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Anharmonic perturbation of neutron-proton pairs by the unpaired neutrons in heavy finite nuclei

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The properties of a finite heavy nucleus in which the number of neutrons N is not equal to the number of protons Z, N > Z, have been studied. It is assumed that the core of the nucleus is composed of proton-neutron pairs and the excess neutrons constitute the surface region of the nucleus. The interaction between a neutron and a proton constituting the neutron-proton pair is assumed to be harmonic. The unpaired neutrons in the surface region are assumed to interact with the neutron–proton pairs in the core of the nucleus anharmonically. Many body perturbation theory has been used to calculate the total energy of the nucleus, and thereby, we have calculated the binding energy per nucleon, called the binding fraction, the specific heat and the transition temperature. Speculation, as to how the alpha, beta and gamma radiations are emitted by a heavy nucleus, is also presented.

Keywords: Anharmonic perturbation, Heavy finite nuclei, Unpaired neutrons

1 Introduction

Since pairing lies at the heart of nuclear physics and the quantum-many body problem, it is necessary to understand the role of pairing phenomena in nuclear matter and finite nuclei. The presence of neutron superfluidity[1] in the crust and inner part of a neutron star is now well established. The star is assumed to be made up of: (i) an outer crust of bare nuclei arranged in lattice with a relativistic electron bath; (ii) an inner crust in which a similar Coulomb lattice of neutron-rich nuclei is embedded in Fermi seas of neutrons and relativistic electrons; (iii) a quantum fluid interior in which neutron, proton and electron fluids co-exist and finally (iv) a core region of uncertain constitution and phase.

The outer part of the star is assumed to be of low-density and neutron superfluidity is expected mainly in the attraction singlet $^1S_0$ channel. However, for densities higher than the saturation density ρ_0, the pairing effect is quenched due to the strong repulsive short range component of this interaction, and consequently, the nuclei in the crust dissolve into a quantum liquid of neutrons and protons in beta equilibrium.It is also possible that when the nucleons are in a superfluid state in one or another region of the star, suppression factors of the form $\exp(-\Delta_F/KT)$ appear in the expression for emissivity, where Δ_F is the average measure of the energy gap at the Fermi surface. The pairing has a major effect on the star's thermal evolution through the suppression of neutrino emission process and the modification of specific heats.

The pairing studies of infinite nuclear matter have already been described. Now, the pairing correlations in heavy finite nuclei by using the extensively available spectroscopic data, have been studied. It is well established that the nuclear force between two neutrons, two protons and a neutron and a proton, is the same. This yields the idea of charge symmetry of nuclear forces and the electric force between the protons is relatively weak. Simplified models of the nucleon-nucleon interaction, such as the seniority model[2], predict a pair[3] condensate in these systems.

In nuclei with N=Z, neutrons and protons occupy the same shell-model orbitals. Consequently, the large spatial overlaps between neutron and proton single-particle wave functions are expected to enhance neutron-proton (*np*) correlations resulting in *np*-pairing. On the other hand, most of our knowledge about nuclear pairing comes from nuclei with a sizable neutron excess (N>Z) where the isotopic spin T=1 neutron-proton (*nm*) and proton-proton (*pp*) pairing dominates. The objective of our study is to find out as to what role *np*-pairing can play in determining the properties of nuclear systems. The possibility of a transition from the BCS pairing to a Bose-Einstein condensation in asymmetric nuclear matter at low densities can also throw some light on the role of *np*-pairing. An analysis by Lombardo and Schuck[4] of triplet $^3S_1$ pairing in low density symmetric and asymmetric nuclear matter indicates that such a transition is possible. As the system is diluted, the BCS state with large overlapping Cooper pairs evolves smoothly into a Bose-Einstein condensation of tightly bound deutrons or neutron-proton pairs. A neutron excess in this low density system does not affect these deutrons due to large spatial separation of the deutrons and neutrons; or the unpaired neutrons in the neutron excess system may weakly affect the deutrons; or we can say that the unpaired neutrons may perturb the *np*-pairs. With all the experimental and theoretical work available so far, it is still not possible to definitely say whether strong neutron proton pairing exists in finite nuclei. On the other hand, we simply cannot discard the existence of *np*-pairing and the role it plays in nuclear theory.

It is well known that it is via the nuclear shell model that we can analyze pairing correlations in finite nuclei. This is done by appropriately defining model spaces and effective interactions. As an example, consider the chain of tin isotopes from ^{100}Sn to ^{132}Sn then define the valence-space or model-space degrees of freedom. We could however, have chosen^{100}Sn as a closed shell core. In this case, neutron particles from ^{101}Sn to ^{132}Sn define the model space.

