OPTIMAL AND EFFICIENT PRODUCTION OF *ROSE COCO* BEANS THROUGH THE TWENTY FOUR POINTS SECOND ORDER ROTATABLE DESIGN

BY
ISAAC KIPKOSGEI TUM
(Msc. Biostatistics)

A THESIS SUBMITTED FOR EXAMINATION IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF DOCTOR OF PHILOSOPHY IN BIOSTATISTICS IN THE DEPARTMENT OF STATISTICS AND COMPUTER SCIENCE, SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES OF MOI UNIVERSITY

2018

DECLARATION

Declaration by the Candidate

This thesis is my original work and has not been presented for a degree in any other university. No part of this thesis may be reproduced without prior written permission of the author and/or Moi University.

Isaac Kipkosgei Tum

Signature: ___________________________________Date: _________________

PHD/BS/003/12

Declaration by the Supervisors

This thesis has been submitted with our approval as the university supervisors

Prof. J. K. Arap Koske

Signature: ________________________________Date: _____________

Department of Statistics and Computer Science,
Moi University,
P.O. Box 3900,
Eldoret, Kenya.

Prof. J.M. Mutiso

Signature: ____Date:_________

Department of Statistics and Computer Science,
Moi University,
P.O. Box 3900,
Eldoret, Kenya.

DEDICATION

I dedicate this work to my lovely wife Viola, son Ryan, daughter Jayleen, my parents; Samuel Tum and Rosemary Tum, grandma and my siblings. Thank you all for special love, support, understanding, encouragement and prayers during the write-up of this thesis.

ACKNOWLEDGEMENT

Firstly, I thank Almighty God for giving me life, the courage and the determination, as well as guidance in furthering my studies, despite all odds. Secondly, I thank my supervisors Prof. J.K. Koske and Prof. J.M. Mutiso, they have totally transformed me. Without their patience and encouragement, this thesis would not have been possible. I sincerely appreciate the invaluable academic support I received from them throughout the writing of this thesis, their kind, detailed and constructive comments were vital to the development of this dissertation. Thirdly, my acknowledgement goes to Moi University staff members in the school of biological and physical sciences (in particular Dean, Prof Kiprop who occasionally sent us letters to show the 'Ph.D progress'), and I am also indebted to my workmates at the department of statistics and computer science. I wish to extend my gratitude to the classmates (2012 class) for their wonderful cooperation, encouragement and moral support, for being together right from the Masters class in the University of Eldoret after which we enrolled together for Ph.D class in the Moi University. Finally, my further gratitude goes to my family members from whom my success is derived from (especially Jebet who assisted much in the greenhouse work); I am truly blessed to have you all as my family. Indeed, your efforts and contributions in my work cannot be exhausted, though God will reward you abundantly. Thank you.

ABSTRACT

Response Surface Methodology (RSM) is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize the response. The yield results of the twenty four points RSM design permitted a response surface to be fitted easily and provided spherical information contours besides the realizations of an optimum combination of the fertilizers in *rose coco* beans, which resulted in economic use of scarce resources for optimal production of *rose coco* beans. In this study an existing A-optimum and D-efficient second order rotatable design in three dimensions was used in the production of *rose coco* beans optimally and efficiently. The general objective of the study was to produce *rose coco* beans (*Phaseolus vulgaris*) optimally and efficiently using an existing A-optimum and D-efficient twenty four points second order rotatable design in three dimensions in a greenhouse setting using three inorganic fertilizers, namely, nitrogen, phosphorus and potassium. Thus the study was accomplished using the calculus optimum value of the free/letter parameter f=1.1072569. The specific objectives were to estimate the linear parameters, thereby making available the yield response of *rose coco* beans at calculus optimum value design for the first time. The generalized variance of the estimated linear parameters was also obtained, fitted and tested the three models adequacies via lack of fit test, and then found the settings of the experimental factors that produces optimal response using contour plots to assist visualizes the response surfaces. This study demonstrated the importance of statistical methods in the optimal and efficient production of *rose coco* beans. The results showed that the three factors: nitrogen, phosphorus, and potassium contributed significantly on the yield of *rose coco* beans ($p<0.05$). The regression coefficients were determined by employing least squares techniques to predict quadratic polynomial models for group 1 greenhouse (GP1G), group 2 greenhouse (GP2G) and group 3 greenhouse (GP3G) for the three fertilizer combinations. In GP1G second order model was inadequate with a p value of 0.3178, in GP2G and GP3G, the second order model was adequate at 1% level of significance with p values of 0.0065 and 0.0034 respectively. The analysis of variance (ANOVA) of response surface for *rose coco* yield showed that this design was adequate due to satisfactory levels of coefficient of determination, R^2, 0.6810 (GP1G), 0.6704 (GP2G), and 0.8066 (GP3G) and coefficient variation, CV was 13.48, 14.47 and 10.30 for GP1G, GP2G, GP3G respectively. The canonical analysis showed that there were saddle points for the three groups, meaning there was no unique optimum; therefore ridge analysis was used to overcome the saddle problem. The results from ridge analysis provided the maximum yield of 58.78grams, 48.36grams and 70.25grams in GP1G, GP2G and GP3G respectively for the various fertilizer combinations at radii of one. We therefore recommend the use of GP3G design since it gave above board the required coefficient of determination (R^2=80.66) and the maximum yield (70.25grams) was achieved.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS AND ACRONYMS

A	Average Variance Criterion
ANOVA	Analysis of Variance
C	Information Matrix
CV	Coefficient of Variation
D	Determinant Criterion
D_1	The twenty four points second order rotatable design in three dimensions
E	Eigen-value Criterion
GP1G	Group 1 greenhouse with N:P:K fertilizer in grams at the ratio of 10:20:30
GP1GA	the first replicate in GP1G
GP1GB	the second replicate in GP1G
GP1GC	the third replicate in GP1G
GP2G	Group 2 greenhouse with N:P:K fertilizer in grams at the ratio of 20:30:40
GP2GA	the first replicate in GP2G
GP2GB	the second replicate in GP2G
GP2GC	the third replicate in GP2G
GP3G	Group 3 greenhouse with N:P:K fertilizer in grams at the ratio of 30:40:50
GP3GA	the first replicate in GP3G
GP3GB	the second replicate in GP3G
GP3GC	the third replicate in GP3G
K	Potassium
MATLAB	Matrix Laboratory
Mg	Milligram
MSE	Mean Square Error
MT	Metric Tonnes
N	Nitrogen
P	Phosphorous
PRESS	Prediction error sum of squares
RSM	Response Surface Methodology
SAS	Statistical Analysis System
T	Trace criterion

CHAPTER 1: INTRODUCTION

1.1 Background Information

Generally, the objective for agricultural researchers is that, they are in constant search for new or improved technologies to increase productivity. The researcher is usually interested in finding maximum or optimal yield using minimum cost. For Kenya to meet the needs of its ever growing population, then new technologies is inevitable in agricultural fields. Since arable land in Kenya is a fraction, there's need to produce maximally in such areas by utilizing design of an experiment like the one in this study. There are two types of supplies for agriculture, specifically fertilizer and pesticides. It can be said that the fertilizer is food and pesticides is medicine for plants in conventional agriculture (Arjumand, 2013). Soil fertility is diminishing gradually due to the erosion, loss of nutrients, accumulation of salts and other toxic elements and unbalanced nutrients compensation. Many efforts are being exercised to combat the adverse consequences of chemical farming (Faheed, 2008). During the last decades *rose coco* bean is becoming increasingly important legume for human nutrition and a major protein and calorie source in the world. The bean crop requires nitrogen in quite high amount in the first stage of development for the emergence of the nodules and builds up of the symbiotic nitrogen fixation. The amount of nitrogen which symbiotically bound depends on the kind of plant, the efficiency of the bacteria inoculated and soil properties (Bildirici and Yilmaz; 2005). Common bean (*Phaseolus vulgaris*), also referred to as dry beans, is an annual leguminous plant that belongs to the genus, *Phaseolus*, with pinnately compound trifoliate large leaves. It is largely a self-pollinated plant, though cross-pollination is possible if the stigma contacts with pollen coated bee when extended. Seeds are non-endospermic and vary greatly in size and colour from the small black wild type to the large white, brown, red, black or mottled seeds of cultivars, which are 7-16 mm long (Cobley and Steele, 1976). The common *rose coco* beans (*Phaseolus vulgaris)* are the most commonly grown grain legumes that come second after maize as a subsistence crop (GoK, 2006). They serve as a source of protein which is relatively cheaper compared to animal proteins to the majority of the population in Kenya (Karanja et al., 2007). Consumption of common bean is high mostly because it is relatively inexpensive compared to meat (Pachico, 1993). Common bean plays a strategic role in alleviating malnutrition. Regular consumption of common bean is now promoted by health organizations because it reduces the risk of diseases such as cancer, diabetes or coronary heart diseases (Leterme and Munoz, 2002). This is because common bean is low in fat and is cholesterol free. It is also an appetite suppressant because it digests slowly and causes a low sustained increase in blood sugar. Bean provides a rich combination of carbohydrates (60-65%), proteins (21-25%), fats (less than 2%), vitamins and minerals (Ensminger *et al.,* 1994). In fact, with increasing health concerns, most people, especially the urban population are reducing consumption of animal proteins, and instead they are turning to pulses such as dry bean due to its low fat content. Hence the rationale for emphasis in more bean research is self-evident. The crop also provides farm households and traders with incomes and is therefore important from both the food security and income-generation. Hence there is a need for increased bean production to enhance exports as well as satisfies domestic market. It is an important staple food in the diet of people of all income categories. The beans are characterized as near perfect food because of their high protein content and generous amounts of iron, folic acid, complex carbohydrates and other diet essentials. The crop also matures within three months, enabling farmers to plant the crop almost three times annually. Therefore, we need to see this important crop doing well, so as to feed and supplement the right nutrients to its ever growing consumers.

The response surface methodology (RSM) emphasizes on finding a particular treatment combination, which causes the maximum or minimum responses. The use of analysis of the quadratic response function or RSM is necessary to obtain the optimum level of fertilizer requirements. In response surface analysis, the eigenvalues could be used to determine whether the solution gives a maximum, minimum or saddle point on the response curve. Moreover, the effects of treatment combinations, which have not been carried out in the experiment might still be estimated. The regression equation was fitted between the response variable, *rose coco* yield and the three fertilizer treatments, nitrogen (N), phosphorus (P) and potassium (K). The expected yield could be described as a continuous function of the application rate factor. When the fertilizer application rates are higher or lower than the optimum application rates, they might result in a reduction in yields. The purpose of implementing this RSM technique was to determine the optimum levels of fertilizer used in order to optimize *rose coco* yields.

