotatable design in $b^4 = \frac{2^{t(m)}[2e-5]}{2}$ and $n_0 = \frac{-[-2v+4][2^{t(m)}(e-1)+2b^2]^2}{2v[2^{t(m)}(e-2)]} - m2^{t(m)} - 2m$

Using (iii) and (iv) above we obtain

$$b^4 = \frac{2^{t(m)}[c(e-2) - (e-1)]}{2} \tag{2.1}$$

We use equation (1.4) to get the number of centre points (n_0) to be added to the design i.e

$$\lambda_4[v(5-c) - (c-3)^2] + \lambda_2^2[v(c-5) + 4] = 0 \text{ where } \lambda_2 = \frac{2^{t(m)}(e-1)+2b^2}{N}; \lambda_4 = \frac{2^{t(m)}(e-2)}{N}$$

on simplification we obtain

$$N = \frac{[v(c-5)+4][2^{t(m)}(e-1)+2b^2]^2}{[v(5-3)-(c-3)^2]2^{t(m)}(e-2)} \tag{2.2}$$

therefore $n_0 = N - m2^{t(m)} - 2m$

$$\therefore n_0 = \frac{-[v(c-5)+4][2^{t(m)}(e-1)+2b^2]^2}{[v(5-c)-(c-3)^2]2^{t(m)}(e-2)} - m2^{t(m)} - 2m$$

Basing on Box –Hunter rotatability criteria [1] we fix c to be 3 in 2.1 and 2.2 and therefore we have

$$b^4 = \frac{2^{t(m)}[2e-5]}{2} \tag{2.3}$$

and therefore,

$$n_0 = \frac{-[-2v+4][2^{t(m)}(e-1)+2b^2]^2}{2v[2^{t(m)}(e-2)]} - m2^{t(m)} - 2m \tag{2.4}$$

A design whose moments do not conform to the moment conditions of rotatability is said to be non-rotatable. If circumstances are such that exact rotatability is unattainable, it is still a good idea to make the design nearly rotatable [5,6]

The article utilizes the measure developed by Park et al. [6] to access the degree of rotatability. If the region of interest is $0 \leq \rho \leq 1$, then the rotatability measure is expressed as

$$P_v[D] = \frac{1}{1+R_v[D]} \tag{2.5}$$

where

$$R_v[D] = \left[\frac{N}{\delta^2} \right]^2 \left\{ \frac{6v[v(b_{ij}) + 2Cov(b_{ii}, b_{ij}) - 2V(b_{ii})]^2 (v-1)}{(v+2)^2 (v+4)(v+6)(v+8)g^8} \right\} \text{ and } g \text{ is a scaling factor.}$$

on simplification the numerator portion of $v(b_{ij}) + 2Cov(b_{ii}, b_{ij}) - 2V(b_{ii})$ become $\frac{(c-3)\delta^2}{(c-1)N\lambda_4}$ thus $R_v[D]$ becomes, $R_v[D] = \left[\frac{N}{\delta^2} \right]^2 \left\{ \frac{6v[(c-3)\delta^2]^2 (v-1)}{(c-1)N\lambda_4)^2 (v+2)^2 (v+4)(v+6)(v+8)g^8} \right\}$ (2.6)

For second order rotatable design, we have $c=3$. Substituting the value of $c=3$ in the above equation we get $R_v[D] = 0$. Hence $P_v[D]$ takes the value of 1 if and only if a design is rotatable and it's smaller than one for non rotatable design.

To compare the design, Park et al. [6] considered the scaling of the design. In this article, the design is scaled in such a way that all the point's lie inside or on the unit sphere [12].