There is an indication that spectra of the chain of tin isotopes point to a link between superfluidity in infinite star matter and the spectra of finite nuclei. This link is provided especially by the $^1S_0$ partial wave of the nucleon-nucleon interaction. There could also exist proton and neutron BCS-like pairs. Such pair correlations are quite strong and reflect the well-known coherence in the ground states of even-even nuclei. But the proton BCS-like pairing fields are not constant within an isotopic chain, or the proton pair matrix elements are not constant within the isotope chain and such a behaviour is mainly caused by isoscalar neutron-proton pairing, showing that there are important proton-neutron correlations present in the ground state The shell closure at N=28 is manifested in the neutron BCS-like pairing. On the other hand, the proton and neutron occupation numbers show a much smoother behaviour with increasing *A*. However, in the nuclear shell model , the isotope ^{132}Sn could be chosen to have a closed-shell-core of ^{100}Sn and neutron particles from ^{101}Sn to ^{132}Sn define model space. We shall use this concept to propose a nuclear model for a finite heavy nucleus.

For a heavy nucleus whose mass number is *A* and is composed of Z protons and N neutrons such that N>Z, we shall assume that this nucleus has a core composed of Z neutron-proton pairs and this core is surrounded by the unpaired neutrons whose number is (N−Z).These neutrons stay in the surface region. Now, the core will have 2Z particles, and hence, the radius R_C of the core can be written as:

$$R_C = R_0 \left(2Z\right)^{1/3}$$

where $R_0 \approx 1.3\times 10^{15}$ m. …(1)

The radius *R* of the nucleus is:

$$R = R_0 \left(A\right)^{1/3} \quad \ldots(2)$$

Hence, the thickness of the surface region R_S will be:

$$R_S = R - R_C \quad \ldots(3)$$

In the calculation[5] on proton-neutron interactions and the new atomic masses, it is assumed that the core is not significantly altered and δV_{PN} which is the interaction of the last proton(s) with the last neutron(s) by construction, largely cancels out the interaction of the last nucleon with the core.

In our opinion, the above method of looking at the possible interaction in a nucleus is an oversimplification of the exact problem. Under no circumstances, it can be assumed that the interaction between the nucleons in the surface region can be treated in isolation without disturbing the core. Since the *np*-interaction plays an important role in determining the properties of finite nuclei, especially the binding energy *B*, we have assumed that the core of the nucleus is composed of *np*-pairs such that the neutron and proton interacts with each other harmonically, and the unpaired neutrons in the surface region interact with the *np*-pairs such that the interaction leads to anharmonicity in the *np*-interaction. Thus, to calculate the energy o f the system, we shall write the perturbed Hamiltonian H as:

$$\mathrm{H} = \mathrm{H}_0 + V \quad \ldots(4)$$

where

$$\mathrm{H}_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2 \quad \ldots(5)$$

is the unperturbed Hamiltonian, *V* is the perturbing potential that causes anharmonicity in the harmonic interactions of the *np*-pairs and *k* is the force constant for harmonic interaction. This part of the Hamiltonian (*V*) can be written as:

$$V = \beta x^3 + \gamma x^4 \quad \ldots(6)$$

where β and γ are the constants of perturbation.

The methods of second quantization and many-body perturbation theory will be used to calculate the total energy E_n of the nucleus.
Knowing E_n we can calculate the specific heat C by the equation,

$$C = \frac{\partial E_n}{\partial T} \qquad \ldots(7)$$

The transition temperature T_c will be given as:

$$\left(\frac{\partial C}{\partial T}\right)_{T=T_C} = 0 \qquad \ldots(8)$$

It should be clearly understood that our model will be valid for heavy finite nuclei in which N>Z.

2 Theory-Energy of the System

The perturbed Hamiltonian of the assembly is given in Eq. (4), the unperturbed harmonic oscillator Hamiltonian is given by Eq. (5) and the perturbation is given by Eq. (6). The creation and annihilation operators are given as:

$$a^+ = \frac{1}{\sqrt{2}}\left(\xi - \frac{\partial}{\partial\xi}\right) = \left(\frac{m\omega}{2\eta}\right)^{1/2}\left(x - \frac{ip}{m\omega}\right) \qquad \ldots(9)$$

$$a = \frac{1}{\sqrt{2}}\left(\xi + \frac{\partial}{\partial\xi}\right) = \left(\frac{m\omega}{2\eta}\right)^{1/2}\left(x + \frac{ip}{m\omega}\right) \qquad \ldots(10)$$

where m is the mass of the particle, ω is a measure of the harmonic oscillator frequency and p is the momentum. Using Eqs (9) and (10), we can write the displacement operator x as:

$$x = \left(\frac{\eta}{2m\omega}\right)^{1/2}\left(a + a^+\right) \qquad \ldots(11)$$