In any treatment arrangement, we sought a treatment or treatment combination that could be used to either reverse existing methodologies, advance the current methods being used in farming in order to maximize the yield using the scarce resources available. In this research, the investigation geared towards searching the optimum combination of treatment combinations to maximize *rose coco* beans in a greenhouse setting using inorganic fertilizers was highlighted. It is seen how different components of fertilizers affect the output of the beans. The study therefore was used to determine; the best possible level of the identified fertilizer to maximize the yield in *rose coco* beans. In the technological context, this response surface methodology was applied to study the measured yield or output of a system as it varies in response to the changing levels of one or more physical input variables. The experimental design aspect deals with the choice of suitable variables and their various levels, and regression analysis enables a mathematical form to be fitted to the observations to "model" the varying yields.

In order to conduct a first-order model, the operating conditions that maximize the yield of *rose coco* beans were studied. There are three independent variables which influence the process yield: *nitrogen, phosphorus* and *potassium* fertilizers. The low-order polynomial terms has been used to describe some part of the response surface. Once the estimated equation was obtained, we were able to use statistical techniques to check for the model adequacy. The objective was to determine the current levels or settings of the *nitrogen, phosphorus* and *potassium* fertilizers that resulted in a value of a response that was optimal or close to the optimum. When the first order model was inadequate, then second-order model was fitted. With the purpose of exploring the second-order model, the statistical modeling techniques to develop an appropriate approximating relationship between the yield and the process variables was used.

1.2 Statement of the problem

The beans are the second commonly grown grain legumes after maize as a subsistence crop. Most farmers plant the crop without fertilizers or they apply the available fertilizer without considering the plant optimal requirement for the optimal production, when the fertilizer application rates are higher or lower than the optimum application rates, reduction in yield is experienced. The fertilizers are wasted if the amount applied is more than the optimum rate and can be harmful to the soils, (Khamis, 2006). Kenya's current annual bean production of approximately 215,000 MT, barely meets half the annual consumption of 450,000 MT. The average production per hectare is 500 kg or less, compared to 1800 to 2000 kg ha^{-1} potential (Africa Agriculture, 2008). The major limitation to bean production in many smallholder farms is declining soil fertility as a result of continuous cropping with minimal inputs or rotation to replenish soil nutrients. Furthermore, there is minimal or no optimal use of the required inorganic fertilizers. The over use of N (nitrogen) relative to P_2O_5 (phosphorus) and K_2O (potassium) has raised concerns in environmental perspective. The phosphorus and potassium fertilizers have been in short supply and farmers have been more steadily adopting the use of nitrogenous fertilizers because of impressive response. There is evidence that soil P_2O_5 and K_2O level are declining. So determining the optimum balance of N, P_2O_5 and K_2O so as to produce high yield of *rose coco* has been of concern that the study tried to address. In the year 2014 and 2015, maize was majorly affected by a disease which swept the large acreage of farms, thus the need to embark on the research for the alternative crop that is *rose coco* bean. The study was geared towards estimating the linear parameters in one of the existing six specific second order rotatable designs in three dimensions in which Mutiso J.M.(1998) calculated the calculus optimal value to be 1.1072569 for the free/letter parameter (f), Koech, F.(2016) calculated the relative efficiencies for the six designs and their optimality criteria, Koech showed that the twenty four point second order rotatable design was the most D-efficient and A-optimal design. Therefore, out of these researches we dwelled on the twenty four points, second order rotatable design and proceeded to have a practical greenhouse experiment to realize the optimal and efficient production of *rose coco* bean using the three fertilizer components, namely, nitrogen, phosphorus and potassium. The study facilitated the estimation of the coefficient β_0, β_1, β_2, β_3, β_{11}, β_{22}, β_{33}, β_{12}, β_{13} and β_{23} in an existing second order rotatable design of twenty four points in three dimensions. Achieving the production efficiency of *rose coco* has not been easy that this research tends to employ twenty four points second order rotatable design approach, focusing on the area of modeling *rose coco* yield production and highlighting the respective point of intercept and gradient levels of fertilizer with which the variety of *rose coco* was capable of delivering the yield production efficiency. The study found out how the yields fitted the second order model and checked its model adequacy. The purpose of implementing the twenty four points' rotatable design was to determine the optimum levels of fertilizer used in order to optimize *rose coco* yields.

Therefore the problem was to achieve an optimal and efficient production of *rose coco* beans through the twenty four point second order rotatable design in three dimensions for parameter estimation.

1.3 Objectives of the study

This section highlights the general objective and specific objectives.

1.3.1 General objective

The general objective was to produce *rose coco* beans optimally and efficiently using an existing A-optimum and D-efficient twenty four point second order rotatable design in three dimensions.

1.3.2 Specific objectives

1) To estimate the linear parameters in an existing A-optimum and D-efficient calculus optimum value second order rotatable design.
2) To obtain the generalized variance of the estimated linear parameters.
3) To fit and test the three models adequacies.
4) To find the settings of the experimental factors that produces the optimal response.

1.4 Significance of the study

The major nutrients, that is, nitrogen (N), phosphorus (P) and potassium (K) are essential for growth and production of bean, (Marschner, 2012). There is unbalanced nutrients compensation in the soil. Therefore, the need to determine the optimum balance of N, P and K so as to realize the maximum potential yield of *rose coco* beans. The study focused on the estimated linear parameters of a specific twenty four point second order rotatable design in three dimensions for the calculus optimal value for the first time, checked the adequacy of the models and obtained the generalized variance for the estimated linear parameters. Furthermore, the response surface for the optimal response yield of *rose coco* bean was found for the three groups. The study would provide the most reliable advice for policy makers on the right range application of fertilizers that are desirable to optimize/maximize the *rose coco* bean crop production.

1.5 Limitation of the study

The r*ose coco* beans were planted simultaneously in a greenhouse (15mx10m) to guard soil contamination from the subsequent planting, we could have had three planting periods at different seasons say January-April, May-August and September-December, or instead built two or three greenhouses, this could have allowed the independent variables to be screened to get the amount of each component of the fertilizers. The choice of each fertilizer starting point in this study was done arbitrarily; there were so many infinite options available at our disposal. The top dressing and foliar spray was not applied, only one time initial fertilizer application was done during planting and left till harvest was done.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter presents some literature relevant to the study objectives. The subsections included are; the *rose coco* beans production in Kenya, organic and inorganic use of fertilizers, response surface methodology, optimal design criteria, D-optimality, Model adequacy.

2.2 The rose coco bean production in Kenya

Kenya's current annual bean production of approximately 215,000 metric tonnes (MT), barely meets half the annual consumption of 450,000 MT. The average production per hectare is 500 kg or less, compared to 1800 to 2000 kg ha^{-1} potential (Africa Agriculture, 2008). The major limitation to bean production in many smallholder farms is declining soil fertility as a result of continuous cropping with minimal inputs or rotation to replenish soil nutrients. Furthermore, there is minimal or no use of the required inorganic fertilizers. Some of the options that are currently being pursued to address low soil fertility include integrated use of organic (e.g., crop residues, animal manures, agro forestry tree pruning) and inorganic (fertilizers) resources. Common beans are grown in pure stands by large scale farmers, but commonly intercropped with maize by smallholder farmers (Kimenju, 2004). Like many legumes, beans thrive well in sufficiently aerated and well drained soils with a pH of 6.5 - 7.5 because they are very sensitive to soil acidity and an optimal amount of organic carbon above 2.4 percent (Baudoin *et al*., 2001). The optimum altitude should exceed 1000 m above sea level. Most common bean cultivars are short-season crops with a maturity period ranging from 65 to 110 days from emergence to physiological maturity, hence two bean harvests per year (Buruchara, 2007).

Common bean production in Kenya is mainly in the highlands and the midlands. About 75 percent of the annual cultivation occurs in four regions, namely: Rift Valley, Western, Nyanza, and Eastern Province (Wortmann, 1998). The Rift Valley contributes the biggest share, accounting for 33 percent of the national production followed by Nyanza and Western province accounting for 22 percent each (Katungi, 2010). Output from the eastern parts of the country and the coast is constrained by adverse climatic conditions. Although Kenya has two seasons for common bean, a significant number of farmers grow the crop once a year because of adverse climatic conditions. In Kenya, some of the Andean beans that are widely cultivated include the red mottled (occurring in different local names such as Rose coco or GLP 2 and Nyayo), red kidneys (such as Canadian Wonder or GLP 24), purple/gray speckled (locally known as Mwezi Moja). *Rose coco* type is the most widely grown followed by Canadian Wonder type. *Rose coco* and *Canadian Wonder* type are high yielding but require heavy rains and high soil fertility to yield well. Soil fertility is diminishing gradually due to the erosion, loss of nutrients, accumulation of salts and other toxic elements and unbalanced nutrients compensation. *Rose coco* bean (*Phaseolus vulgaris*), is an important legume for human nutrition and a major protein and calorie source. Bean production is declining in Kenya due to various factors; virus diseases and optimum combination of the needed component of fertilizers are some of the major yield reduction factors in bean production. The *rose coco* hardly meets the demand; therefore the need to search for improved methods.

2.3 Organic and inorganic use of fertilizers

The animal manures are valuable sources of nutrients and the yield-increasing effect of manure is well established. Apart from the nutrients in manure, its effects on the improvement of soil organic matter, soil structure and the biological life of the soil are well recognized. There is also some evidence that it may contain other growth-promoting substances like natural hormones and B vitamins (Leonard, 1986). Plants can only use the nutrients that are in an inorganic form. Manure N and P are present in organic and inorganic forms, and are not totally available to plants. The organic forms must be mineralized or converted into inorganic forms over time before they could be used by plants. The organic manure could then be applied to plants as a source of nitrogen, phosphorus and potassium (N, P & K) which are the macro nutrients that limit crop growth (Kwabiah, 2003; Wasonga, 2008). The term “organic” describes production systems that optimize natural processes. Organic farming systems rely on ecologically-based practices such as cultural and biological pest management, and virtually exclude the use of synthetic chemicals in crop production and prohibit the use of antibiotics and hormones in livestock production. The chemical fertilizers are used in modern agriculture to correct known plant-nutrient deficiencies; to provide high levels of nutrition, which aid plants in withstanding stress conditions; to maintain optimum soil fertility conditions; and to improve crop quality. Adequate fertilization programs supply the amounts of plant nutrients needed to sustain maximum net returns (Leonard, 1986). In essence, fertilizers are used to make certain that soil fertility is not a limiting factor in crop production.