If we have a set of points $\mathbf{x}'_i = (x_{1u}, x_{2u}, \dots, x_{vu})$ $i=1,2,\dots,N$ then the scaled point, gx_i should satisfy

$$0 \leq g\sqrt{(x_{1u})^2 + (x_{2u})^2 + \dots + (x_{vu})^2} \leq 1 \quad i=1,2,\dots,N \quad (2.7)$$

One advantage of this is that, when we add center point, the remaining points do not have to be rescaled. To construct the measure we will use equation (2.5) developed by park et al. [6] and replace v with m because in supplementary difference sets we have $m = \frac{v-1}{2}$ factors. We therefore obtain the following expressions $P_m[D] = \frac{1}{1+R_m[D]}$

$$\text{Where } R_m[D] = \left[\frac{N}{\delta^2} \right]^2 \left\{ \frac{6m[(c-3)\delta^2]^2 (m-1)}{(c-1)N\lambda_4)^2 (m+2)^2 (m+4)(m+6)(m+8)g^8} \right\} \quad (2.8)$$

$$c = \frac{\sum_{i=1}^N x_{iu}^4}{\sum_{i=1}^N x_{iu}^2 x_{ju}^2} = \frac{2^{t(m)}(e-1) + 2b^4}{2^{t(m)}(e-2)}, \quad b^4 = \frac{2^{t(m)}[2e-5]}{2}, \quad \lambda_4 = \frac{2^{t(m)}(e-2)}{N}$$

$$g = \begin{cases} \frac{1}{b} & \text{if } b \leq \sqrt{2^{t(m)-1} + m} \\ 1 \\ \sqrt{\sqrt{2^{t(m)-1} + m}} & \text{if } b > \sqrt{2^{t(m)-1} + m} \end{cases}$$

$$\text{On simplifying 2.8 we have } R_m[D] = \frac{6m(m-1)(c-3)^2}{(m+2)^2 (m+4)(m+6)(m+8)\lambda_4^2 (c-1)^2 g^8} \quad (2.9)$$

3 Results, Discussion and Conclusion

For a second order rotatable design, we have $c=3$. Substituting the value of $c=3$ in (2.6), we get $R_m[D] = 0$. Hence $P_m[D]$ takes the value of 1 if the design is rotatable and less than 1 if it is not rotatable.

Illustration

Let us consider 3-[7:2:1] SDS. In this case we have, $v = 7, m = e = \frac{v-1}{2} = 3$, and $\lambda_4 = \frac{2^{t(m)}(e-2)}{N} = \frac{2^2(3-2)}{21} = \frac{4}{21} = 0.1905$

Using (2.9) we have $R_m[D] = \frac{6m(m-1)(c-3)^2}{(m+2)^2(m+4)(m+6)(m+8)\lambda_4^2(c-1)^2 g^8} = 0.05727 \frac{(c-3)^2}{(c-1)^2 g^8}$

For rotatable design, $c=3$ meaning that $R_m[D]=0$ and therefore $P_m[D] = \frac{1}{1+R_m[D]} = 1$. Suppose $c= 3.92$, We use (2.8) to get the value of g . In this case $g = \frac{1}{b} = 0.7143$

$$R_m[D] = 0.05727 \frac{(c-3)^2}{(c-1)^2 g^8} = 0.05727 \frac{(3.92-3)^2}{(3.92-1)^2 g^8} = 8.3886 \times 10^{-2}$$

$$\text{Therefore } P_m[D] = \frac{1}{1+R_m[D]} = \frac{1}{1+8.3886 \times 10^{-2}} = 0.9226.$$

The above value is less than one but very close to one meaning that the design is nearly rotatable. Suppose $c=13.71$, $R_m[D] = 0.05727 \frac{(13.71-3)^2}{(13.71-1)^2 g^8} = 22.3328$

$$\text{Therefore } P_m[D] = \frac{1}{1+R_m[D]} = \frac{1}{1+22.3328} = 4.285 \times 10^{-2}$$

The above value is less than one but not close to one meaning that the design is not rotatable.

The expressions for calculating $R_m[D]$ for different class of supplementary difference sets are given in the following equations. The results of the measure for different class of SDS are as shown in tables 1 to 10 in the appendix.