The perturbation V can be written using Eq. (11) as:

$$V = \beta\left(\frac{\eta}{2m\omega}\right)^{3/2}\left(a + a^+\right)^3 + \gamma\left(\frac{\eta}{2m\omega}\right)^2\left(a + a^+\right)^4 \qquad \ldots(12)$$

and the total Hamiltonian H becomes:

$$\mathrm{H} = \sum_{i=1}^{A}\varepsilon a_i^+ a + \frac{1}{2}\sum_{i=1}^{Z}\eta\omega_i\left(a_i + a_i^+\right)^2 + \sum_{i=1}^{N-Z}\left[\beta\left(\frac{\eta}{2\mu\omega_i}\right)^{3/2}\left(a_i + a_i^+\right)^3 + \gamma\left(\frac{\eta}{2\mu\omega_i}\right)^2\left(a_i + a_i^+\right)^4\right] \qquad \ldots(13)$$

where ε_i are the single-particle energies; $k=\mu\omega^2$ where

$$\mu = \frac{m_n m_p}{m_n + m_p} \qquad \ldots(14)$$

is the reduced mass of a neutron-proton pair. The first summation is the kinetic energy of the system, the second summation is the potential energy due to the neutron-proton pairs, while the third summation is the perturbation energy due to the interaction of the unpaired neutrons in the surface region with neutron-proton pairs constituting the core of the heavy nucleus. It is this term that causes anharmonicity of the otherwise harmonic interaction of the neutron-proton pair.

The eigenvalues and eigenfunctions of the unperturbed harmonic oscillator Hamiltonian, H_0 are well known.

$$\mathrm{H}_0\left|n\right\rangle = \mathrm{E}_n^0\left|n\right\rangle \qquad \ldots(15)$$

where

$$\left|n\right\rangle = \left(\frac{1}{n!2^n\sqrt{\pi}}\right)^{1/2}\widehat{\mathrm{H}}_n(\xi)e^{-\frac{\xi^2}{2}} \qquad \ldots(16)$$

$$\mathrm{E}_n^0 = \left(n + \frac{1}{2}\right)\eta\omega, \qquad n = 0,1,2,3,\ldots \qquad \ldots(17)$$

where $\widehat{\mathrm{H}}_n$ are Hermite polynomials.
When the system is perturbed, the eigenvalue problem that needs to be solved becomes:

$$\mathrm{H}\left|n\right\rangle = \left(\mathrm{H}_0 + \mathrm{H}'\right)\left|n\right\rangle = \left(\mathrm{E}_n^0 + \mathrm{E}_n^1\right)\left|n\right\rangle \qquad \ldots(18)$$

where

$$\mathrm{H}' = \beta x^3 + \gamma x^4 \qquad \ldots(19)$$

To solve Eq. (18), the following creation and annihilation harmonic oscillator operators are defined as:

$$a = \frac{1}{2^{1/2}}\left(\alpha x + \frac{1}{\alpha}\frac{\partial}{\partial x}\right); \quad a^+ = \frac{1}{2^{1/2}}\left(\alpha x - \frac{1}{\alpha}\frac{\partial}{\partial x}\right) \qquad \ldots(20)$$

where $\alpha = \left(\frac{\mu\omega}{\eta}\right)^{1/2}$

These operators have the following properties:

$$a\left|n+1\right\rangle = (n+1)^{1/2}\left|n\right\rangle; \; a^+\left|n\right\rangle = (n+1)^{1/2}\left|n+1\right\rangle \qquad \ldots(21)$$

Using these operators, the displacement operator x becomes:

$$x = \frac{1}{2^{1/2}\alpha}\left(a + a^{+}\right) \quad \ldots(22)$$

and the perturbation becomes:

$$H' = \frac{\beta}{8^{1/2}\alpha^3}\left(a + a^{+}\right)^3 + \frac{\gamma}{4\alpha^4}\left(a + a^{+}\right)^4 \quad \ldots(23)$$

Now , the total energy to second order can be written as:

$$E_n = \left(n + \frac{1}{2}\right)\eta\omega + \langle n|H'|n\rangle + \langle n|H'\frac{1}{E_0 - H_0}H'|n\rangle \quad \ldots(24)$$

which after lengthy calculations give:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega + \frac{3\gamma\eta^2}{2\mu^2\omega^2}\left(n^2 + n + \frac{1}{2}\right) - \frac{15\beta^2\eta^2}{2\mu^3\omega^4}\left(n^2 + n + \frac{11}{30}\right) \quad \ldots(25)$$

where $n = 0, 1, 2, 3, \ldots$

Since the unperturbed energy E_n^0 is due to the neutron-proton pairs and the number of such pairs is equal to the proton number Z, which is also the number of harmonic oscillators, the unperturbed energy is given as ZE_n^0. Similarly, since the perturbation is due to the (N–Z) unpaired neutrons, the energy due to perturbation will be given as $(N - Z)E_n'$.