2.4 Response Surface Methodology

The response Surface Methodology (RSM) is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response (Montgomery 2005). In most RSM problems, the form of the relationship between the response and the independent variables is unknown. Thus the first step in RSM is to find a suitable approximation for the true functional relationship between the response and the set of independent variables which are subject to the control of the scientist or engineer. Usually, a low-order polynomial in some region of the independent variables is employed. If the response is well modeled by a linear function of the independent variables, then the approximating function is the first-order model and if there is curvature in the system, then a polynomial of higher degree such as the second order is used. The broad aims of the RSM are to investigate the nature of the response surface over a region of interest and to identify operating conditions associated with maximum or minimum responses. RSM is a sequential procedure and is generally conducted in three phases, as emphasized in (Myers and Montgomery, 2002). Phase 1 involves the screening of explanatory variables when the number of factors is large or when experimentation is expensive to identify those which have a significant effect or the most influential on the response(s) being investigated; phase 2 is concerned with the location of optimum operating conditions by conducting a sequence of suitable experiments; and phase 3 involves the fitting of an appropriate empirical model, usually a second-order polynomial model, in order to examine the nature of the response surface in the vicinity of the optimum. In order to get the most efficient result in the approximation of polynomials the proper experimental design must be used to collect data. Once the data are collected, the method of least squares was used to estimate the parameters in the polynomials. The response surface analysis was performed by using the fitted surface. The response surface designs are the types of designs for fitting response surface. The fundamental methods for quantitative variables involve fitting first-order (linear) or second-order (quadratic) functions of the predictors to one or more response variables, and then examining the characteristics of the fitted surface to decide what action was appropriate. The **β** could be estimated by using the least squares method as: **β** = $(\mathbf{x'x})^{-1}\mathbf{x'y}$. The measure of accuracy of the column of estimators, **β**, is the variance-covariance matrix which is defined as; V(**β**) = $\sigma^2(\mathbf{x'x})^{-1}$ (Myers, 1971), (Unal, 1996), (Craig, 1978), (Mitchell, 1974) where σ^2 is the variance of the error. The V(**β**) matrix is a statistical measure of the goodness of fit. The V(**β**) is a function of $(\mathbf{X'X})^{-1}$ and therefore, one would want to minimize $(\mathbf{X'X})^{-1}$ to improve the quality of the fit. Statisticians have shown that minimizing $(\mathbf{X'X})^{-1}$ is equivalent to maximizing the determinant of **X'X** (Mitchell, 1974, Montgomery, 1991, Box and Draper, 1974). Therefore, generating a design matrix which enables the construction of a good least square approximation model translates to maximizing the determinant of the X'X matrix and experimental designs that maximize |**X'X**| are referred to as D optimal designs (Unal, et al 1996, Craig, 1978, Box and Draper, 1974) Here, "D" stands for the determinant of the **X'X** matrix associated with the model. This analysis could easily be extended to the quadratic model with the same conclusions for D-optimality.

2.4.1 Optimal Design Criteria

The design optimality criteria are often called the alphabetical optimality criteria because they are named by some of the letters of the alphabet. Kiefer and Wolfowitz (1959) were among the first authors who developed these optimality criteria. There are many types of optimal criteria available to generate experimental designs which include the determinant criterion D-; the average-variance criterion A-; the eigenvalue criterion E-; and the trace criterion T-;. Concerning optimality, the smallest value among the matrix means is usually taken thus the identification of optimality criteria. D-optimum design minimizes the content of the ellipsoidal confidence region for the parameter of the linear model. In terms of eigenvalues D-minimizes the generalized variance of the parameter estimates. A-optimality minimizes the sum (or average) of the variance of the parameter estimates (Atkinson and Donev 1992). The smallest among the criteria determines the optimality for the generalized variance of the parameter estimates of the information matrix. These criteria measure the desirability of a design, E-reduces the variance of each individual parameter estimate, and T- optimum design is one that has not enjoyed much use because of the linearity aspect of the T-criterion. Evaluation of these criteria showed that the more homogeneous the design the higher optimal it becomes. The ultimate purpose of any optimality criterion is to measure 'largeness' of a non-negative definite information matrix C_{sxs}. At this point, the specific optimality values of the design by utilizing the methods of evaluation of the particular criteria as outlined by Pukelsheim (1993).

The determinant criterion, D-, $\phi_0(C) = (\det C)^{1/s}$.

The average variance criterion, A-, $\phi_{-1}(C) = \left(\frac{1}{s} trace C^{-1}\right)^{-1}$ (if C is positive definite)

The consideration is a three dimensional specific rotatable design of order two. The relation C= $(K'M^{-1}K)^{-1}$ is used for the second order model, where M=1/N(**X'X**) is the moment matrix. The information matrix is utilized in the determination of the optimality criteria and the exact values of the criteria.

2.4.2 D-optimality

When considered historically, D-optimality (Kiefer, 1958) was the first alphabetical optimality criterion developed. It is also still among the most popular because of its simple computation, and the many available algorithms. In *D*-optimal designs, the goal is to minimize the generalized variance of the regression model parameter estimates. This is achieved by maximizing the determinant of the information matrix, |**X′X**|. It turns out that the information matrix is inversely proportional to the variance-covariance matrix of the least squares estimates of the linear parameters of a model. Thus, by maximizing the information matrix, the generalized variance of the model parameters is minimized. The twenty four point second order rotatable design is *D*-optimal.

The focus of D-optimality is an estimation of model parameters through good attributes of the moment matrix, which is defined as $M = \frac{\mathbf{X'X}}{N}$ where **X'X** is the information matrix, and *N*, the total number of runs, is used as a penalty for larger designs. D-optimality requires one to maximize the determinant of the moment matrix, that is, a D-optimal design is the design, *D**, in the design space W such that the determinant of M, $|M(D^*)| = Max_{D\in\Omega}|M(D)|$ Under the standard normality assumptions, |***X'X***| is inversely proportional to the square of the volume of the confidence region for the regression coefficients. The larger the determinant of ***X'X*** then better the estimation of the model parameters.

2.5 Model adequacy

The large value of R^2 does not necessarily imply that the regression model is good one, adding a variable to the model always increases R^2, regardless of whether the additional variable is statistically significant or not. Thus, it is possible for models that have large values of R^2 to yield poor predictions of new observations or estimates of the mean response. Because R^2 always increases as we add terms to the model, some regression model builders prefer to use an adjusted R^2 statistic defined as

$$R^2_{adj} = 1 - \frac{SS_E/(n-p)}{SS_T/(n-1)} = 1 - \frac{n-1}{n-p}(1-R^2) \tag{2.1}$$

In general, the adjusted R^2 statistic is not always increasing as variables are added to the model. In fact, when unnecessary terms are added, the value of R^2_{adj} often decreases. When R^2 and R^2_{adj} differ dramatically, there is a good chance that non-significant terms have been included in the model. We are frequently interested in testing hypotheses on the individual unknown parameters. Such tests would be useful in determining the value of each of the regressor variables in the response surface model. For example, the model might be more effective with the inclusion of additional variables, or perhaps with the deletion of one or more of the variables already in the model. Adding a variable to the regression model always causes the sum of squares for regression to increase and the error sum of squares to decrease. We must decide whether the increase in the regression sum of squares is sufficient to warrant using the additional variable in the model. Furthermore, adding an unimportant variable to the model can actually increase the mean square error, thereby decreasing the usefulness of the model (Carley et al, 2004).

The significant terms in the model were found by analysis of variance (ANOVA) for each response. The significance was judged by determining the probability level that the F-statistic calculated from the data was less than 5%. The model adequacies were checked by R^2, adjusted-R^2. The analysis included examining normal probability plots. The maximization and minimization of the polynomials thus fitted was usually performed by the mapping of the fitted responses.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This chapter gives explanations on how each one of the four specific objectives were achieved. The research was done with the combination of independent treatment factors of inorganic fertilizers to check how factors influence the optimal yield of *rose coco* beans. The experiment was carried out in a greenhouse of size 15m x 10m during the period of February and July 2016 on the twenty four points second order rotatable design in three dimensions.

3.2 Estimation of the linear parameters in an existing A-optimum and D-efficient calculus optimum value second order rotatable design.

This section gives the twenty four points' calculus optimum value experimental design, screening experiment, experiment layout, planting *rose coco* beans and the method of estimating the parameters.

3.2.1 Twenty Four Points Calculus Optimum Value Experimental Design

The design of twenty four points second order rotatable design has the free parameter(s) f which has been determined using differential calculus by Mutiso J.M (1998). The study was geared towards estimation of the linear parameters in one of the existing six specific second order rotatable designs in three dimensions in which Mutiso J.M (1998) calculated the calculus optimal value to be 1.1072569 for the free/letter parameter, the other values of c_1 and c_2 were given as 0.7829487 and 1.2735263 respectively. We went further to obtain the yield for the calculus optimal value design in the greenhouse setting after which linear parameters were estimated. We consider a set of twenty four point's rotatable designs as highlighted by Mutiso (1998) and Koske et al (2008) is given as:

$$D_1 = [\frac{1}{2} G(f, f, 0) + \frac{1}{4} G(c_1, 0, 0) + \frac{1}{4} G(c_2, 0, 0)] \tag{3.1}$$

Table 3. 1: Twenty four point second order rotatable design in three dimensions.

1)	-f	-f	0	13)	c_1	0	0	19)	c_2	0	0
2)	f	-f	0	14)	$-c_1$	0	0	20)	$-c_2$	0	0
3)	-f	f	0	15)	0	0	c_1	21)	0	0	c_2
4)	f	f	0	16)	0	0	$-c_1$	22)	0	0	$-c_2$
5)	-f	0	-f	17)	0	c_1	0	23)	0	c_2	0
6)	-f	0	f	18)	0	$-c_1$	0	24)	0	$-c_2$	0
7)	f	0	-f								
8)	f	0	f								
9)	0	-f	-f								
10)	0	-f	f								
11)	0	f	-f								
12)	0	f	f								

3.2.2 Screening experiment

The critical preliminary step was the screening stage at which we sought to identify the key list of factors that influence the bean production process. The goal of a screening experiment was to identify those factors which had an influence on the response, and then work with those significant factors in the study. The three straight fertilizers (nitrogen, phosphorus and potassium) of inorganic were screened, to determine the amount each could contribute to optimal yield of *rose coco* beans. Each one of the straight inorganic fertilizer was applied at the following amounts; 10grams, 20grams, 30grams, 40grams, and 50grams per hole to *rose coco* plant. We then checked what level of inorganic fertilizer gave maximum yield independently. Thereafter, the constructed rotatable design of twenty four points was used to find out the maximum optimal combination of the treatment factors.

