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Manufacturing Lot Size Optimization Under Demand Uncertainty: A Stochastic Goal Programming Approach

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Abstract: *Optimization has become a standard phenomenon in the majority of organizations and establishments. Many Manufacturing companies operate under uncertainties which affect the system performance. Product demand is one of the common kinds of uncertainty that characterizes production environments. One of the challenges faced by manufacturing companies that use cost analyses is product demand uncertainty that often affects the manufacturing system performance and decision making. Manufacturing Lot size problems are normally related to proficient production planning of a given product. If a manufacturing firm wants to compete within the market, it must make the right decisions regarding lot-sizing problems and this can be a critical decision for any manufacturer. In this paper, an optimization model for the manufacturing lot size was developed using Markov chains in conjunction with stochastic goal programming. The goal constraints, deviation variables, priorities and objective function were defined to determine the over-achievement or underachievement of the manufacturing lot size for aggregate production planning, the different states of demand for the product being represented by states of a Markov chain. The model was solved using the linear programming solver in MATLAB™ to determine the quantity of product plan for manufacturing within the first quarter of the year when demand changes from one state to another.*

Keywords: Optimization, manufacturing lot size, demand uncertainty, production planning, goals

1. Introduction

Uncertainties present an unavoidable concern associated with a continuous operation of the manufacturing system, a state of insufficient information, and this can be seen in three forms: inexactness, unreliability, and border with ignorance [1].

One of the challenges faced by manufacturing companies that use cost analyses is product demand uncertainty, as it may influence the manufacturing system performance hence the final decision on utilizing a manufacturing system at the initial stages [2]. When assessing the risk related to a decision, understanding these uncertainties and their impacts, which can make it difficult to predict performance, are of major concern [3].

Production planning is the pillar of any manufacturing operation, with the main aim of determining the amount of products to be manufactured considering the level of inventory to be shifted from one period to another with the objective to minimize both the total costs of production and the inventory, meeting the customers' demand [4]. In production planning, making the right decisions about the lot size is very important as it directly affects the system performance and productivity [5] and this is key for any manufacturing firm that wants to compete in the Market.

Lot sizing problems have got a direct effect on the system performance and productivity. Manufacturing Lot sizing can be defined as determining the quantity of a given product that needs to be manufactured in a specified period of time. Manufacturing Lot sizing problems are normally associated with proficient production planning of a given product. Each production plan has got the main problem of determining the manufacturing lot size for each product. In order to have efficient production planning lot allocation issues must be solved based on the demand that needs to be achieved and the availability of inventory stock minimizing production costs by determining the optimal production quantity[6].

The smaller the manufacturing lot size, the less the holding cost but raises the ordering cost whereas the larger the manufacturing lot size, the more the holding cost but reducing the ordering cost. Based on the concepts of lean production, it is preferable to have a small lot size as it prevents the accumulation of inventory which comes with management and holding costs. The lot size recommended by a mathematical manufacturing lot size model would be the best as it accounts for the tradeoff between the costs involved [5].

Optimization is the process of finding (activity of choosing [7]) the best possible solution to a given problem by examining several alternatives (assessed after a predefined criterion) [8] and can be done by adjusting the inputs to or characteristics of a device, mathematical process, or experiment to find the minimum or maximum output [9].

The optimization problem contains three basic parameters needed to be considered, that is, the objective function, a set of variables and a set of constraints [10].

The objective of the optimization model depends on certain characteristics of the system, called variables or unknowns with the goal of finding the values of these variables that optimize the objective, although these variables are often restricted, or constrained in one way or the other. Brahimi et al. grouped optimization problems into four categories: process planning, layout design, reconfigurability and planning and scheduling.

Manufacturing lot size is in the category of planning and scheduling. Manufacturing companies must have the ability to adjust scalable production capacities and to respond rapidly to market demands making planning and scheduling become complex in such a dynamic environment [11].

Stochastic analysis and goal programming are introduced into the framework to handle uncertainties in real-world manufacturing systems.