Thus, the total energy of the system becomes:

$$E_n = Z\left(n + \frac{1}{2}\right)\eta\omega + (N - Z)\left[\frac{3\gamma\eta^2}{2\mu^2\omega^2}\left(n^2 + n + \frac{1}{2}\right) - \frac{15\beta^2\eta^2}{4\mu^3\omega^4}\left(n^2 + n + \frac{11}{30}\right)\right] \quad \ldots(26)$$

3 Binding Fraction

The binding energy per nucleon or the binding fraction is derived by dividing Eq.(26) by A, that is:

$$f = \frac{E_n}{A} \quad \ldots(27)$$

4 Specific Heat and Transition Temperature

At the transition temperature, the probability amplitude Greens function, which according to quantum statistical mechanics is equivalent to the thermal activation factor $\exp\left(-\frac{\Delta E}{\kappa T}\right)$, where $\Delta E = \eta\omega$, and κ is Boltzmann constant. The level density ρ_n is given as:

$$\rho_n = \exp\left(-\frac{\Delta E}{\kappa T}\right) \text{ and } \sum_n \rho_n = 1 \quad \ldots(28)$$

Eq. (26) can ,thus, be written as:

$$E_n = Z\left(n + \frac{1}{2}\right)\eta\omega + (N - Z)\left[\frac{3\gamma\eta^2}{2\mu^2\omega^2}\left(n^2 + n + \frac{1}{2}\right) - \frac{15\beta^2\eta^2}{4\mu^3\omega^4}\left(n^2 + n + \frac{11}{30}\right)\right]\exp-\left(\frac{\eta\omega}{\kappa T}\right) \quad \ldots(29)$$

The specific heat is given by the following equation:

$$C = \frac{\partial E_n}{\partial T} \quad \ldots(30)$$

and hence

$$C = (N - Z)\left[\frac{3\gamma\eta^2}{2\mu^2\omega^2}\left(n^2 + n + \frac{1}{2}\right) - \frac{15\beta^2\eta^2}{4\mu^3\omega^4}\left(n^2 + n + \frac{11}{30}\right)\right]\frac{\eta\omega}{\kappa T^2}\exp-\left(\frac{\eta\omega}{\kappa T}\right) \quad \ldots(31)$$

The transition temperature T_C of the system is obtained from the condition that:

$$\left(\frac{\partial C}{\partial T}\right)_{T = T_C} = 0 \quad \ldots(32)$$

Substituting Eq.(31) in Eq. (32) , we get:

$$\frac{-2}{T_C^3} + \frac{\eta\omega}{\kappa T_C^4} = 0 \quad \ldots(33)$$

or

$$T_C = \frac{\eta\omega}{2\kappa} \quad \ldots(34)$$

The value of T_C given by Eq. 34) is in kelvin (K); in nuclear theory it is to be converted to MeV.

5 Entropy

The expression relating entropy S to temperature T is:

$$dS = \frac{dQ}{T} \quad \text{or} \quad \int dS = \int \frac{dQ}{T} = \int \frac{mCdT}{T} \quad \ldots(35)$$

where m= mass of nucleus = Zm_p+Nm_p
Substituting Eq. 31) in Eq. (35) and integrating we get:

$$S(\mathrm{T}) = (\mathrm{N}-\mathrm{Z})\left[\frac{3\gamma\eta^2}{2\mu^2\omega^2}\left(n^2+n+\frac{1}{2}\right) - \frac{15\beta^2\eta^2}{4\mu^3\omega^4}\left(n^2+n+\frac{11}{30}\right)\right]\left(\frac{1}{\mathrm{T}}e^{\frac{-\eta\omega}{\kappa\mathrm{T}}} + \frac{\kappa}{\eta\omega}e^{\frac{-\eta\omega}{\kappa\mathrm{T}}}\right) \quad \ldots(36)$$

6 Numerical Calculations

Since βx^4 and γx^4 must have the dimensions of energy $ML^2\mathrm{T}^{-2}$, the dimensions of β and γ should be $ML^{-1}\mathrm{T}^{-2}$ and $ML^{-2}\mathrm{T}^{-2}$, respectively, since x which is the displacement operator has the dimension of length L. Therefore, a parameter a_0 which is assumed to be fundamental to the perturbation parameters β and γ has been introduced. This parameter a_0 is defined as the bond length between the nucleons in the nucleus. The bond length is taken as:

$$a_0 = 1.3\times10^{-13} A^{1/3}\,\mathrm{cm}. \quad \ldots(37)$$

The perturbation parameters can, therefore, be defined as:

$$\beta = \frac{\eta\omega}{a_0^3} \text{ and } \gamma = \frac{\eta\omega}{a_0^4} \quad \ldots(38)$$

Substituting these in Eq.(26), we get:

$$\mathrm{E_n} = \mathrm{Z}\left(n+\frac{1}{2}\right)\eta\omega + (\mathrm{N}-\mathrm{Z})\left[\frac{3\eta^3}{2\mu^2\omega a_0^4}\left(n^2+n+\frac{1}{2}\right) - \frac{15\eta^4}{4\mu^3\omega^2 a_0^6}\left(n^2+n+\frac{11}{30}\right)\right] \quad \ldots(39)$$

where $n = 0,1,2,3,....$

The following values for different physical quantities have been used.