$$3\text{-}[7:2:1] \text{ SDS} - R_m[D] = 5.7258 \times 10^{-2} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$4\text{-}[9:2:1] \text{ SDS} - R_m[D] = 7.9991 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$5\text{-}[11:2:1] \text{ SDS} - \text{Taking half replicate of factorial part} - R_m[D] = 1.1111 \times 10^{-2} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$5\text{-}[11:2:1] \text{ SDS} - R_m[D] = 8.4266 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$6\text{-}[13:2:1] \text{ SDS} - \text{Taking } \frac{1}{4} \text{ replicate of factorial part} - R_m[D] = 7.5599 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$7\text{-}[15:2:1] \text{ SDS} - \text{Taking } \frac{1}{4} \text{ replicate of factorial part} - R_m[D] = 4.3792 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$8\text{-}[17:2:1] \text{ SDS} - \text{Taking } \frac{1}{8} \text{ replicate of factorial part} - R_m[D] = 3.3859 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$9\text{-}[19:2:1] \text{ SDS} - \text{Taking } \frac{1}{16} \text{ replicate of factorial part} - R_m[D] = 2.6592 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$10\text{-}[21:2:1] \text{ SDS} - \text{Taking } \frac{1}{32} \text{ replicate of factorial part} - R_m[D] = 2.1586 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

$$11\text{-}[23:2:1] \text{ SDS} - \text{Taking } \frac{1}{64} \text{ replicate of factorial part} - R_m[D] = 1.7802 \times 10^{-3} \frac{(c-3)^2}{(c-1)^2 g^8}$$

A measure of rotatability of modified second-order rotatable design was endorsed by Ng'eno, H. M. [7]. The variance function of a second-order response design and an infinite class of supplementary difference sets is used in coming up with the design. Koukouvinos et al. [2] also present a method for constructing second-order rotatable designs in order to explore and optimize response surfaces based on an infinite class of supplementary difference sets. The produced designs achieve both properties of rotatability and estimation efficiency. Also, they possess good predictive properties [2].

Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX

Table 1. Measure of rotatability for five level SORD for 3-[7:2:1] SDS $v=7$, $m=3$, $N = 21$ $\lambda_4= 0.1905$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 2.50 | 1.0000 | 0.006362 | 0.9937 |
| 1.19 | 3.00 | 0.8403 | 0.0000 | 1.0000 |
| 1.20 | 3.04 | 0.8333 | 0.00094686 | 0.9999 |
| 1.40 | 3.92 | 0.7143 | 0.083886 | 0.9226 |
| 1.60 | 5.28 | 0.6250 | 0.6988 | 0.5889 |
| 1.80 | 7.25 | 0.5556 | 2.9158 | 0.2554 |
| 2.00 | 10.00 | 0.5000 | 8.8672 | 0.1013 |
| 2.20 | 13.71 | 0.4545 | 22.3283 | 0.04287 |
| 2.40 | 18.58 | 0.4472 | 28.1139 | 0.034235 |
| 2.60 | 24.85 | 0.4472 | 30.0435 | 0.032213 |
| 2.80 | 32.73 | 0.4472 | 31.4249 | 0.030834 |
| 3.00 | 42.50 | 0.4472 | 32.4282 | 0.029915 |
| 3.20 | 54.43 | 0.4472 | 33.1656 | 0.029269 |
| 3.40 | 68.81 | 0.4472 | 33.7148 | 0.028806 |
| 3.60 | 85.98 | 0.4472 | 34.1302 | 0.028466 |
| 3.80 | 106.26 | 0.4472 | 34.4478 | 0.02821 |
| 4.00 | 130.00 | 0.4472 | 34.6939 | 0.028016 |
| 4.20 | 157.58 | 0.4472 | 34.8866 | 0.027866 |
| 4.40 | 189.40 | 0.4472 | 35.0393 | 0.027748 |