Stochastic Goal Programming is a multi-criteria decision support model that gives "satisficing" solutions to a linear system under an uncertainty case from the normally expected utility viewpoint [12], [13]. Most real-world optimization problems consist of various inexact information estimates and goals, conflicting criteria. In such situations, the stochastic goal programming method suggests an analytical structure aid in modelling and solving such problems. Stochastic goal programming can deal with the inherent uncertainty and has been applied in different fields including Portfolio selection, project selection, resource allocation, Healthcare management, transportation, marketing [14], cash management [15], wealth management [16], economic development, energy consumption, workforce allocation, and greenhouse gas emissions[17], forest planning [18]. Not many applications are seen in production planning in manufacturing systems hence the need for manufacturing lot size optimization under demand uncertainty. This can be considered as a guideline for production planners and practitioners used to solve specific decision-making problems (optimal manufacturing lot size). Manufacturing companies will minimize on overproduction when demand is actually low or under-producing when demand is actually high.

Due to the fluctuating and uncertainties in demand, manufacturing companies over and over again face the challenge of establishing optimal manufacturing lot sizes in production planning systems. Manufacturing companies are continuously looking for efficiency to overcome the challenges associated with the market dynamics. One of the common types of uncertainty that characterizes production environments is uncertainty in product demand. It is therefore important that these uncertain parameters be considered in the production planning process when developing a robust production plan because when neglected, production efficiency and system performance will be affected [19].

Manufacturing industries establish their production plans based on external demands with the core aim of determining the quantity (lot size) to be produced given each period while satisfying the demands and minimizing total costs [20]. In production planning, making the right decisions about the lot size is very important as it directly affects the system performance and productivity [5] and this is key for any manufacturing firm that wants to compete on market.

As this is complex as well as important, it has been highly studied although, there is still a gap about showing the contributions to clarify the suitability of those methods used concerning each kind of underlying manufacturing environment (regarding variations in demand and peaks of seasonality) [21].

Therefore the present study aimed at developing an optimization model for the manufacturing lot size under demand uncertainty, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired for aggregate production planning.

2. Mathematical model formulation

A manufacturing company producing products with fluctuations and uncertainties in demand was considered. The demand for these products during each time period over a finite fixed planning horizon was described as either favorable or unfavorable.

The Markov chain approach ([22], [23], [24], [25], [26]) in conjunction with stochastic goal programming ([13], [27], [14], [28], [18], [15]) was adopted and the states of a Markov chain represent possible states of demand for the finished products with the notations shown in Table 1.

Table 1: Notations used in the Markov models

i, j	Set of states of demand	M	Manufacturing lot-size
F	Favorable demand	$X_{ij}\ (p, q)$	Quantity of product p to be manufactured in quarter q
U	Unfavorable demand	N	Customer matrix
Q	Demand transition matrix	C_p	Unit production cost
p	Product	C_h	Unit holding cost
q	Quarter of the year	C_s	Unit shortage cost
FF, FU, UF, UU	State transitions	D	Demand matrix
Z	Value of the objective function	V	Inventory matrix
P_k	Preemptive priority of the kth goal	C	Production-Inventory cost matrix
d_k^+	Over achievement of the kth goal	B	Beginning Inventory
d_k^-	Under achievement of the kth goal	E	Ending Inventory

Average on-hand inventory, $V = (B + E)/2$ (1)

Consider the customer matrix:

$$N(p,q) = \begin{bmatrix} N_{FF}(p,q) & N_{FU}(p,q) \\ N_{UF}(p,q) & N_{UU}(p,q) \end{bmatrix} \quad (2)$$

2.1 Demand transition probability

As demand changes from state *i* to state *j* for $i, j \in \{F, U\}$, the associated demand transition probabilities are calculated as:

$$Q_{ij}(p,q) = \frac{N_{ij}(p,q)}{N_{if}(p,q) + N_{iu}(p,q)} \quad (3)$$

This yields the demand transition matrix:

$$Q(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & Q_{FF}(p,q) & Q_{FU}(p,q) \\ \boldsymbol{U} & Q_{UF}(p,q) & Q_{UU}(p,q) \end{matrix} \quad (4)$$

Then the demand matrix, the inventory matrix and the production-inventory cost matrix.