Planck's constant/2π =η is given as 1.054×10^{-27} erg-s;The neutron-proton reduced mass μ is given as 8.369×10^{-25}g; Boltzmann's constant κ is given as 1.3807×10^{-16} erg/K;The angular frequency/2π = ω=$6\times10^{22}\mathrm{S}^{-1}$

7 Variation of *f* with A, Z, N and η

Giving different values to A, Z and N for different heavy nuclei and using Eq. (39) for E_n, we get the value of f and these are presented in Table 1. The variation of binding fraction f with A is shown in Fig. 1. The graph shows that for medium heavy and heavy nuclei our value for f fit quite well with those obtained by the semi-empirical mass formula and also the experimental data[6-8]. Thus, the theory developed by us is able to predict the values for f that are known so far. Our theory clearly confirms that the binding energy per nucleon or the binding fraction f reduces with increase in mass number for heavy nuclei[9]. Figs 2 and 3 show the variation of the binding fraction

Table 1 — Variation of binding fraction *f* with A, Z and N

A	*Z*	*N*	*f* (MeV)
52	24	28	9.317
70	31	39	8.769
80	35	45	8.662
101	44	57	8.622
115	49	66	8.433
33	55	78	8.184
144	60	84	8.245
150	62	88	8.179
163	66	97	8.012
175	71	104	8.027
181	73	108	7.98
190	76	114	7.914
223	87	136	7.718
227	89	138	7.756
238	92	146	7.647

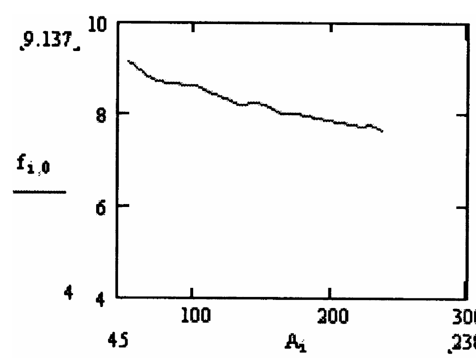

Fig. 1 — Variation of binding fraction f with with mass number A

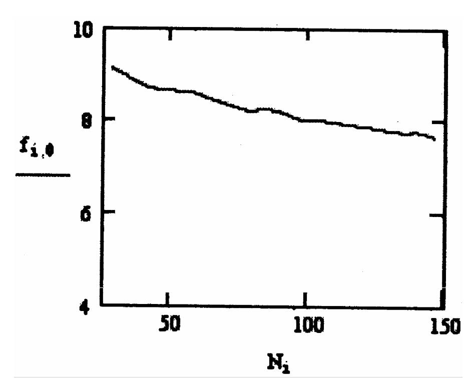

Fig. 2 — Variation of binding fraction *f* with neutron number N

Table 2 — Variation of binding fraction f with the neutron excess parameter η

A	Z	N	η	f (MeV)
52	24	28	0.077	9.137
70	31	39	0.114	8.769
80	35	45	0.125	8.662
101	44	57	0.129	8.622
115	49	66	0.148	8.433
133	55	78	0.173	8.184
144	60	84	0.167	8.245
150	62	88	0.173	8.179
163	66	97	0.19	8.012
175	71	104	0.189	8.027
181	73	108	0.193	7.98
190	76	114	0.2	7.914
223	87	136	0.22	7.718
227	89	138	0.216	7.756
238	92	146	0.227	7.647

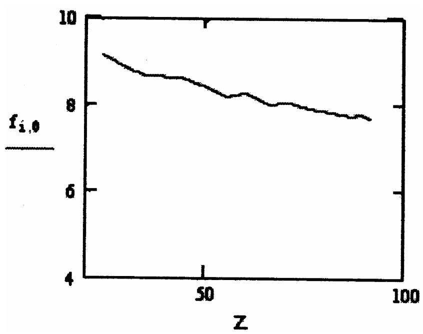

Fig. 3 — Variation of binding fraction f with proton number Z

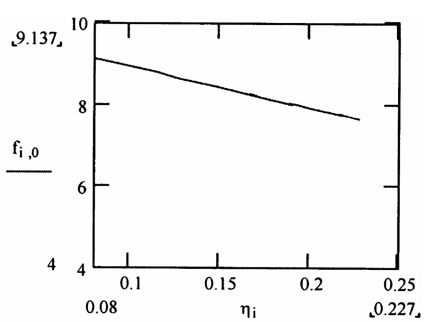

Fig. 4 — Variation of binding fraction f with neutron excess parameter η

f with the neutron number N and the proton number Z, respectively.

Further, we can define a neutron excess parameter η such that:

$$\eta = \frac{N - Z}{A}$$

For different values of A, Z and N, we get the value of the neutron excess parameter η along with f, and these values are presented in Table 2. The variation of f with η is shown in Fig. 4. The variation of f with η is linear indicating that the heavy nuclei remain stable even when Z becomes large.