Rotatable b = 1.19

Table 2. Measure of rotatability for five level SORD for 4-[9:2:1] SDS $m= 4$, $v=9$, $N = 47$, $\lambda_4= 0.5106$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.63 | 1.0000 | 0.037788 | 0.9636 |
| 1.20 | 1.76 | 0.8333 | 0.091496 | 0.9161 |
| 1.40 | 1.98 | 0.7143 | 0.1277 | 0.8867 |
| 1.60 | 2.32 | 0.6250 | 0.09108 | 0.9165 |
| 1.80 | 2.81 | 0.5556 | 0.096972 | 0.9904 |
| 1.86 | 3.00 | 0.5376 | 0.0000 | 1.0000 |
| 2.00 | 3.50 | 0.5000 | 0.0881827 | 0.9244 |
| 2.20 | 4.43 | 0.4545 | 0.7628 | 0.5673 |
| 2.40 | 5.65 | 0.4167 | 2.8549 | 0.2594 |
| 2.60 | 7.21 | 0.3846 | 7.6719 | 0.1153 |
| 2.80 | 9.18 | 0.3571 | 17.2483 | 0.054799 |
| 3.00 | 11.63 | 0.3536 | 21.5503 | 0.044345 |
| 3.20 | 14.61 | 0.3536 | 23.7928 | 0.040334 |
| 3.40 | 18.20 | 0.3536 | 25.5346 | 0.037687 |
| 3.60 | 22.49 | 0.3536 | 26.8937 | 0.035851 |
| 3.80 | 27.56 | 0.3536 | 27.9576 | 0.034533 |
| 4.00 | 35.50 | 0.3536 | 29.0153 | 0.033316 |
| 4.20 | 40.39 | 0.3536 | 29.4604 | 0.032829 |
| 4.40 | 48.35 | 0.3536 | 29.9925 | 0.032266 |

Rotatable b = 1.86

Table 3. Measure of rotatability for five level SORD for 5-[11:2:1] SDS m= 5($\frac{1}{2}$ replicate), v=11, N=58, $\lambda_4= 0.4138$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.42 | 1.0000 | 0.1573 | 0.8641 |
| 1.20 | 1.51 | 0.8333 | 0.4079 | 0.7102 |
| 1.40 | 1.65 | 0.7143 | 0.7073 | 0.5857 |
| 1.60 | 1.88 | 0.6250 | 0.7731 | 0.5639 |
| 1.80 | 2.21 | 0.5556 | 0.5216 | 0.6571 |
| 2.00 | 2.67 | 0.5000 | 0.1110 | 0.9000 |
| 2.12 | 3.00 | 0.4717 | 0.0000 | 1.0000 |
| 2.20 | 3.29 | 0.4545 | 0.097877 | 0.91088 |
| 2.40 | 4.09 | 0.4167 | 1.5212 | 0.3966 |
| 2.60 | 5.14 | 0.3846 | 6.2023 | 0.1388 |
| 2.80 | 6.16 | 0.3571 | 15.7609 | 0.059663 |
| 3.00 | 8.08 | 0.3333 | 37.5668 | 0.025929 |
| 3.20 | 10.07 | 0.3333 | 44.3371 | 0.022057 |
| 3.40 | 12.47 | 0.3333 | 49.7412 | 0.019708 |
| 3.60 | 15.33 | 0.3333 | 54.0227 | 0.018174 |
| 3.80 | 18.71 | 0.3333 | 57.4194 | 0.017118 |
| 4.00 | 22.67 | 0.3333 | 60.1221 | 0.016361 |
| 4.20 | 27.26 | 0.3333 | 62.2781 | 0.015803 |
| 4.40 | 32.57 | 0.3333 | 64.0171 | 0.015380 |

Rotatable b = 2.12

Table 4. Measure of rotatability for five level SORD for 5-[11:2:1] SDS m= 5, v=11, N=101, $\lambda_4= 0.4752$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.38 | 1.0000 | 0.1531 | 0.8672 |
| 1.20 | 1.42 | 0.8333 | 0.5129 | 0.6609 |
| 1.40 | 1.49 | 0.7143 | 1.1808 | 0.4583 |
| 1.60 | 1.61 | 0.6250 | 1.8792 | 0.3473 |
| 1.80 | 1.77 | 0.5556 | 2.3680 | 0.2969 |
| 2.00 | 2.00 | 0.5000 | 2.1572 | 0.3167 |
| 2.20 | 2.31 | 0.4545 | 1.2839 | 0.4378 |
| 2.40 | 2.72 | 0.4167 | 0.2457 | 0.8028 |
| 2.50 | 2.96 | 0.4000 | 0.053552 | 0.9947 |
| 2.52 | 3 | 0.3968 | 0.0000 | 1.0000 |
| 2.60 | 3.24 | 0.3846 | 0.2021 | 0.8319 |
| 2.80 | 3.89 | 0.3571 | 3.0221 | 0.2486 |
| 3.00 | 4.71 | 0.3333 | 11.7548 | 0.078402 |
| 3.20 | 5.70 | 0.3125 | 30.5762 | 0.031669 |
| 3.40 | 6.90 | 0.2941 | 65.7838 | 0.014974 |
| 3.60 | 8.33 | 0.2778 | 125.6152 | 0.0078980 |
| 3.80 | 10.02 | 0.2778 | 143.89873 | 0.069010 |
| 4.00 | 12.00 | 0.2778 | 159.0360 | 0.0062490 |
| 4.20 | 14.30 | 0.2778 | 173.4942 | 0.005797 |
| 4.40 | 16.95 | 0.2778 | 181.7284 | 0.005473 |