Demand matrix;

$$D(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & D_{FF}(A,1) & D_{FU}(A,1) \\ \boldsymbol{U} & D_{UF}(A,1) & D_{UU}(A,1) \end{matrix} \quad (5)$$

Inventory matrix;

$$V(A,1) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & V_{FF}(A,1) & V_{FU}(A,1) \\ \boldsymbol{U} & V_{UF}(A,1) & V_{UU}(A,1) \end{matrix} \quad (6)$$

Production-inventory cost matrix;

When demand outweighs the amount produced then,

$$C(p,q) = \begin{bmatrix} C_p \\ + \\ C_h \\ + \\ C_s \end{bmatrix} [D(p,q) - V(p,q)] \quad (7)$$

Similarly, when the demand is less than the amount produced then,

$$C(p,q) = C_h[V(p,q) - D(p,q)] \quad (8)$$

Hence, as demand changes from state *i* to state *j* ($i, j \in \{F, U\}$)

$$C(p,q) = \begin{matrix} & \boldsymbol{F} & \boldsymbol{U} \\ \boldsymbol{F} & C_{FF}(p,q) & C_{FU}(p,q) \\ \boldsymbol{U} & C_{UF}(p,q) & C_{UU}(p,q) \end{matrix} \quad (9)$$

where $C(p,q) =$ production-inventory cost matrix.

2.2 Expected demand, inventory, production-inventory costs and manufacturing lot-size

Expected demand

Favorable Demand $E[D_F(p,q)] = Q_{FF}(p,q)D_{FF}(p,q) + Q_{FU}(p,q)D_{FU}(p,q)$ (10)

Unfavorable Demand $E[D_U(p,q)] = Q_{UF}(p,q)D_{UF}(p,q) + Q_{UU}(p,q)D_{UU}(p,q)$ (11)

Expected inventory

Favorable Demand $E[V_F(p,q)] = Q_{FF}(p,q)V_{FF}(p,q) + Q_{FU}(p,q)V_{FU}(p,q)$ (12)

Unfavorable Demand $E[V_U(p,q)] = Q_{FU}(p,q)V_{UF}(p,q) + Q_{UU}(p,q)V_{UU}(p,q)$ (13)

Expected production-inventory costs

Favorable Demand $E[C_F(p,q)] = Q_{FF}(p,q)C_{FF}(p,q) + Q_{FU}(p,q)C_{FU}(p,q)$ (14)

Unfavorable Demand $E[C_U(p,q)] = Q_{UF}(p,q)C_{UF}(p,q) + Q_{UU}(p,q)C_{UU}(p,q)$ (15)

Expected manufacturing lot-size

Favorable demand

$$E[M_F(p,q)] = \begin{cases} E[D_F(p,q)] - E[V_F(p,q)] & if\ E[D_F(p,q)] > E[V_F(p,q)] \\ 0 & otherwise \end{cases} \quad (16)$$

Unfavorable demand

$$E[M_U(p,q)] = \begin{cases} E[D_U(p,q)] - E[V_U(p,q)] & if\ E[D_U(p,q)] > E[V_U(p,q)] \\ 0 & otherwise \end{cases} \quad (17)$$

2.3 Stochastic goal programming formulation

The stochastic goal programming model was formulated by setting priorities, defining the objective function and formulating the goal constraints as follows:

Set priorities

- P_1: Produce a batch of $E[M_F(p,q)]$ units when demand is favorable
- P_2: Produce a batch of $E[M_U(p,q)]$ units when demand is unfavorable
- P_3: Total production-inventory cost must not exceed $E[C_F(p,q)]$ when demand is favorable
- P_4: Total production-inventory cost must not exceed $E[C_U(p,q)]$ when demand is unfavorable

Objective function

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \quad (18)$$

Goal constraints

P1: Manufacturing lot-size $E[M_F(p,q)]$ - favorable demand

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \quad (18.1)$$

P2: Manufacturing lot-size $E[M_U(p,q)]$ - unfavorable demand

$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \quad (18.2)$$

P3: Total production-inventory cost – favorable demand

$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad (18.3)$$

P4: Total production-inventory cost – unfavorable demand

$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad (18.4)$$

2.4 Stochastic goal programming model for manufacturing lot-size

$$Minimise\ Z = \sum_{k=1}^{4} \sum_{p=1}^{3} \sum_{q=1}^{3} P_k(p,q)\,[d_k^+ + d_k^-] \quad (19)$$

Subject to:

$$X_{FF}(p,q) + X_{FU}(p,q) + d_1^- - d_1^+ = E[M_F(p,q)] \quad (19.1)$$
$$X_{UF}(p,q) + X_{UU}(p,q) + d_2^- - d_2^+ = E[M_U(p,q)] \quad (19.2)$$
$$C_{FF}(p,q)X_{FF}(p,q) + C_{FU}(p,q)\,X_{FU}(p,q) - d_3^+ = E[C_F(p,q)] \quad (19.3)$$
$$C_{UF}(p,q)X_{UF}(p,q) + C_{UU}(p,q)\,X_{UU}(p,q) - d_4^+ = E[C_U(p,q)] \quad (19.4)$$
$$X_{FF}(p,q), X_{FU}(p,q), X_{UF}(p,q), X_{UU}(p,q), d_1^-, d_1^+, d_2^-, d_2^+, d_3^+, d_4^+ \geq 0 \quad (19.5$$