Table 3 — Variation of specific heat $C4_{0,T}$ with excitation energy $E4_{0,T}$ for ^{161}Dy

$C4_{0,T}$	$E4_{0,T}$
0	1.30647×10^{3}
3.4991×10^{-8}	1.30647×10^{3}
1.1413×10^{-5}	1.30647×10^{3}
1.7391×10^{-4}	1.30647×10^{3}
8.0573×10^{-4}	1.30647×10^{3}
2.0939×10^{-3}	1.30647×10^{3}
3.9485×10^{-3}	1.30647×10^{3}
6.1303×10^{-3}	1.30648×10^{3}
8.3942×10^{-3}	1.30649×10^{3}
0.0106	1.30650×10^{3}
0.0125	1.30651×10^{3}
0.0142	1.30652×10^{3}
0.0156	1.30654×10^{3}
0.0167	1.30655×10^{3}
0.0176	1.30657×10^{3}
0.0182	1.30659×10^{3}

Table 4 — Variation of specific heat $C5_{0,T}$ with excitation energy $E5_{0,T}$ for ^{163}Dy

$C5_{0,T}$	$E5_{0,T}$
0	1306.47
3.682×10^{-8}	1306.47
1.201×10^{-5}	1306.47
1.83×10^{-4}	1306.4701
8.478×10^{-4}	1306.4705
2.203×10^{-3}	1306.472
4.155×10^{-3}	1306.4751
6.45×10^{-3}	1306.4804
8.833×10^{-3}	1306.4881
0.011	1306.4981
0.013	1306.5102
0.015	1306.5243
0.016	1306.54
0.018	1306.557
0.018	1306.575
0.019	1306.5938

8 Variation of *C* with E_n

Using Eq. (29) in Eq. (31), we get:

$$C = \frac{\eta\omega}{\kappa T^2}\left[E_n - Z\left(n + \frac{1}{2}\right)\eta\omega\right] \quad \ldots(40)$$

Eq. (40) can now be used to calculate the variation of C with E_n. Eq. (40) shows that C varies directly as Z and it should be so since as Z increases, repulsive energy between the protons increases and this changes the total energy of the nucleus. The variations of C with E_n for the heavy nuclei (Tables 3 and 4) ^{161}Dy and ^{163}Dy are shown in Figs 5 and 6, respectively.

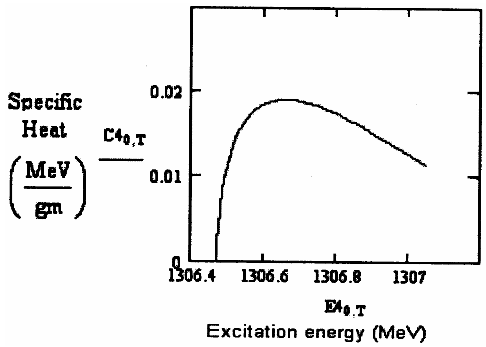

Fig. 5 — Variation of the specific heat C4 with excitation energy E4 for ^{161}Dy

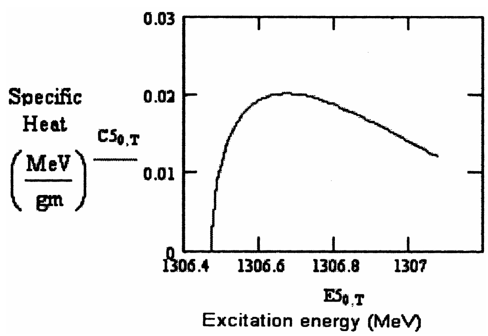

Fig. 6 — Variation of specific heat C5 with excitation energy E5 for ^{163}Dy

Table 5 — Specific heat for ^{161}Dy ($C4_{0,T}$) and ^{163}Dy ($C5_{0,T}$) with temperature T (T=1,2,3.........16 MeV)

	$T=4_{0,T}=$	$C5_{0,T}=$
1	0	0
2	3.499×10^{-8}	3.682×10^{-8}
3	1.141×10^{-5}	1.201×10^{-5}
4	1.739×10^{-4}	1.83×10^{-4}
5	8.057×10^{-4}	8.478×10^{-4}
6	2.094×10^{-3}	2.203×10^{-3}
7	3.949×10^{-3}	4.155×10^{-3}
8	6.13×10^{-3}	6.45×10^{-3}
9	8.394×10^{-3}	8.833×10^{-3}
10	0.011	0.011
11	0.013	0.013
12	0.014	0.015
13	0.016	0.016
14	0.017	0.018
15	0.018	0.018
16	0.018	0.019

Using Eq. (31), we have calculated the variation of C with T for the heavy nuclei ^{161}Dy and ^{163}Dy. Table 5 presents the variation of C with T for both the nuclei and Fig. 7 shows the variation of C with T for the same nuclei.The value of the transition temperature $T_C\kappa=(\eta\omega)/2$, and this turns out to be 19.602 MeV. The specific heat curves are S-shaped and these results are similar to the ones obtained earlier[1,10].