3.2.3 Experiment Layout

This section gives the experimental layout of GP1G, GP2G and GP3G using the inorganic fertilizers.

3.2.3.1 Layout of calculus optimum value design

The r*ose coco* beans were subjected to inorganic fertilizers N, P, K at different levels of a twenty four point second order rotatable design according to table 3.1. Therefore, each of the three groups (that is, GP1G, GP2G and GP3G) was given three replications each, for example in GP1G we had GP1GA, GP1GB and GP1GC.

3.2.3.2 Group 1 Greenhouse-(GP1G)

In GP1G a combination of 10grams of nitrogen, 20grams of phosphorus and 30grams of potassium were the initial fertilizers applied to r*ose coco* beans and acted as the center point. This group was given the straight N, P & K fertilizers, such that GP1GA, GP1GB and GP1GC were the three replications with each having twenty four *rose coco* plants.

3.2.3.3 Group 2 Greenhouse-(GP2G)

In GP2G a combination of 20grams of nitrogen, 30grams of phosphorus and 40grams of potassium were the initial fertilizers applied to *rose coco* beans and acted as the center point. This group was given the straight N, P & K fertilizers, such that GP2GA, GP2GB and GP2GC were the three replications with each having twenty four *rose coco* plants.

3.2.3.4 Group 3 Greenhouse-(GP3G)

In GP3G a combination of 30grams of nitrogen, 40grams of phosphorus and 50grams of potassium were the initial fertilizers applied to *rose coco* beans and acted as the center point. This group was given the straight N, P & K fertilizers, such that GP3GA, GP3GB and GP3GC were the three replications with each having twenty four *rose coco* plants.

In the study some of the cases (plants) were lost due to some environmental factors such as drying up or any other cause, so dead plants were replaced to allow the study to go on. The yields for the *rose coco* plant that died after one month from the day of planting were not obtained.

3.2.4 Planting *Rose coco* Beans

In the greenhouse, organic matter on the soil surface was cleared, and then prepared by *Jembe* ploughing followed by harrowing until fine tillage was obtained. The *rose coco* had three block groups with each group having three replications of twenty four design points. Certified, viable and uniform seeds of *rose coco* beans were planted in the plots in February 2016. Beans were planted as a pure stand in a greenhouse. Before planting, the bean seeds were dressed with Aldrin at the rate of 5g per kg of seeds, to control soil pests especially bean fly

(Melanargromyza phaseoli). Furadan (5% carbofuran) were applied at a rate of 2g m^{-1} in the bean rows at sowing to control cutworms *(Agrotis ipsilon).* The bean seeds were surface sterilized using 3% sodium hypochlorite and pre-germinated on a nutrient free agar media before planting. Bean plant population density was 24 plants per line, meaning 72 plants per block of three replications (75 cm between row spacing and an intra-row spacing of 30 cm). All the plots were given a drip irrigation using drip line pipes. The plots received inorganic fertilizers at different combination. The inorganic fertilizers (nitrogen, phosphorus, potassium) combinations were applied in each experimental unit before planting two seeds (pre-germinated) of *rose coco* beans, and one week after germination, they were thinned to one plant per experimental unit. The twenty four *rose coco* bean plants per plot were retained after thinning. The first weeding was carried out on the greenhouse at two weeks after emergence. The second weeding was carried out four weeks later. The third weeding was done before flowering six weeks later, no other supplement were added either by top dressing or foliar spray.

3.2.5 Data collection

The harvesting of each *rose coco* plant was carried out after 85 days in stages to ensure pods mature well and it was done in a timely manner to avoid self-explosion, the harvest was placed in labeled white polythene bags. The yield of each harvest was weighed using sensitive weighing scale. Careful recording of the harvest was done to ensure correct yield for each *rose coco* plant crop was captured. The labeling was to assist for easy administration and management of the plants and the yields. The bean pods were shelled and grains dried to 12.5%

moisture content, after which weighing was done for each *rose coco* plant and yield recorded in grams for further analysis.

3.2.6 Method of estimating the parameters

In order to get the most efficient result in the approximation of polynomials the proper experimental design must be used to collect data. Once the data are collected, the method of least squares was used to estimate the parameters in the polynomials. The representation $\mathbf{y} = \mathbf{x}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ was given where $\mathbf{y}$ is a vector of observations, $\boldsymbol{\varepsilon}$ is the vector of errors, $\mathbf{x}$ is the design matrix and $\boldsymbol{\beta}$ is a vector of unknown model coefficients. The design matrix was a set of combinations of the values of the coded variables, which specifies the settings of the design parameters to be performed during experimentation. The $\boldsymbol{\beta}$ could be estimated by using the least squares method as: $\boldsymbol{\beta} = (\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{y}$.

3.3 Obtaining the generalized variance of the estimated linear parameters

This section gives residual sum of squares and variance of parameter estimates.

3.3.1 Residual sum of squares and variance

The method of least squares produces an unbiased estimator of the parameter $\boldsymbol{\beta}$ in the multiple linear regression models. The important parameter was the sum of squares of the residuals

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} e_i^2 = \mathbf{e}'\mathbf{e} \tag{3.2}$$

Because $\mathbf{X}'\mathbf{X}\mathbf{b} = \mathbf{X}'\mathbf{y}$, we could have a computational formula for SSE:

$$\text{SSE} = \mathbf{y}'\mathbf{y} - \mathbf{b}'\mathbf{X}'\mathbf{y} \tag{3.3}$$

equation 3.3 is called the error or the residual sum of squares.

The unbiased estimator of σ^2 is

$$\sigma^2 = \frac{SSE}{n-p} \quad (3.4)$$

where n is a number of observations and p is a number of unknown parameters. Optimal experimental designs are a class of experimental designs that is optimal with respect to some statistical criterion of choice. The statistical criterion is in relation to any type of variance that is minimized, such as the generalized variance in the model parameter estimates or variance in predicted values of the response variable. Optimizing with respect to some statistical criterion allows the parameters to be estimated without bias with minimum variance or increase the precision of predicted values. The goal of optimal design was to eliminate the option of conducting a non-optimal experimental design, which would result in an increased number of experimental runs to estimate parameters with the same amount of precision compared to an optimal design (Wikipedia contributors). In general, each of the criteria deal with some aspect of either generalized variance of the model parameters or with the variance of predicted values.

3.4 Fitting and testing the three models adequacies

This section gives the design of fitting the first order model, the second order model, the model adequacy and residual analysis.

3.4.1 Design of fitting the first-order model

In most RSM problems, the true response function is unknown; we therefore need to approximate the response function. In order to develop an appropriate approximation for response function, we model the data by starting with a first-order polynomial. When the response could be defined by a linear function of independent variables, that is, there is no curvature then the approximating function is a first-order model. The linear regression model with k independent variables takes the form;

$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik} + \varepsilon_i$, (i = 1, 2, 3, ..., n)

$$y_i = \beta_0 + \sum_{i=1}^{k} \beta_i x_{ij} + \varepsilon_i \quad (3.5)$$

The parameter β_i measures the expected change in response y per unit increase in x_i when the other independent variables are held constant.

The eventual objective of RSM is to determine the optimum operating conditions for the system, or to determine a region of the factor space in which the operating specifications are satisfied. The general model relationship with $X_1, X_2, ..., X_k$ independent variables is given by

$$y = f(X_1, X_2, ..., X_k) + \varepsilon; \quad (3.6)$$

where the form of the true response function *f* is unknown, and *ε* is a term that represents other sources of variability not accounted for in *f*. Usually *ε* includes effects such as measurement error in the response, background noise, the effect of other variables, and so on. Usually *ε* is treated as a statistical error, often assuming it to have a normal distribution with mean zero and variance σ^2. Then

$$E(y) = \overline{y} = E[f(X_1, X_2, ..., X_k)] + E(\varepsilon) = f(X_1, X_2, ..., X_k); \quad (3.7)$$

The variables $X_1, X_2, ..., X_k$ in equation (3.7) are usually called the natural variables, because they are expressed in the natural units of measurement. In much RSM work it is convenient to transform the natural variables to coded variables $x_1, x_2, ..., x_k$, which are usually defined to be dimensionless with mean zero and the same standard deviation (Myers et al. 1989). In terms of the coded variables, the response function (3.7) is written as

$$\hat{y} = f(x_1, x_2, ..., x_k); \quad (3.8)$$

The successful use of RSM is critically dependent upon the experimenter's ability to develop a suitable approximation for *f*, because the form of the true response function *f* is unknown, we must approximate it. Usually a low-order polynomial in some relatively small region of the independent variable space is appropriate. In many cases, either a first-order or a second order model is used. The first-order model is likely to be appropriate when the experimenter is interested in approximating the true response surface over a relatively small region of the independent variable space in a location where there is little curvature in *f*. In the case of three independent variables, the first-order model in terms of the coded variables is

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3; \quad (3.9)$$

The form of the first-order model in equation (3.9) is sometimes called a main effects model, because it includes only the main effects of the three variables x_1, x_2 and x_3. If there is an interaction between these variables, it can be added to the model easily as follows:

$$\hat{y} = \beta o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 \quad (3.10)$$

This is the model with interaction. The addition of the interaction term introduces curvature into the response function. When the curvature in the true response surface is strong enough then the first-order model is inadequate. A second-order model is likely required in these situations.

The general expected first-order model is given by;

$$\hat{y} = \beta o + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k \quad (3.11)$$

The low and high factor settings are coded as negative and positive, the midpoint coded as 0. Finally, let's note that there is a close connection between RSM and linear regression analysis. For example, consider the model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon \quad (3.12)$$

The β's are a set of unknown parameters.

The experimenter tries to quantify the relationship between a set of k predictor variables $X' = (X_1, X_2, \ldots, X_k)$ and the response variable y. Often the goal of the experiment has been to maximize or minimize E(y), the expected value of the response. In most cases, the X_i are transformed into coded x_i by $(X_i - X_{i0})/(sc)_i$ i=1,2,…,k where X_{i0} and $(sc)_i > 0$ are the centering and scaling constants, respectively.

The model in equation 3.5 could be represented in matrix form as;

$$\mathbf{Y} = \mathbf{X\beta} + \boldsymbol{\varepsilon} \tag{3.13}$$

In which

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ . \\ . \\ . \\ \beta_k \end{bmatrix} \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ . \\ \varepsilon_n \end{bmatrix} \tag{3.14}$$

where,

Y is an (n × 1) vector of observations,

X is an (n × k) design matrix,

β is a (k × 1) vector of unknown parameters, and

ε is a (n × 1) vector of independently identically distributed random variables with mean zero and variance σ^2, (Montgomery 2005).