3. Case study

In this section, a real case application from Movit Products Uganda limited was used to demonstrate the applicability of the proposed mathematical models. The manufacturing industry manufactures, distributes and sells skincare, hair & nail care products. The numerical illustration contains real data for the first quarter of the year, which was collected and then reduced to usable dimensions as shown in Table 2. Data classification by state of demand was made, analyzed and used in the proposed mathematical model.

Considering a product A, for a given week, demand is favorable (state F) if $N_{ij} > 12$ otherwise demand is unfavorable (state U) if $N_{ij} \leq 12$ as shown in Table 2.

Table 2: Data classification by state of demand for product A

Month	**Week**	**Customers (N)**	**Demand (D) (x10³)**	**On hand inventory (V) (x10³)**	**State of demand (*i*)**
1	1	9	3937	6076	U
	2	12	4668	4687	U
	3	8	2485	6306	U
	4	17	7955	10160	F
2	1	1	110	4525	U
	2	15	3832	5681	F
	3	7	2870	4363	U
	4	20	3824	6028	F

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3	1	4	758	2018	U
	2	16	6125	4149	F
	3	14	2625	4163	F
	4	17	3685	6279	F

Table 3a, 3b and 3c shows the over stocking or under stocking of product A with the corresponding holding or shortage costs in the first quarter of the year.

Table 3a: Overstocking and understocking with holding and shortage costs for 1st month

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	3937	6076	2139	231.6537
2	4668	4687	19	2.0577
3	2485	6306	3821	413.8143
4	7955	10160	2205	238.8015

Table 3b: Overstocking and understocking with holding and shortage costs for 2nd month

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	110	4525	4415	478.1445
2	3832	5681	1849	200.2467
3	2870	4363	1493	161.6919
4	3824	6028	2204	238.6932

Table 3c: Overstocking and understocking with holding and shortage costs for 3rd month

Week	Demand (D) ($x10^3$)	On hand inventory (V) ($x10^3$)	over/under stocking	Holding/shortage costs
1	758	2018	1260	136.458
2	6125	4149	-1976	1569.734
3	2625	4163	1538	166.5654
4	3685	6279	2594	280.9302

Figure 1: Over stocking and under stocking of product A

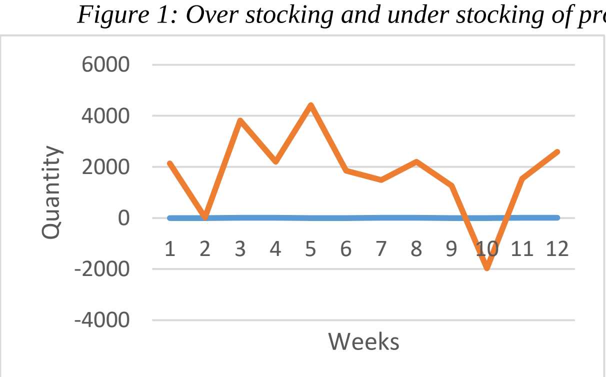
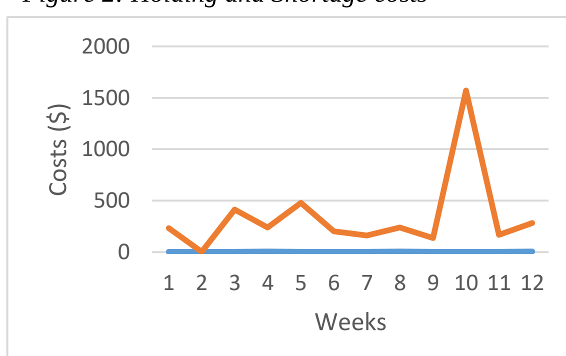


Figure 2: Holding and Shortage costs



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3.1 State transitions and on-hand inventory

For a particular state transition, given the beginning and ending inventory, the average on-hand inventory was calculated as presented in Table 4.