Rotatable b = 2.52

Table 5. Measure of rotatability for five level SORD for 6-[13:2:1] SDS m= 6($\frac{1}{4}$ replicate), v=13, N=68, $\lambda_4= 0.4706$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.31 | 1.0000 | 0.2247 | 0.8166 |
| 1.20 | 1.38 | 0.8333 | 0.5909 | 0.6286 |
| 1.40 | 1.49 | 0.7142 | 1.0604 | 0.4853 |
| 1.60 | 1.66 | 0.6250 | 1.3383 | 0.4277 |
| 1.80 | 1.91 | 0.5556 | 1.1944 | 0.4557 |
| 2.00 | 2.25 | 0.50 | 0.6967 | 0.5894 |
| 2.20 | 2.71 | 0.4545 | 0.1194 | 0.8933 |
| 2.30 | 3.00 | 0.4348 | 0.0000 | 1.0000 |
| 2.40 | 3.32 | 0.4167 | 0.1582 | 0.8634 |
| 2.60 | 4.11 | 0.3846 | 2.0115 | 0.3321 |
| 2.80 | 5.09 | 0.3571 | 7.4646 | 0.1181 |
| 3.00 | 6.31 | 0.3333 | 19.2869 | 0.049292 |
| 3.20 | 7.80 | 0.3162 | 37.6919 | 0.025845 |
| 3.40 | 9.60 | 0.3162 | 44.5528 | 0.021953 |
| 3.60 | 11.75 | 0.3162 | 50.1168 | 0.019563 |
| 3.80 | 14.28 | 0.3162 | 54.5766 | 0.017993 |
| 4.00 | 17.25 | 0.3162 | 58.1711 | 0.0169 |
| 4.20 | 20.70 | 0.3162 | 61.0659 | 0.016112 |
| 4.40 | 24.68 | 0.3162 | 63.4073 | 0.015526 |

Rotatable b = 2.30

Table 6. Measure of rotatability for five level SORD for 7-[15:2:1] SDS m= 7($\frac{1}{4}$ replicate), v=15, N=139, $\lambda_4= 0.5755$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.23 | 1.0000 | 0.2593 | 0.7941 |
| 1.20 | 1.25 | 0.8333 | 0.9229 | 0.5200 |
| 1.40 | 1.30 | 0.7143 | 2.0773 | 0.3252 |
| 1.60 | 1.36 | 0.6250 | 3.9033 | 0.2039 |
| 1.80 | 1.46 | 0.5556 | 5.4054 | 0.1561 |
| 2.00 | 1.60 | 0.5000 | 6.1037 | 0.1408 |
| 2.20 | 1.79 | 0.4545 | 5.6421 | 0.1506 |
| 2.40 | 2.01 | 0.4177 | 4.5405 | 0.1805 |
| 2.60 | 2.34 | 0.3846 | 2.2192 | 0.3106 |
| 2.80 | 2.74 | 0.3571 | 0.3698 | 0.7301 |
| 2.90 | 2.97 | 0.3448 | 0.005084 | 0.9949 |
| 2.91 | 3.00 | 0.3436 | 0.0000 | 1.0000 |
| 3.00 | 3.23 | 0.3333 | 0.3059 | 0.7658 |
| 3.20 | 3.82 | 0.3125 | 4.0712 | 0.1972 |
| 3.40 | 4.54 | 0.2941 | 14.8072 | 0.06363 |
| 3.60 | 5.40 | 0.2778 | 36.7330 | 0.026502 |
| 3.80 | 6.41 | 0.2632 | 75.5483 | 0.013064 |
| 4.00 | 7.60 | 0.2582 | 107.6900 | 0.0092005 |
| 4.20 | 8.98 | 0.2582 | 124.4928 | 0.0079686 |
| 4.40 | 10.57 | 0.2582 | 138.7124 | 0.0071576 |