When the **X'X** is invertible, that is, there's a determinant, then the linear system of equation (3.13) has a unique least squares solution given by;

$$\hat{\beta} = (\mathbf{X'X})^{-1}\mathbf{X'y} \tag{3.15}$$

The total sum of squares is

$$\text{SST} = y'y - \frac{(\sum_{i=1}^{n} y_i)^2}{n} = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n} \tag{3.16}$$

3.4.2 The second-order model

With the purpose of exploring the second-order model, we intend to use a statistical modeling to develop an appropriate approximating relationship between the yield and the process variables: the *nitrogen, phosphorus* and *potassium* of inorganic fertilizers. We fitted a first order model, and then we justify it with the second-order model. The second-order model is very flexible; consequently we could experiment with a wide variety of functional forms. The method of least squares was used to estimate the parameters. Once an appropriate approximating model was obtained, it was analyzed to determine the optimum conditions. When we found curvature in the response surface the first-order model was insufficient. The second-order model was useful in approximating a portion of the true response surface with curvature. The second-order model includes all the terms in the first-order model, and quadratic and cross product terms. It is usually represented as

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \beta_{11} x_1^2 + \ldots + \beta_{kk} x_k^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \ldots + \beta_{k-1,k} x_{k-1} x_k + \varepsilon$ or

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i}^{k} \beta_{ii} x\, x_i^2 + \sum\sum_{i<j} \beta_{ij} x_i x_j + \varepsilon \tag{3.17}$$

where β_i is an unknown parameter and ε is a random error.

The general expected second-order model is given by;

$$\bar{y} = \beta_0 + \sum_{i}^{k} \beta_i x_i + \sum_{i}^{k} \beta_{ii} x_i^2 + \sum \sum_{i<j=2}^{k} \beta_{ij} x_i x_j \tag{3.18}$$

where $\overline{y}$ is the measured response, β_0 is the intercept term, β_i are the linear coefficients, β_{ij} is the logarithmic coefficient, β_{ii} are the quadratic coefficients and x_i are the coded independent variables.

In the case of three variables, the second-order model is given by:

$$\hat{y} = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x^2_1 + \beta_{22} x^2_2 + \beta_{33} x^2_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 \quad (3.19)$$

Where, $\hat{y}$ is the response variable (dependent variable), β_0 is intercept (constant), β_1, β_2 and β_3 are linear coefficients, β_{12}, β_{13}, and β_{23} are interaction coefficients, β_{11}, β_{22} and β_{33} are quadratic coefficients and x_1, x_2, x_3, x^2_1, x^2_2, x^2_3, x_1x_2 , x_1x_3 and x_2x_3 are the level of coded independent variables. This model would likely be useful as an approximation to the true response surface in a relatively small region. The second-order model is widely used in response surface methodology for several reasons:

1. The second-order model is very flexible. It can take on a wide variety of functional forms, so it often works well as an approximation to the true response surface.

2. It is easy to estimate the parameters (the β's) in the second-order model. The method of least squares could be used for this purpose.

3. There is considerable practical experience indicating that second-order models work well in solving real response surface problems.

Often, a second order model fit to the experimental data, including all linear, quadratic and cross product terms for the x_i. The second order model, usually fits by ordinary least squares is represented for a single response y_u, u=1,…, N . Experimental data were fitted to, a first and second-order polynomial model to obtain the regression coefficients. The yield of *rose coco* beans was taken as the dependent response variable. The data obtained from *rose coco* on response variables were subjected to the analysis of variance (ANOVA). The mean values of the triplicate trials were fitted to a second-order polynomial according to twenty four point second order rotatable design.

3.4.3 Model adequacy checking

The goodness of the fit of the developed regression models was tested. It is always necessary to: 1. Examine the fitted model to ensure that it provides an adequate approximation to the true system; 2. Verify that none of the least squares regression assumptions are violated. We consider several techniques for checking model adequacy this includes: the lack-of-fit significance, coefficient of determination, R^2, coefficient of variation (CV), mean square error (MSE) and model significance were used to judge adequacy of model fit. The statistical significance of the model equation was determined by Fisher's test value (p value) and significance of each coefficient was determined using t-test, and the extent of variance that could be explained by the model was determined by the multiple coefficient of determination, R squared (R^2) value, this assess the fitness of the polynomial model (Myers et al, 2008). When R^2 approaches unity, the better the empirical model fits the actual data. The smaller the value of R^2, the less relevant the dependent variables in the model have in explaining the behavior variation (Little and Hill, 1978). In general, the R^2 measures percentage of the variation of y around $\overline{y}$ that is explained by the regression equation.

Then the coefficient of multiple determination, R^2 is defined as

$$R^2 = 1 - \frac{SS_E}{SS_T} \tag{3.20}$$

From inspection of the analysis of variance identity (equation 3.20) we could see that $0 \leq R^2 \leq 1$. One can gauge the fit of the model using the R^2, which is usually defined as the proportion of variance of the response that could be explained by the independent variables. The higher values of R^2 are generally taken to indicate a better model. In fact, though, when the R^2 is low there are two possible conclusions. The first is that the independent variables and the dependent variables have very little relation to each other; the second is that the model fit is poor. Further, there are a number of possible causes of this poor fit, some outliers; an incorrect form of the independent variables; may be even incorrect assumptions about the errors. The errors may not be normally distributed, or indeed the independent variables may not be fixed. To confound the issues further, the low R^2 may be any combination of the above aspects (Simpson P. et al, 2004). According to Myers et al (2008), for a good fit of a model, the correlation coefficient should be at least 0.80.

3.4.4 The residual analysis

The residuals from the least squares fit, defined by $e_i = y_i - \hat{y}_i$, i = 1, 2,…, n, play an important role in judging model adequacy. The PRESS computes and displays the predicted residual sum of squares (PRESS) statistic for each dependent variable in the model. The large difference between the ordinary residual and the PRESS residual indicates a point where the model fits the data well, but a model built without that point predicts poorer. The significant terms in the model was found by analysis of variance (ANOVA) for each response. Significance was judged by determining the probability level that the F-statistic calculated from the data is less than 5%. The model adequacies were checked by R^2 and the prediction error sum of squares (PRESS). A good model has a large predicted R^2, and a low PRESS. This analysis included examining diagnostic plots such as normal probability plots. To analyze the multiple regression and variance, develop a regression equation between variables and response and numerically optimize the procedure; the SAS statistical package (version 9.0, SAS Institute Inc., Cary, NC, USA) was employed.

3.5 The settings of the experimental factors that produces the optimal response

The experiment was run using the optimum values for the variables given by response optimization in order to validate the predicted optimum values of variable response of the *rose coco*. The *rose coco* beans were labeled in the greenhouse to facilitate the identification during the entire procedure. The average yield was computed for GP1G, GP2G and GP3G. The SAS software was used to generate the critical value of the responses and also to generate response surfaces, while holding one variable constant in the second-order polynomial model. The R-software was used to come up with contour plots. Canonical analysis (Khuri and Cornell, 1996) was carried out to determine the location and nature of the stationary point of the model. When the results showed a saddle point in response surfaces the ridge analysis of SAS RSREG procedure was employed to compute the estimated ridge of the maximum response for increasing radii from the center of original design. The fitted polynomial equations were generated to response surface and contour plots so as to visualize the relationship between the response and experimental levels of each factor. The regression analysis and contour graphs were obtained by using SAS software (Version 9.3, SAS Institute, Cary, NC) and R-software.

3.5.1 Coding the factor variables

For the results of the canonical and ridge analyses to be interpretable, the values of different factor variables should be comparable. This is because the canonical and ridge analyses of the response surface are not invariant with respect to differences in scale and location of the factor variables. The analysis of variance is not affected by these changes. Although the actual predicted surface does not change, its parameterization does. The usual solution to this problem is to code each factor variable so that its minimum in the experiment is –1 and its maximum is 1 and to carry through the analysis with the coded values instead of the original ones. This practice has the added benefit of making 1 a reasonable boundary radius for the ridge analysis since 1 represents approximately the edge of the experimental region. By default, PROC RSREG computes the linear transformation to perform this coding as the data are initially read in, and the canonical and ridge analyses are performed on the model fit to the coded data. The actual form of the coding operation for each value of a variable is given by:

$$x_{iu} = \frac{\psi_{iu} - \psi_{i\bullet}}{s_i}, \qquad \psi_{i\bullet} = \frac{\sum_{u=1}^{N} \psi_{iu}}{N} \quad \text{and} \quad s_i = \left[\frac{\sum_{u=1}^{N} (\psi_{iu} - \psi_{i\bullet})^2}{N} \right]^{\frac{1}{2}} \tag{3.21}$$

$$\psi_{iu} = x_{iu} s_i + \psi_{i\bullet}$$

For $\sum_{u=1}^{N} x_{iu}^2 = N$ and $\sum_{u=1}^{N} x_{iu} = 0$.

The second-order models illustrate quadratic surfaces such as minimum, maximum and saddle. The graphical visualization is very helpful in understanding the second-order response surface. Specifically, contour plots can help characterize the shape of the surface and locate the optimum response roughly.

3.5.2 The stationary point

The sign of the stationary point is determined from the signs of the eigenvalues of the matrix **B**. The standard quadratic model could be written in matrix notation as:-

$$y = \hat{\beta}_0 + \mathbf{x'b} + \mathbf{x'Bx} + \varepsilon \tag{3.22}$$

where $\mathbf{x}$ is a fixed combination of the levels of the k input variables, $\hat{\beta}_0$, $\mathbf{b}$ and $\mathbf{B}$ contains estimates of the intercept, linear and second order coefficients, respectively.