Table 4: Average on-hand inventory for product A

State transitions (i, j)	Beginning inventory (B)	Ending inventory (E)	Average on-hand inventory $V = (B + E)/2$
FF	4163	6279	5221
FU	4525	2018	3271.5
UF	10160	4149	7154.5
UU	4687	6306	5496.5

From Equation (1) section 2, the average on-hand inventory was calculated giving;

$$V_{FF}(A,1) = 5221 \qquad V_{FU}(A,1) = 3271.5 \qquad V_{UF}(A,1) = 7154.5 \qquad V_{UU}(A,1) = 5496.5$$

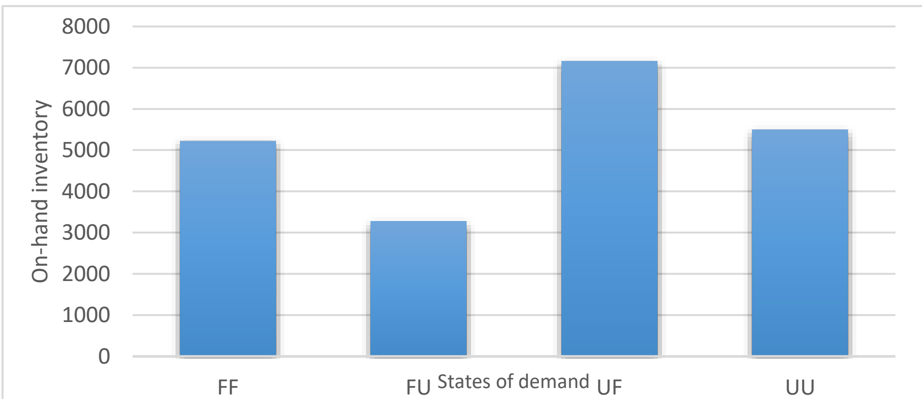

Figure 3: Average on-hand inventory and state transitions

3.2 Demand transition probabilities

Data classification by state-transition was done as illustrated in Table 5 and then used to calculate the demand transition probabilities for the product

Table 5: *Data classification by state-transition for product A*

Month	State transition (i, j)	Number of customers $N_{ij}(A, 1)$	Demand $D_{ij}(A, 1)$
1	FF	0	0
	FU	0	0
	UF	25	10440
	UU	41	15758
2	FF	0	0
	FU	22	6702
	UF	43	10636
	UU	0	0
3	FF	61	15060
	FU	0	0
	UF	20	6883
	UU	0	0

From Table 5, the Totals for customers and demand as it changes from one state to another are;

Customers: $N_{FF}(A,1) = 0 + 0 + 61 = 61$ $\qquad N_{FU}(A,1) = 0 + 22 + 0 = 22$

$N_{UF}(A,1) = 25 + 43 + 20 = 88$ $\qquad N_{UU}(A,1) = 41 + 0 + 0 = 41$

Demand: $D_{FF}(A,1) = 0 + 0 + 15060 = 15060$ $\quad D_{FU}(A,1) = 0 + 6702 + 0 = 6702$

$D_{UF}(A,1) = 10440 + 10636 + 6883 = 27959$

$D_{UU}(A,1) = 15758 + 0 + 0 = 15758$

From Equation (3) in section 2, the demand transition probabilities are;

$$Q_{FF}(A,1) = \frac{N_{FF}(A,1)}{N_{FF}(A,1)+N_{FU}(A,1)} = \frac{61}{61+22} = 0.7349$$
$$Q_{FU}(A,1) = \frac{N_{FU}(A,1)}{N_{FF}(A,1)+N_{FU}(A,1)} = \frac{22}{61+22} = 0.2651$$
$$Q_{UF}(A,1) = \frac{N_{UF}(A,1)}{N_{UF}(A,1)+N_{UU}(A,1)} = \frac{88}{88+41} = 0.6822$$
$$Q_{UU}(A,1) = \frac{N_{UU}(A,1)}{N_{UF}(A,1)+N_{UU}(A,1)} = \frac{41}{88+41} = 0.3178$$

Hence the demand transition matrix as from equation (4),

$$Q(A,1) = \begin{array}{c} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{array} \begin{array}{c} \begin{array}{cc} \boldsymbol{F} & \boldsymbol{U} \end{array} \\ \begin{pmatrix} 0.7349 & 0.2651 \\ 0.6822 & 0.3178 \end{pmatrix} \end{array}$$

3.3 Demand matrix, inventory matrix and production-inventory cost matrix

The demand matrix, the inventory matrix and the production-inventory cost matrix were developed as follows.