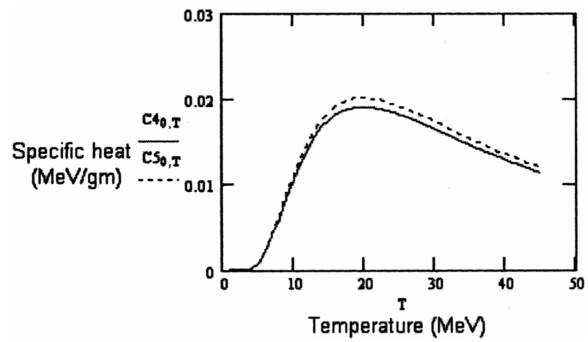

Fig.7 — Variation of specific heat C4 and C5 for ^{161}Dy (solid line) and ^{163}Dy (dotted line) respectively with temperature T

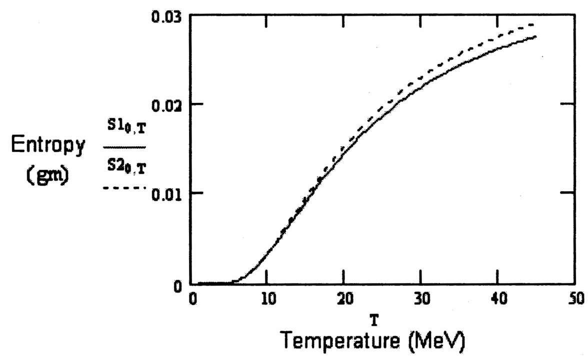

Fig. 8 — Variation of entropy S for ^{161}Dy (solid line) and ^{163}Dy (dotted line), respectively against temperature T (MeV)

The S-shaped curve is interpreted as a fingerprint of a phase-transition-like behaviour in finite systems. Extensive studies of nuclear multi fragmentation for the last about two decades has been strongly stimulated by the idea that this process is related to a liquid-gas phase transition. The liquid-gas phase transition in nuclear matter[11,12] and atomic nucleus[3] have been known since long. The values of C are positive and this is mainly due to the Coulomb interaction[9]. The values of T_C vary[11,13] between 10-20 MeV and[14,15] T_C=18 MeV .

The results obtained by us strongly support that the magnitude of T_C should be greater than 15 MeV. Fig. 7 shows that at any temperature the specific heat of ^{163}Dy is more than that of ^{161}Dy. The variation of f with η is linear indicating that the heavy nuclei remain stable even when Z becomes large. Thus , in a pure neutron system (a neutron star), the specific heat will be larger than the specific heat of a system composed of neutrons, protons and may be electrons. In ^{163}Dy, there are two additional neutrons and these neutrons stay in the surface region and contribute to the perturbation of the core resulting in the increase of perturbation energy and hence increase of entropy of ^{163}Dy when compared to ^{161}Dy which is shown in Figs 8-10.

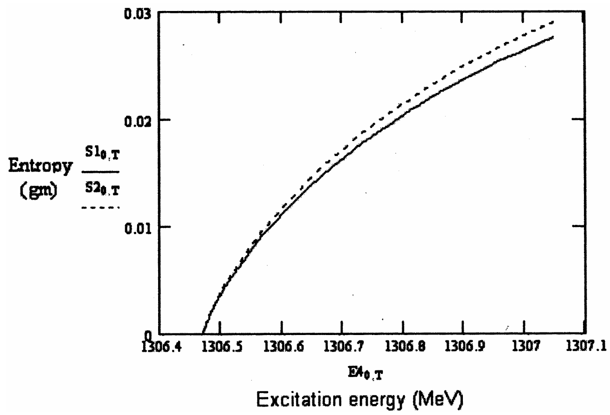

Fig. 9 — Variation of entropy S1 and S2 against excitation energy E4 for ^{161}Dy (solid) and ^{163}Dy (dotted)

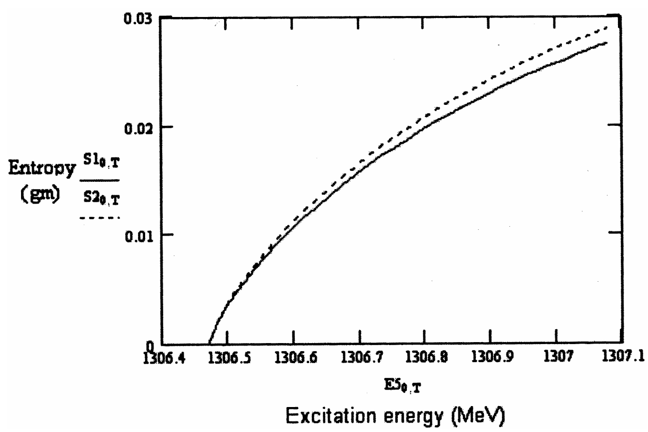

Fig. 10 — Variation of entropy S1 and S2 against excitation energy E5 for ^{161}Dy (solid) and ^{163}Dy (dotted)

9 Variation of S with T

Using Eq. (36), we have calculated the variation of S with T for the heavy nuclei ^{161}Dy and ^{163}Dy. Table 6 presents the variation of S with T for both the nuclei and Fig. 8 shows the variation of S with T for the same nuclei.