Where $\mathbf{b} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} \hat{\beta}_{11} & \frac{1}{2}\hat{\beta}_{12} & \cdots & \frac{1}{2}\hat{\beta}_{1k} \\ \frac{1}{2}\hat{\beta}_{12} & \hat{\beta}_{22} & \cdots & \frac{1}{2}\hat{\beta}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2}\hat{\beta}_{1k} & \frac{1}{2}\hat{\beta}_{2k} & \cdots & \hat{\beta}_{kk} \end{bmatrix}$ (3.23)

x'= $[x_1, x_2, ..., x_k]$ and **B** is the k x k symmetric matrix and ε is the random error. The stationary point is the combination of design variables where the surface is at either a maximum or a minimum in all directions. If the stationary point is a maximum in some direction and minimum in another direction, then the stationary point is a *saddle point*. When the surface curves in one direction, but is fairly constant in another direction, then this type of surface is called *ridge* system (Oehlert, 2000). The stationary point could be found by using matrix algebra. The fitted second-order model in matrix form is as follows:

$$\hat{y} = \hat{\beta}_0 + \mathbf{x'b} + \mathbf{x'Bx} \tag{3.24}$$

The derivative of $\hat{y}$ with respect to the elements of the vector $\mathbf{x}$ is

$$\frac{\partial \hat{y}}{\partial x} = \mathbf{b} + 2\mathbf{B}x \tag{3.25}$$

Therefore, setting the derivative vector to 0 yields the stationary point of the system:

$$x_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b} \tag{3.26}$$

This might be a maximum, minimum, or a saddle point of the fitted surface. The eigenvalues (call them λs) and eigenvectors of **B** are the key to characterizing the shape. The $\mathbf{x}_s$ are a point of maximum if all λ's are negative, the point of minimum if all λ's are positive and saddle point if λ's are of mixed sign.

$$\text{Where } \mathbf{b} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \hat{\beta}_{11} & \frac{1}{2}\hat{\beta}_{12} & \frac{1}{2}\hat{\beta}_{13} \\ & \hat{\beta}_{22} & \frac{1}{2}\hat{\beta}_{23} \\ sym & & \hat{\beta}_{33} \end{bmatrix} \tag{3.27}$$

b is a (3x1) vector of the first-order regression coefficients and **B** is a (3x3) symmetric matrix whose main diagonal elements are the quadratic coefficients and whose off-diagonal elements are one-half the mixed quadratic coefficients (Montgomery, 2005). The estimated response value at the stationary point is

$$\hat{y}_s = \hat{\beta}_0 + \frac{1}{2} x_s^{'} \mathbf{b} \tag{3.28}$$

3.5.3 The canonical analysis

Canonical analysis is a mathematical approach used to examine the overall shape of the response surface and to determine if the estimated response point is a maximum, minimum or a saddle point. If the stationary point is a maximum or minimum, a corresponding increase or decrease results in the response. In the case of a saddle point, the response may increase or decrease when we move away from the stationary point, depending on which direction is taken. The eigenvalues and eigenvectors of the matrix of second-order characterize the shape of the response surface. The eigenvectors point in the directions of principal orientation of the surface, and the signs and magnitudes of the associated eigenvalues give the shape of the surface in these directions. Positive eigenvalues indicate directions of upward curvature, and negative eigenvalues indicate directions of downward curvature. The larger an eigenvalue is in absolute value, the more pronounced is the curvature of the response surface in the associated direction. Often, all the coefficients of an eigenvector except for one are relatively small, indicating that the vector points roughly along the axis associated with the factor corresponding to the single large coefficient. In this case, the canonical analysis could be used to determine the relative sensitivity of the predicted response surface to variations in that factor (SAS user guide 2013).

3.5.4 The ridge analysis

Ridge analysis in RSM is a contour-based technique for investigating a quadratic response surface. It is a search for a new stationary point S_R on a given radius R such that the second order model has a minimum at this stationary point. Then, the maximum or minimum response value at different locations from the design center could be determined by comparing each 'constrained' stationary point (Myers and Montgomery 2002 and Khuri and Cornell 1996). This procedure of RIDGE was conducted by the SAS program (SAS, version 9.0). A RIDGE statement computes the ridge of optimum response. The ridge starts at a given point x_0, and the point on the ridge at radius R from x_0 is the collection of factor settings that optimizes the predicted response at this radius. The ridge analysis could be used as a tool to help interpret an existing response surface or to indicate the direction in which further experimentation should be performed. The default starting point, x_0, has each coordinate equal to the point midway between the highest and lowest values of the factor in the design. The default radii at which the ridge is computed are 0, 0.1, ..., 0.9, 1. If the ridge analysis is based on the response surface fit to coded values for the factor variables, then these results in a ridge that starts at the point with a coded zero value for each coordinate and extends toward, but not beyond, the edge of the range of experimentation. Alternatively, both the center point of the ridge and the radii at which it is to be computed could be specified. The starting point should be well inside the range of experimentation. The coded radii give the distances from the ridge starting point at which to compute the optimal (SAS user guide 2013).

CHAPTER 4: RESULTS AND DISCUSSIONS

4.1 Introduction

This chapter presents the results and discussions of the study in line with the four specific objectives.

4.2 Estimation of the linear parameters in an existing A-optimum and D-efficient calculus optimum value second order rotatable design

This section gives the data collected on *rose coco* beans, twenty four points rotatable design of coded levels and natural levels with the yields of *rose coco* beans for GP1G, GP2G and GP3G.

Table 4. 1: The yield of 30 *rose coco* beans with straight fertilizers: nitrogen, phosphorus and potassium.

	ROSE COCO YIELD UNDER INDIVIDUAL FERTILIZER: N, P, K.		
FERTILIZERS IN GRAMS	Potassium (K)	Phosphorus (P)	Nitrogen (N)
10	38	52	25
20	30	65	25
30	30	45	25
40	30	40	24
50	50	70	20
60	25	48	12
70	25	50	20
80	25	40	25
90	20	42	26
100	15	45	15

The table 4.1 shows the *rose coco* yield of the straight fertilizers, nitrogen, phosphorus and potassium. Each of the fertilizers was initially started with 10grams then increased by 10grams up to 100grams for each *rose coco* plant.

Table 4. 2: Descriptive Analysis of straight fertilizers of N, P and K on *rose coco* beans mean yield.

Descriptives

Weight *rose coco*

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Potassium (K)	10	28.8000	9.71597	3.07246	21.8496	35.7504	15.00	50.00
Phosphorus (P)	10	49.7000	10.25291	3.24226	42.3655	57.0345	40.00	70.00
Nitrogen(N)	10	21.7000	4.85455	1.53514	18.2273	25.1727	12.00	26.00
Total	30	33.4000	14.67487	2.67925	27.9203	38.8797	12.00	70.00

Table 4.2 shows that phosphorus has more influence on the rose coco beans followed by potassium then nitrogen.

Table 4. 3: Analysis of variance for the yield of *rose coco* beans on straight fertilizers of N, P and K of table 4.1.

ANOVA

Weight rosecoco

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	4237.400	2	2118.700	28.491	.000
Within Groups	2007.800	27	74.363		
Total	6245.200	29			

In table 4.3 the p=0. 000 is less than α=0. 05, therefore, we reject the null hypothesis that states that the nitrogen, phosphorus and potassium has the same effects on *rose coco* beans and conclude that the treatments differ significantly, meaning that all the three fertilizers do not have the same effects on the *rose coco* beans.

Means Plots

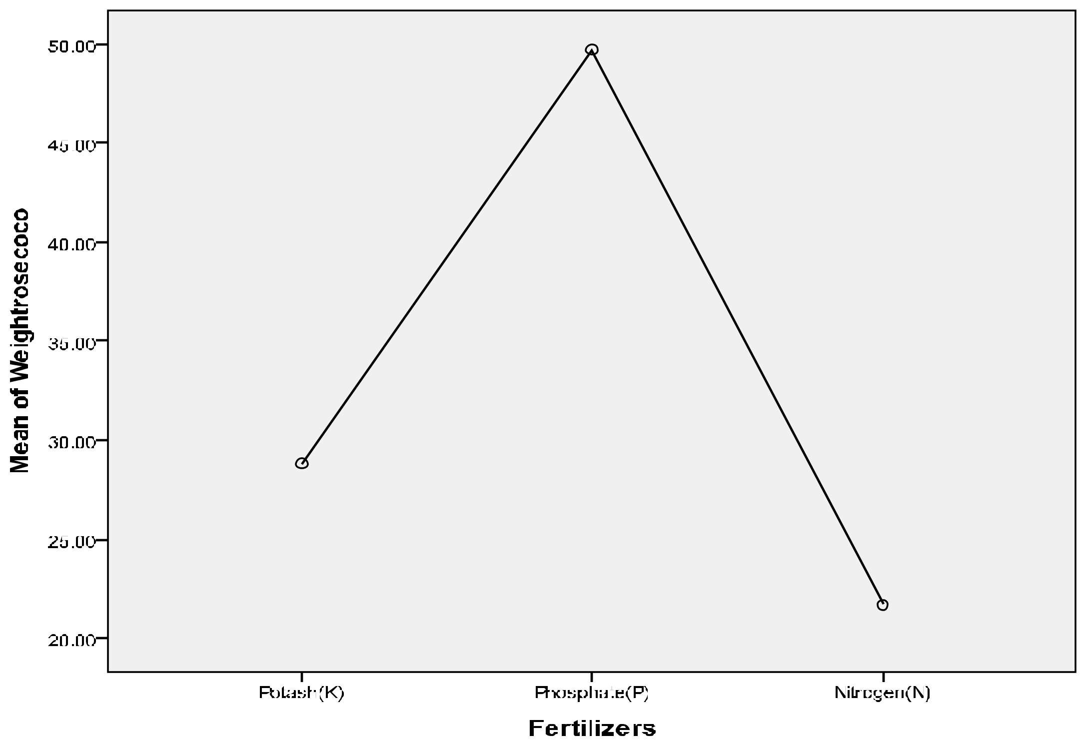


Figure 4. 1: The plot of mean of yield of *rose coco* beans under each of the three fertilizers; N, P, K.