From Equation (5), the demand matrix becomes;

$$D(A,1) = \begin{array}{c} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{array} \begin{array}{c} \begin{array}{cc} \boldsymbol{F} & \boldsymbol{U} \end{array} \\ \begin{pmatrix} 15060 & 6702 \\ 27959 & 15758 \end{pmatrix} \end{array}$$

From Equation (6), the Inventory matrix becomes;

$$V(A,1) = \begin{array}{c} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{array} \begin{array}{c} \begin{array}{cc} \boldsymbol{F} & \boldsymbol{U} \end{array} \\ \begin{pmatrix} 5221 & 3271.5 \\ 7154.5 & 5496.5 \end{pmatrix} \end{array}$$

Production-inventory cost matrix

The production-inventory cost matrix is then computed for the product From Equations (7), (8) and (9).

Unit production cost, $C_p(A) = \$\ 7.2222$

Unit holding cost, $C_h(A) = \$\ 0.1083$

Unit shortage cost, $C_s(A) = \$\ 0.7944$

$C_{FF}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{FF}(A,1) - V_{FF}(A,1)\right)$

$C_{FF}(A,1) = (7.2222 + 0.1083 + 0.7944)(15060 - 5221) = 79940.9$

$C_{FU}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{FU}(A,1) - V_{FU}(A,1)\right)$

$C_{FU}(A,1) = (7.2222 + 0.1083 + 0.7944)(6702 - 3271.5) = 27872.5$

$C_{UF}(A,1) = \left(C_p(A) + C_h(A) + C_s(A)\right)\left(D_{UF}(A,1) - V_{UF}(A,1)\right)$

$C_{UF}(A,1) = (7.2222 + 0.1083 + 0.7944)(27959 - 7154.5) = 169034.5$

$C_{UU}(A,1) = \left(C_h(A)\right)\left(D_{UU}(A,1) - V_{UU}(A,1)\right)$

$C_{UU}(A,1) = (0.1083)(15758 - 5496.5) = 1111.3$

Hence,

$$C(A,1) = \begin{array}{c} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{array} \begin{array}{c} \begin{array}{cc} \boldsymbol{F} & \boldsymbol{U} \end{array} \\ \begin{pmatrix} C_{FF}(A,1) & C_{FU}(A,1) \\ C_{UF}(A,1) & C_{UU}(A,1) \end{pmatrix} \end{array}$$

$$C(A,1) = \begin{array}{c} \\ \boldsymbol{F} \\ \boldsymbol{U} \end{array} \begin{array}{c} \begin{array}{cc} \boldsymbol{F} & \boldsymbol{U} \end{array} \\ \begin{pmatrix} 79940.9 & 27872.5 \\ 169034.5 & 1111.3 \end{pmatrix} \end{array}$$

3.4 Expected demand, inventory, production-inventory costs and manufacturing lot-size

Expected demand

After generating the demand transition matrix and formulating the production-inventory cost matrix, the expected demand, expected inventory and expected production-inventory costs are computed for the product considering both favorable and unfavorable demand as shown below;

Favorable demand (F) was computed from equation (10)

$E[D_F(A,1)] = Q_{FF}(A,1) * D_{FF}(A,1) + Q_{FU}(A,1) * D_{FU}(A,1)$

$E[D_F(A,1)] = (0.7349 * 15{,}060) + (0.2651 * 6{,}702)$

$E[D_F(A,1)] = 12{,}844.3\ units$

Unfavorable demand (U) was computed from equation (11)

$$E[D_U(A,1)] = Q_{UF}(A,1) * D_{UF}(A,1) + Q_{UU}(A,1) * D_{UU}(A,1)$$
$$E[D_U(A,1)] = (0.6822 * 27{,}959) + (0.3178 * 15{,}758)$$
$$E[D_U(A,1)] = 24{,}081.5\ units$$

Expected Inventory

Computation of the expected inventory considering both favorable and unfavorable demand for the product was computed From equation (12) as follows:

Favorable demand (F)

$$E[V_F(A,1)] = Q_{FF}(A,1) * V_{FF}(A,1) + Q_{FU}(A,1) * V_{FU}(A,1)$$
$$E[V_F(A,1)] = (0.7349 * 5221) + (0.2651 * 3271.5)$$
$$E[V_F(A,1)] = 4{,}704.2\ units$$