10 Variation of S with E_n

Using Eq. (36) and Eq. (39), we have calculated the variation of S with E_n for the heavy nuclei ^{161}Dy and ^{163}Dy. Tables 7 and 8 presents the variation of S with E_n for both the nuclei and Fig. 9 shows the variation of S with E_n for the same nuclei.

11 Discussion

From the results, a large finite nucleus can be assumed to be composed of Z neutron-proton pairs that reside in the core of the nucleus and the unpaired neutrons can be assumed to reside in the surface region of nucleus. The *np*-pairs have harmonic interaction. When the unpaired neutrons in the surface

Table 6 — Variation of entropy S with temperature T for ^{161}Dy ($S1_{0,T}$) and ^{163}Dy ($S2_{0,T}$), respectively for T =1,2,3,............16 MeV

T=	$S1_{0,T}$=	$S2_{0,T}$=
1	0	0
2	1.857×10^{-9}	1.954×10^{-9}
3	9.304×10^{-7}	9.79×10^{-7}
4	1.935×10^{-5}	2.036×10^{-5}
5	1.146×10^{-4}	1.206×10^{-4}
6	3.654×10^{-4}	3.845×10^{-4}
7	8.216×10^{-4}	8.645×10^{-4}
8	1.489×10^{-3}	1.567×10^{-3}
9	2.342×10^{-3}	2.464×10^{-3}
10	3.34×10^{-3}	3.514×10^{-3}
11	4.439×10^{-3}	4.671×10^{-3}
12	5.601×10^{-3}	5.894×10^{-3}
13	6.793×10^{-3}	7.148×10^{-3}
14	7.99×10^{-3}	8.407×10^{-3}
15	9.172×10^{-3}	9.651×10^{-3}
16	0.01	0.01

Table 7 — Variation of entropy $S1_{0,T}$ against excitation energy $E4_{0,T}$ for ^{161}Dy

$S1_{0,T}$=	$E4_{0,T}$=
0	1306.47
1.857×10^{-9}	1306.47
9.304×10^{-7}	1306.47
1.935×10^{-5}	1306.47007
1.146×10^{-4}	1306.47051
3.654×10^{-4}	1306.4719
8.216×10^{-4}	1306.47489
1.489×10^{-3}	1306.47991
2.342×10^{-3}	1306.48717
3.34×10^{-3}	1306.49666
4.439×10^{-3}	1306.50821
5.601×10^{-3}	1306.52158
6.793×10^{-3}	1306.53648
7.99×10^{-3}	1306.55264
9.172×10^{-3}	1306.56978
0.01	1306.58767

Table 8 — Variation of entropy $S2_{0,T}$ against excitation energy $E5_{0,T}$ for ^{163}Dy

$S2_{0,T}$=	$E5_{0,T}$=
0	1306.47
1.954×10^{-9}	1306.47
9.79×10^{-7}	1306.47
2.036×10^{-5}	1306.47007
1.206×10^{-4}	1306.47054
3.845×10^{-4}	1306.472
8.645×10^{-4}	1306.47514
1.567×10^{-3}	1306.48043
2.464×10^{-3}	1306.48807
3.514×10^{-3}	1306.49806
4.671×10^{-3}	1306.51021
5.894×10^{-3}	1306.52427
7.148×10^{-3}	1306.53995
8.407×10^{-3}	1306.55695
9.651×10^{-3}	1306.57499
0.011	1306.59382

region interact with these *np*-pairs, it leads to anharmonicity in the *np*-pair interaction. The binding energy and the binding fraction values compare nicely with the experimental values known so far. Eq. (29) can be used to determine the energy level scheme for some heavy nuclei. The level scheme, thus, obtained can be compared with the corresponding level schemes known so far. We have chosen the nuclei $_{66}Dy^{161,163}$ only, but we could obtain the energy level scheme for any heavy nucleus. It could be interesting to design an experiment to ascertain whether the core of a heavy nucleus is really composed of *np*-pairs and whether the unpaired neutrons stay in the surface region of the nucleus, what may be the actual radius of the core and what may be the actual thickness of the surface region. May be a neutron beam or a deutron beam with specific energy could be used to collide with a nucleus to get some answer.

According to this model, two *np*-pairs come together to form an alpha particle that will pass through the surface region before leaving the nucleus. This alpha particle will collide with the neutrons in the surface region resulting in the following reaction, $n \rightarrow p + \beta$. The β particle will be emitted from the nuclear surface region and the proton will combine with a neutron in the surface region to form an *np*-pair that will enter the core. The formation of the *np*-pair and its subsequent entry into the core will be accompanied by the emission of a gamma ray that will leave the surface of the nucleus. According to this model of the heavy nucleus, the explanation for the natural emission of alpha, beta and gamma radiation from a heavy nucleus is given in the present paper.

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