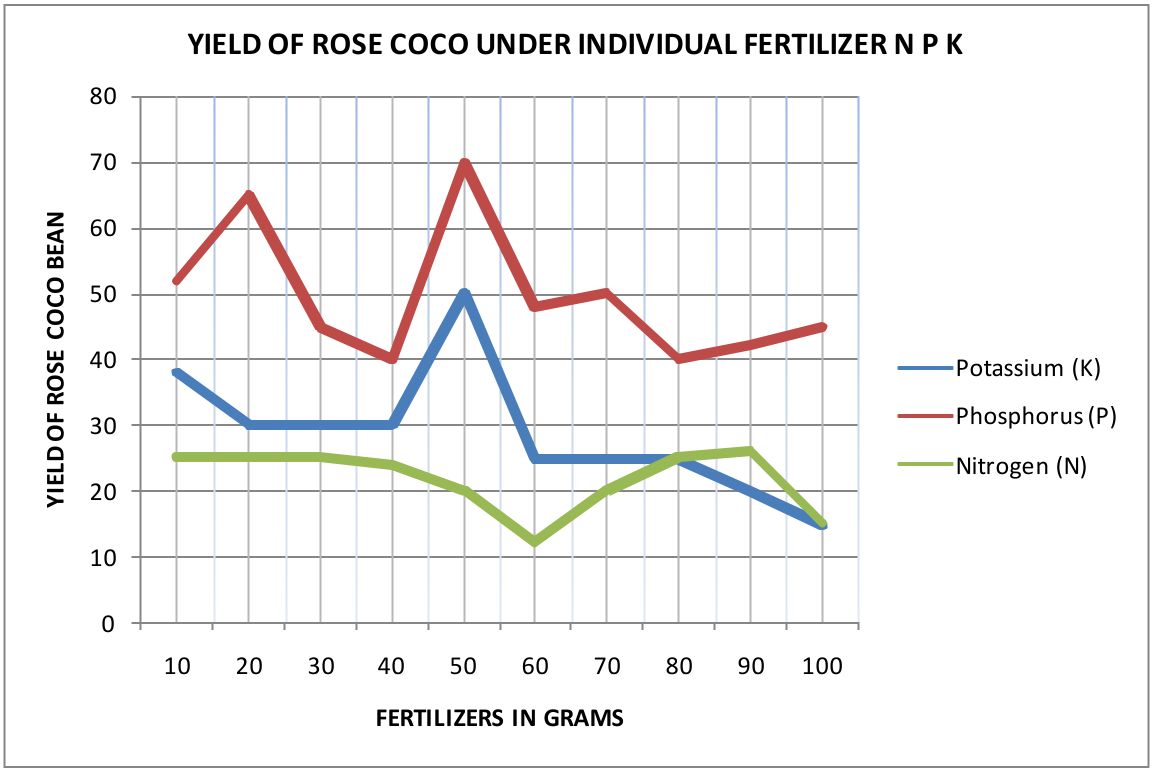


Figure 4. 2: The yield of *rose coco* beans under each of the three straight fertilizers; N, P, K of table 4.1.

Figure 4.2 shows the effect of individual fertilizers applied to *rose coco* beans. The fertilizers were applied once during the planting period and no other kind of fertilizers was applied after the initial application. Weeding and pest control was done. The phosphorus fertilizer had more influence on *rose coco* bean followed by potassium and then nitrogen.

4.2.2 Twenty four points rotatable design of coded levels and natural levels with the yields of *rose coco* beans-GP1G

The x_{1u}, x_{2u} and x_{3u} are coded values while Ψ_{1u}, Ψ_{2u} and Ψ_{3u} are natural values-GP1G. The natural values (Ψ_{iu}) of fertilizers N, P, K, were measured using a sensitive weighing scale and applied to *rose coco* beans in a greenhouse which gave the observed yield-y_i. Predicted yield $\hat{y}$ was calculated using a second order model of GP1G (equation 4.54).

Table 4. 4: The 24-point's rotatable design of coded levels and natural levels with the yield of *rose coco* beans-GP1G at the ratio 10:20:30 of N: P: K fertilizers.

(x_{1u}	x_{2u}	x_{3u})	Ψ_{1u}	Ψ_{2u}	Ψ_{3u}	Observed Yield- y_i	Predicted yield- $\hat{y}$
1.1072569	1.1072569	0	10.553628	20.332177	30	63	56.4342
-1.1072569	1.1072569	0	9.446372	20.332177	30	71	61.1068
1.1072569	-1.1072569	0	10.553628	19.667823	30	35	35.8807
-1.1072569	-1.1072569	0	9.446372	19.667823	30	56	53.5533
1.1072569	0	1.1072569	10.553628	20	31.107257	37	44.3546
-1.1072569	0	1.1072569	9.446372	20	31.107257	62	69.5273
1.1072569	0	-1.1072569	10.553628	20	28.892743	57	55.7932
-1.1072569	0	-1.1072569	9.446372	20	28.892743	54	52.9659
0	1.1072569	1.1072569	10	20.332177	31.107257	52	55.3883
0	-1.1072569	1.1072569	10	19.667823	31.107257	52	47.8349
0	1.1072569	-1.1072569	10	20.332177	28.892743	53	59.3270
0	-1.1072569	-1.1072569	10	19.667823	28.892743	40	38.7735
0.7829487	0	0	10.391474	20	30	43	45.8615
-0.7829487	0	0	9.608526	20	30	58	53.7617
0	0	0.7829487	10	20	30.782949	58	50.0108
0	0	-0.7829487	10	20	29.217051	42	48.1996
0	0.7829487	0	10	20.234885	30	48	52.1156
0	-0.7829487	0	10	19.765115	30	44	42.1783

1.2735263	0	0	10.636763	20	30	50	47.3699
-1.2735263	0	0	9.363237	20	30	53	60.2203
0	0	1.2735263	10	20	31.273526	60	53.3991
0	0	-1.2735263	10	20	28.726474	56	50.4532
0	1.2735263	0	10	20.382058	30	52	54.8270
0	-1.2735263	0	10	19.617942	30	32	38.6632

4.2.2.1The three replication yield of twenty four points second order rotatable design, Group 1-GP1G

Table 4.5 below shows the three replication yields of twenty four points *rose coco* beans (from table 4.4) under fertilizers 10grams of nitrogen, 20grams of phosphorus and 30grams of potassium.

Table 4. 5: The yield of 24-points *rose coco* beans (from table 4.4) under fertilizers at the ratio of 10grams of nitrogen, 20grams of phosphorus and 30grams of potassium

N:P:K

10:20:30

Bean No.	GP1GA Yield	GP1GB Yield	GP1GC Yield	AVERAGE YIELD GP1G
1	41	48	100	63
2	13	170	30	71
3	20	35	50	35
4	20	88	60	56
5	63	20	28	37

6	76	70	40	62
7	76	60	35	57
8	42	75	45	54
9	84	32	40	52
10	38	58	60	52
11	34	60	65	53
12	50	40	30	40
13	27	70	32	43
14	70	70	34	58
15	_	36	80	58
16	83	15	28	42
17	56	50	38	48
18	32	50	50	44
19	53	55	42	50
20	65	42	52	53
21	42	98	40	60
22	28	95	45	56
23	41	40	75	52
24	41	30	25	32
	GP1G-10grams nitrogen, 20grams phosphorus, 30grams potassium			

Blank space indicated by a dash, the *rose coco* plant died so no yield was obtained.

4.2.3 Twenty four points rotatable design of coded levels and natural levels with the yield of rose coco beans-GP2G

The x_{1u}, x_{2u} and x_{3u} are coded values while Ψ_{1u}, Ψ_{2u} and Ψ_{3u} are natural values-GP2G. The natural values (Ψ_{iu}) of fertilizers N, P, K, were measured using a sensitive weighing scale and planted in a greenhouse which gave the observed yield-y_i. Predicted yield $\hat{y}$ was calculated using a second order model of GP2G (equation 4.62).

Table 4. 6: The 24-point's rotatable design of coded levels and natural levels with the yield of *rose coco* beans GP2G at the ratio of 20:30:40 N: P: K fertilizers.

$(x_{1u}$	x_{2u}	$x_{3u})$	Ψ_{1u}	Ψ_{2u}	Ψ_{3u}	Observed Yield- y_i	Predicted yield- $\hat{y}$
1.1072569	1.1072569	0	20.553628	30.332177	40	49	53.3803
-1.1072569	1.1072569	0	19.446372	30.332177	40	30	35.1918
1.1072569	-1.1072569	0	20.553628	29.667823	40	44	39.9260
-1.1072569	-1.1072569	0	19.446372	29.667823	40	55	51.7374
1.1072569	0	1.1072569	20.553628	30	41.107257	59	55.9145
-1.1072569	0	1.1072569	19.446372	30	41.107257	53	45.7260
1.1072569	0	-1.1072569	20.553628	30	38.892743	37	42.8240
-1.1072569	0	-1.1072569	19.446372	30	38.892743	45	46.6354
0	1.1072569	1.1072569	20	30.332177	41.107257	64	58.5578
0	-1.1072569	1.1072569	20	29.667823	41.107257	56	57.6034
0	1.1072569	-1.1072569	20	30.332177	38.892743	57	49.9673
0	-1.1072569	-1.1072569	20	29.667823	38.892743	54	54.0129

0.7829487	0	0	20.391474	30	40	34	37.6742
-0.7829487	0	0	19.608526	30	40	39	35.4195
0	0	0.7829487	20	30	40.782949	37	43.6884
0	0	-0.7829487	20	30	39.217051	42	39.3817
0	0.7829487	0	20	30.234885	40	47	39.6306
0	-0.7829487	0	20	29.765115	40	47	40.7235
1.2735263	0	0	20.636763	30	40	44	39.0672
-1.2735263	0	0	19.363237	30	40	30	35.3998
0	0	1.2735263	20	30	41.273526	46	53.9337
0	0	-1.2735263	20	30	38.726474	46	46.9286
0	1.2735263	0	20	30.382058	40	41	45.9492
0	-1.2735263	0	20	29.617942	40	41	47.7269

4.2.3.1 The three replication yield of twenty four points second order rotatable design, Group 2-GP2G

Table 4.7 below shows the three replication yields of twenty four points rotatable design of *rose coco beans* (from table 4.6) under fertilizers at the ratio of 20grams of nitrogen, 30 grams of phosphorus and 40 grams of potassium.

Table 4. 7: The yield of 24-points *rose coco* beans (from table 4.6) under fertilizers at the ratio of 20 grams of nitrogen, 30 grams of phosphorus and 40 grams of potassium

N:P:K

20:30:40

Bean No.	GP2GA	GP2GB	GP2GC	AVERAGE YIELD GP2G
1	42	60	45	49
2	47	18	25	30
3	40	62	30	44
4	60	35	70	55
5	36	82	_	59
6	51	50	58	53
7	69	22	20	37
8	73	20	42	45
9	75	72	45	64
10	58	58	52	56
11	51	65	55	57
12	72	50	40	54
13	22	32	48	34
14	50	32	35	39
15	43	18	50	37
16	36	35	55	42
17	46	55	40	47
18	56	35	50	47
19	32	58	42	44
20	30	40	20	30
21	77	21	40	46

22	60	60	18	46
23	44	38	41	41
24	31	60	32	41
	GP2G-20 grams Nitrogen, 30 grams Phosphorus, 40 grams Potassium			

Blank space indicated by a dash, the *rose coco* plant died, so no yield was obtained.

4.2.4 Twenty four points rotatable design of coded levels and natural levels with the yield of rose coco beans-GP3G

The x_{1u}, x_{2u} and x_{3u} are coded values while Ψ_{1u}, Ψ_{2u} and Ψ_{3u} are natural values-GP3G. The natural values (Ψ_{iu}) of fertilizers N, P, K, were measured using a sensitive weighing scale and planted in a greenhouse which gave the observed yield-y_i. Predicted yield $\hat{y}$ was calculated using a second order model of GP3G (equation 4.67).