Unfavorable demand (U) was computed from equation (13) as follows

$$E[V_U(A,1)] = Q_{UF}(A,1) * V_{UF}(A,1) + Q_{UU}(A,1) * V_{UU}(A,1)$$
$$E[V_U(A,1)] = (0.6822 * 7154.5) + (0.3178 * 5496.5)$$
$$E[V_U(A,1)] = 6{,}627.6\ units$$

Expected production-Inventory costs

The expected production-Inventory costs are then computed for the product considering both favorable and unfavorable demand results were computed from equations (14) and (15) as follows;

Favorable demand (F)

$$E[C_F(A,1)] = Q_{FF}(A,1) * C_{FF}(A,1) + Q_{FU}(A,1) * C_{FU}(A,1)$$
$$E[C_F(A,1)] = (0.7349 * 79940.9) + (0.2651 * 27872.5)$$
$$E[C_F(A,1)] = \$\ 66{,}137.6$$

Unfavorable demand (U)

$$E[C_U(A,1)] = Q_{UF}(A,1) * C_{UF}(A,1) + Q_{UU}(A,1) * C_{UU}(A,1)$$
$$E[C_U(A,1)] = (0.6822 * 169034.5) + (0.3178 * 1111.3)$$
$$E[C_U(A,1)] = \$\ 115{,}668.5$$

Expected manufacturing lot size

Computation of the expected manufacturing lot size considering both favorable and unfavorable demand for the product yields was computed from equations (16) and (17) as follows:

Favorable demand (F)

$$E[M_F(A,1)] = \begin{pmatrix} E[D_F(A,1)] - E[V_F(A,1)]\ if\ E[D_F(A,1)] > E[V_F(A,1)] \\ 0 \qquad\qquad otherwise \end{pmatrix}$$
$$E[M_F(A,1)] = E[D_F(A,1)] - E[V_F(A,1)]$$
$$E[M_F(A,1)] = 12{,}844.3 - 4{,}704.2 = 8{,}140.1\ units$$

Unfavorable demand (U)

$$E[M_U(A,1)] = \begin{pmatrix} E[D_U(A,1)] - E[V_U(A,1)]\ if\ E[D_U(A,1)] > E[V_U(A,1)] \\ 0 \qquad\qquad otherwise \end{pmatrix}$$
$$E[M_U(A,1)] = E[D_U(A,1)] - E[V_U(A,1)]$$
$$E[M_U(A,1)] = 24{,}081.5 - 6{,}627.6 = 17{,}453.9\ units$$

3.5 Stochastic goal programming model

The stochastic goal programming model for the product was formulated by setting priorities, defining the objective function and formulating the goal constraints as follows:

Priorities set

P_1: $Produce\ a\ batch\ of$ 8,140.1 $units\ when\ demand\ is\ initially\ favorable$

P_2: $Produce\ a\ batch\ of$ 17,453.9 $units\ when\ demand\ is\ initially\ unfavorable$

P_3: $Total\ production\_inventory\ costs\ must\ not\ exceed$ \$ 66,137.6 $when\ demand\ is\ favorable$ P_4: $Total\ production\_inventory\ costs\ must\ not\ exceed$ \$ 115,668.5 $when\ demand\ is\ unfavorable$

Objective function

$$Minimize\ Z = \sum_{k=1}^{4}[P_K(A,1)d_k^+ + P_K(A,1)d_k^-]$$

Goal constraints

Manufacturing lot size

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1\ (Favorable\ demand)$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9\ (Unfavorable\ demand)$$

Total production-Inventory costs

$$79940.9X_{FF}(A,1) + 27872.5X_{FU}(A,1) - d_3^+ = 66{,}137.6\ (Favorable\ demand)$$

$$169034.5X_{UF}(A,1) + 1111.3X_{UU}(A,1) - d_4^+ = 115{,}668.5\ (Unfavorable\ demand)$$

Non negativity

$$X_{FF}(A,1), X_{FU}(A,1), X_{UF}(A,1), X_{UU}(A,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

3.6 Stochastic goal programming model for manufacturing lot size

The stochastic goal programming model for manufacturing lot size was then developed for the product as below. This determines the quantity of the product to manufacture in the first quarter of the year when demand changes from state i to state j for $i, j \in \{F, U\}$, establishing the over-achievement or under achievement of the manufacturing lot size priorities desired.