Rotatable b = 2.91

Table 7. Measure of rotatability for five level SORD for 8-[17:2:1] SDS m= 8 ($\frac{1}{8}$ replicate), v=17, N=158, $\lambda_4= 0.6076$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.19 | 1.0000 | 0.3073 | 0.7649 |
| 1.20 | 1.21 | 0.8333 | 1.0581 | 0.4859 |
| 1.40 | 1.25 | 0.7143 | 2.4508 | 0.2900 |
| 1.60 | 1.30 | 0.6250 | 4.6697 | 0.1764 |
| 1.80 | 1.39 | 0.5556 | 6.3548 | 0.1360 |
| 2.00 | 1.50 | 0.5000 | 7.8011 | 0.1136 |
| 2.20 | 1.65 | 0.4545 | 8.0213 | 0.1108 |
| 2.40 | 1.86 | 0.4167 | 6.5448 | 0.1325 |
| 2.60 | 2.12 | 0.3846 | 4.3665 | 0.1863 |
| 2.80 | 2.45 | 0.3571 | 1.8422 | 0.3518 |
| 3.00 | 2.85 | 0.3333 | 0.1462 | 0.8725 |
| 3.06 | 3.00 | 0.3268 | 0.0000 | 1.0000 |
| 3.20 | 3.35 | 0.3125 | 0.8258 | 0.5477 |
| 3.40 | 3.95 | 0.2941 | 6.2736 | 0.1375 |
| 3.60 | 4.67 | 0.2778 | 19.7659 | 0.048156 |
| 3.80 | 5.51 | 0.2632 | 45.5388 | 0.021487 |
| 4.00 | 6.50 | 0.2500 | 89.8597 | 0.011006 |
| 4.20 | 7.65 | 0.2500 | 108.4967 | 0.0091328 |
| 4.40 | 8.98 | 0.2500 | 124.6094 | 0.0079612 |

Rotatable b = 3.06

Table 8. Measure of rotatability for five level SORD for 9-[19:2:1] SDS m= 9 ($\frac{1}{16}$ replicate), v=19, N=176, $\lambda_4= 0.6364$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.16 | 1.0000 | 0.3517 | 0.7398 |
| 1.20 | 1.18 | 0.8333 | 1.1693 | 0.4609 |
| 1.40 | 1.21 | 0.7143 | 2.8508 | 0.2597 |
| 1.60 | 1.26 | 0.6250 | 5.1152 | 0.1635 |
| 1.80 | 1.33 | 0.5556 | 7.4999 | 0.1176 |
| 2.00 | 1.43 | 0.5000 | 9.0751 | 0.099254 |
| 2.20 | 1.56 | 0.4545 | 9.6567 | 0.093837 |
| 2.40 | 1.74 | 0.4167 | 8.4809 | 0.1055 |
| 2.60 | 1.96 | 0.3846 | 6.5193 | 0.1329 |
| 2.80 | 2.24 | 0.3571 | 3.7776 | 0.2093 |
| 3.00 | 2.59 | 0.3333 | 1.1610 | 0.4627 |
| 3.10 | 2.79 | 0.3226 | 0.3120 | 0.7622 |
| 3.19 | 3.00 | 0.3135 | 0.0000 | 1.0000 |
| 3.20 | 3.02 | 0.3125 | 0.0028662 | 0.9971 |
| 3.40 | 3.53 | 0.2941 | 2.0849 | 0.3242 |
| 3.60 | 4.14 | 0.2778 | 9.8820 | 0.091894 |
| 3.80 | 4.87 | 0.2632 | 26.9603 | 0.035765 |
| 4.00 | 5.71 | 0.2500 | 57.6936 | 0.017037 |
| 4.20 | 6.70 | 0.2425 | 93.6938 | 0.001056 |
| 4.40 | 7.84 | 0.2425 | 111.3340 | 0.0089019 |