$$Minimize\ Z = \sum_{k=1}^{4}[P_K(A,1)d_k^+ + P_K(A,1)d_k^-]$$

Subject to:

$$X_{FF}(A,1) + X_{FU}(A,1) + d_1^- = 8{,}140.1$$

$$X_{UF}(A,1) + X_{UU}(A,1) + d_2^- = 17{,}453.9$$

$$79940.9X_{FF}(A,1) + 27872.5X_{FU}(A,1) - d_3^+ = 66{,}137.6$$

$$169034.5X_{UF}(A,1) + 1111.3X_{UU}(A,1) - d_4^+ = 115{,}668.5$$

$$X_{FF}(A,1), X_{FU}(A,1), X_{UF}(A,1), X_{UU}(A,1), d_1^-, d_2^-, d_3^+, d_4^+ \geq 0$$

Where:

d_1^-, d_2^- = slack variables

d_3^+, d_4^+ = surplus variables

$X_{FF}(A,1)$ – manufacturing lot size of product A when initially favorable demand remains favorable

$X_{FU}(A,1)$ - manufacturing lot size of product A when initially favorable demand becomes unfavorable

$X_{UF}(A,1)$ - manufacturing lot size of product A when initially unfavorable demand becomes favorable

$X_{UU}(A,1)$ - manufacturing lot size of product A when initially unfavorable demand remains unfavorable

4. Results and Discussions

In this study, the stochastic goal programming model for the product was solved using the using the linear programming (linprog) solver in MATLAB™ ([29], [30])., an optimal solution was obtained with the values as shown in Table 6:

Table 6: Optimal solution from MATLAB

Variables	X_{FF} (A,1)	X_{FU} (A,1)	X_{UF} (A,1)	X_{UU} (A,1)	d_1^-	d_2^-	d_3^+	d_4^+
values	0	2.3729	0	104.0840	8137.7	17350	0	0

The results highlight the optimal values of the manufacturing lot size of product A in the first quarter of the year as demand changes from one state to another. The results were analyzed and discussed based on the priorities set and the optimal values achieved as seen from Table 6.

The improvement of the solution from the case is establishing the over-achievement and under achievement of the manufacturing lot size priorities desired during production planning. An expansion in this case is incorporating in Markov chains which considers changes form one state to another.

As seen from Table 6, for cases where initially demand is favorable and unfavorable, more products shouldn't be manufactured but use what is already in stock as it is enough to meet the demand since the model predicts 0 manufacturing lot size of product A in the first quarter of the year.

The model also predicts the manufacturing lot size of product A of 2.3729 units and 104.0840 units when initially favorable demand becomes unfavorable and unfavorable demand remains unfavorable respectively. Meaning these number of products should be produced to meet demand.

Table 7: Expected goal values and actual stochastic solution with over and under achievement

Goals/ priorities	Expected value from Goal	Value of the stochastic solution	Deviation	Over-achievement	Under-achievement
1	8140.1	8140.07	0.03		8137.7
2	17453.9	17454.08	0.18		17350
3	66137.6	66138.66	1.06	0	
4	115668.5	115668.55	0.05	0	

With the set priorities and expected values from each goal, the results from Table 7 show the importance of utilizing the available sources of information when generating a plan.

As observed from Table 7, Priority 1 and 2 can be fully achieved however, an underachievement of 8137.7 units 17350 units respectively is realized in the first quarter when demand is initially favorable (state F) and unfavorable (state U) respectively.

Priority 3 is partially achieved as the actual stochastic solution is slightly higher than the expected goal value targeted production-inventory costs in the first quarter when demand is initially favorable (state F). And priority 4 is fully achieved in the first quarter when demand is initially unfavorable (state U). Both priority 3 and 4 have no over-achievement.

5. Conclusion

A stochastic goal programming model that optimizes the manufacturing lot size under demand uncertainty was presented in this paper. The model determines the quantity of the product (with demand uncertainty) to be produced in the first quarter of the year when demand changes from state i to state j for $i, j \in \{F, U\}$, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired. The decision of whether or not to produce more units is modelled using Markov chains in conjunction with stochastic goal programming. The model was solved with the help of MATLAB software environment and the results indicate the optimal manufacturing lot sizes as demand changes from one state to another, establishing the over-achievement or underachievement of the manufacturing lot size priorities desired.

Further research is sought to extend the proposed model in order to handle multiple products under demand and price uncertainty. In addition, weighted goal programming can be introduced to improve computational efficiency while handling pre-emptive priorities of the product.

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