Rotatable b = 3.19

Table 1. Measure of rotatability for five level SORD for 10-[21:2:1] SDS m= 10 ($\frac{1}{3^2}$ replicate), v=21, N=195, $\lambda_4= 0.6564$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.14 | 1.0000 | 0.3810 | 0.7241 |
| 1.20 | 1.16 | 0.8333 | 1.2279 | 0.4488 |
| 1.40 | 1.19 | 0.7143 | 2.8905 | 0.2570 |
| 1.60 | 1.23 | 0.6250 | 5.4906 | 0.1541 |
| 1.80 | 1.29 | 0.5556 | 8.2655 | 0.1079 |
| 2.00 | 1.38 | 0.5000 | 10.0432 | 0.090553 |
| 2.20 | 1.49 | 0.4545 | 11.2580 | 0.081579 |
| 2.40 | 1.64 | 0.4167 | 10.7226 | 0.08533 |
| 2.60 | 1.84 | 0.3846 | 8.5991 | 0.1042 |
| 2.80 | 2.09 | 0.3571 | 5.6896 | 0.1495 |
| 3.00 | 2.39 | 0.3333 | 2.7297 | 0.2681 |
| 3.20 | 2.76 | 0.3125 | 0.4413 | 0.6938 |
| 3.30 | 2.98 | 0.3030 | 0.030998 | 0.9967 |
| 3.31 | 3.00 | 0.3021 | 0.0000 | 1.0000 |
| 3.40 | 3.21 | 0.2941 | 0.3482 | 0.7417 |
| 3.60 | 3.75 | 0.2778 | 4.5266 | 0.1809 |
| 3.80 | 4.38 | 0.2632 | 15.6246 | 0.060152 |
| 4.00 | 5.13 | 0.2500 | 37.6280 | 0.025888 |
| 4.20 | 5.99 | 0.2357 | 81.3648 | 0.012141 |
| 4.40 | 6.98 | 0.2357 | 100.3828 | 0.0098636 |

Rotatable b = 3.31

Table 10. Measure of rotatability for five level SORD for 11-[23:2:1] SDS m= 11 ($\frac{1}{64}$ replicate), v=23, N=214, $\lambda_4= 0.6729$

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 1.00 | 1.13 | 1.0000 | 0.3684 | 0.7308 |
| 1.20 | 1.14 | 0.8333 | 1.3515 | 0.4253 |
| 1.40 | 1.16 | 0.7143 | 3.4739 | 0.2235 |
| 1.60 | 1.20 | 0.6250 | 6.1932 | 0.1390 |
| 1.80 | 1.26 | 0.5556 | 8.7806 | 0.1022 |
| 2.00 | 1.33 | 0.5000 | 11.6712 | 0.07892 |
| 2.20 | 1.44 | 0.4545 | 12.2897 | 0.075247 |
| 2.40 | 1.57 | 0.4167 | 12.3255 | 0.075045 |
| 2.60 | 1.75 | 0.3846 | 10.3298 | 0.088264 |
| 2.80 | 1.96 | 0.3571 | 7.9009 | 0.1123 |
| 3.00 | 2.24 | 0.3333 | 4.3911 | 0.1855 |
| 3.20 | 2.57 | 0.3125 | 1.4683 | 0.4051 |
| 3.40 | 2.97 | 0.2941 | 0.007376 | 0.9927 |
| 3.41 | 3.00 | 0.2933 | 0.0000 | 1.0000 |
| 3.60 | 3.44 | 0.2778 | 1.6321 | 0.3799 |

| b | c | g | $R_m[D]$ | $P_m[D]$ |
|----------|----------|----------|----------------------------|----------------------------|
| 3.80 | 4.01 | 0.2632 | 8.7035 | 0.1031 |
| 4.00 | 4.67 | 0.2500 | 24.1574 | 0.039750 |
| 4.20 | 5.43 | 0.2381 | 51.8560 | 0.018919 |
| 4.40 | 6.32 | 0.2294 | 90.4012 | 0.010941 |

Rotatable b = 3.41

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