Table 4. 8: The 24-point's rotatable design of coded levels and natural levels with the yield of *rose coco* beans-GP3G at the ratio of 30:40:50 N: P: K fertilizers.

(x_{1u}	x_{2u}	x_{3u})	Ψ_{1u}	Ψ_{2u}	Ψ_{3u}	Observed Yield- y_i	Predicted yield- $\hat{y}$
1.1072569	1.1072569	0	30.553628	40.332177	50	47	53.3952
-1.1072569	1.1072569	0	29.446372	40.332177	50	82	75.6594
1.1072569	-1.1072569	0	30.553628	39.667823	50	68	75.4937
-1.1072569	-1.1072569	0	29.446372	39.667823	50	50	44.7579
1.1072569	0	1.1072569	30.553628	40	51.107257	69	63.6074
-1.1072569	0	1.1072569	29.446372	40	51.107257	53	59.8717
1.1072569	0	-1.1072569	30.553628	40	48.892743	87	78.5079

-1.1072569	0	-1.1072569	29.446372	40	48.892743	70	73.7722
0	1.1072569	1.1072569	30	40.332177	51.107257	67	60.8035
0	-1.1072569	1.1072569	30	39.667823	51.107257	73	67.9020
0	1.1072569	-1.1072569	30	40.332177	48.892743	84	86.7041
0	-1.1072569	-1.1072569	30	39.667823	48.892743	67	70.8025
0.7829487	0	0	30.391474	40	50	60	55.2395
-0.7829487	0	0	29.608526	40	50	51	52.2443
0	0	0.7829487	30	40	50.782949	51	53.2638
0	0	-0.7829487	30	40	49.217051	72	63.4465
0	0.7829487	0	30	40.234885	50	51	56.6047
0	-0.7829487	0	30	39.765115	50	54	53.4924
1.2735263	0	0	30.636763	40	50	57	59.4536
-1.2735263	0	0	29.363237	40	50	55	54.5818
0	0	1.2735263	30	40	51.273526	55	60.9417
0	0	-1.2735263	30	40	48.726474	75	77.5047
0	1.2735263	0	30	40.382058	50	63	63.0059
0	-1.2735263	0	30	39.617942	50	58	57.9435

4.2.4.1 The three replication yield of twenty four point second order rotatable design, Group 3-GP3G

Table 4.9 shows the three replication yields of twenty four *rose coco* beans (from table 4.8) under fertilizers at the ratio of 30 grams of nitrogen, 40 grams of phosphorus and 50 grams of potassium.

Table 4. 9: The yield of 24-points *rose coco* bean (from table 4.8) under fertilizers at the ratio of 30 grams of Nitrogen, 40 grams of Phosphorus and 50 grams of potassium

N:P:K

30:40:50

Bean No.	GP3GA	GP3GB	GP3GC	AVERAGE YIELD GP3G
1	61	40	40	47
2	136	40	70	82
3	84	80	40	68
4	72	38	40	50
5	77	80	50	69
6	67	48	44	53
7	93	80	88	87
8	82	60	68	70
9	21	100	80	67
10	80	58	81	73
11	100	80	72	84
12	_	72	62	67
13	78	70	32	60
14	58	65	30	51
15	71	50	32	51
16	64	74	78	72

17	78	50	25	51
18	67	55	40	54
19	60	75	36	57
20	32	68	65	55
21	65	55	45	55
22	60	125	40	75
23	67	80	42	63
24	64	80	30	58
	GP3G-30grams nitrogen, 40grams phosphorus, 50grams potassium			

Blank space indicated by a dash, the *rose coco* plant died, so no yield was obtained.

Below is a figure 4.3 that shows the summary graph for the twenty four *rose coco* beans average yield for the three groups.

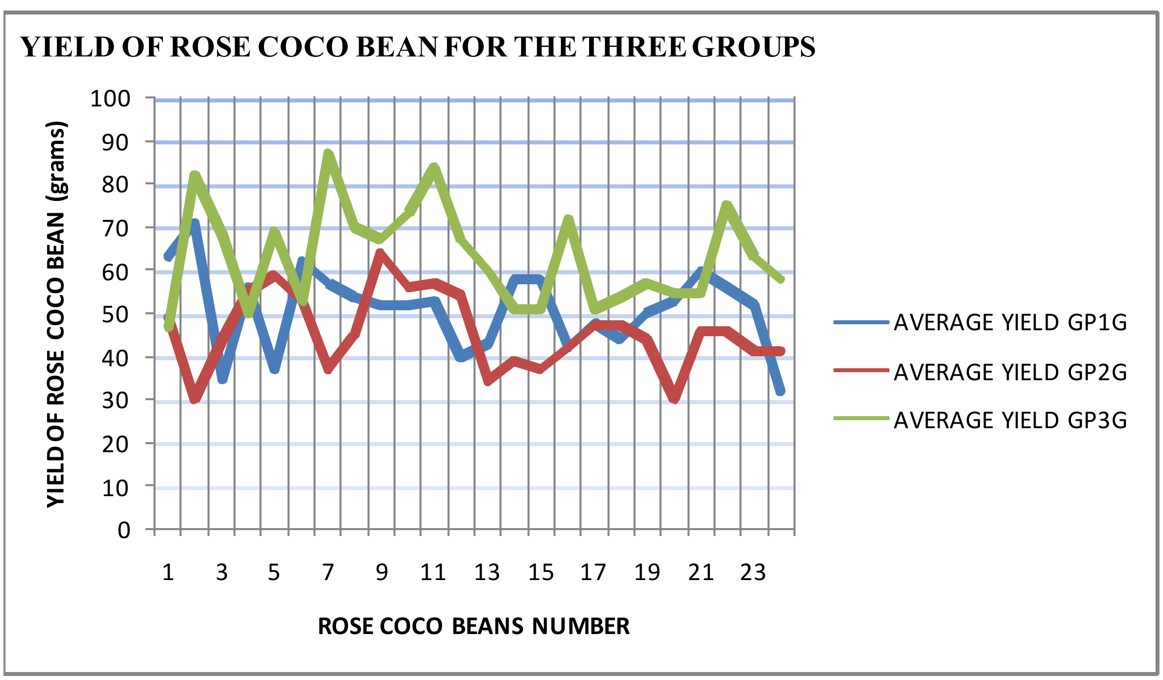


Figure 4. 3: The average yield of rose coco bean for the three groups.

We saw in figure 4.3 that GP3G has the highest yield, the two groups GP1G and GP2G are more less the same, GP1G being slightly higher than GP2G.

4.2.5 Parameter estimation for the first order model

This section gives linear parameters of first order models for GP1G, GP2G and GP3G. The design matrix **X** is given as in equation (3.14) and the equation (4.1) is obtained.

$$X = \begin{array}{c} \\ \end{array}\begin{matrix} x_0 & x_1 & x_2 & x_3 \end{matrix}$$

$$X = \begin{bmatrix} 1 & 1.1072569 & 1.1072569 & 0 \\ 1 & -1.1072569 & 1.1072569 & 0 \\ 1 & 1.1072569 & -1.1072569 & 0 \\ 1 & -1.1072569 & -1.1072569 & 0 \\ 1 & 1.1072569 & 0 & 1.1072569 \\ 1 & -1.1072569 & 0 & 1.1072569 \\ 1 & 1.1072569 & 0 & -1.1072569 \\ 1 & -1.1072569 & 0 & -1.1072569 \\ 1 & 0 & 1.1072569 & 1.1072569 \\ 1 & 0 & -1.1072569 & 1.1072569 \\ 1 & 0 & 1.1072569 & -1.1072569 \\ 1 & 0 & -1.1072569 & -1.1072569 \\ 1 & 0.7829487 & 0 & 0 \\ 1 & -0.7829487 & 0 & 0 \\ 1 & 0 & 0 & 0.7829487 \\ 1 & 0 & 0 & -0.7829487 \\ 1 & 0 & 0.7829487 & 0 \\ 1 & 0 & -0.7829487 & 0 \\ 1 & 1.2735263 & 0 & 0 \\ 1 & -1.2735263 & 0 & 0 \\ 1 & 0 & 0 & 1.2735263 \\ 1 & 0 & 0 & -1.2735263 \\ 1 & 0 & 1.2735263 & 0 \\ 1 & 0 & -1.2735263 & 0 \end{bmatrix} \qquad (4.1)$$

The first-order model with three independent variables was expressed as in equation (3.9). On fitting the model in equation (3.9) to the data, the parameter estimates are derived.

4.2.5.1 Linear parameters of first-order model for GP1G

The model parameter estimates are calculated using the least squares method. In matrix notation, the least squares estimates of the model parameters are:-

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{Y}\,. \qquad (4.2)$$

In order to find the least squares estimates, calculate $(\mathbf{X'X})^{-1}$ and $\mathbf{X'y}$:

$$(X'X)=\begin{bmatrix}24 & 0 & 0 & 0\\ 0 & 14.2779 & 0 & 0\\ 0 & 0 & 14.2779 & 0\\ 0 & 0 & 0 & 14.2779\end{bmatrix}\quad (X'X)^{-1}=\begin{bmatrix}0.0417 & 0 & 0 & 0\\ 0 & 0.0700 & 0 & 0\\ 0 & 0 & 0.0700 & 0\\ 0 & 0 & 0 & 0.0700\end{bmatrix}\quad (X'Y)=\begin{bmatrix}1228\\ -72\\ 90.6\\ 16.5\end{bmatrix} \qquad (4.3)$$

Thus, the least squares estimates of the parameters for the experimental model are;

$$\hat{B} = (X'X)^{-1}X'Y_{GP1G} = \begin{bmatrix}51.1667\\ -5.0452\\ 6.3461\\ 1.1566\end{bmatrix} \qquad (4.4)$$

4.2.5.2 Linear parameters of first-order model for GP2G

We fit first-order model of GP2G with three independent variables. On fitting the model to the data, we derive the parameter estimates as indicated below. In order to find the least square estimates, we calculated $(\mathbf{X'X})^{-1}$ (given in equation 4.3) and $\mathbf{X'y}$ using equation (4.1), then equation (4.2) is given as follows:

$$(X'Y) = \begin{bmatrix} 1097 \\ 20.6 \\ -10 \\ 39.3 \end{bmatrix} \qquad \hat{B} = (X'X)^{-1}X'Y_{GP2G} = \begin{bmatrix} 45.7083 \\ 1.4399 \\ -0.6980 \\ 2.7503 \end{bmatrix} \tag{4.5}$$

4.2.5.3 Linear parameters of first-order model for GP3G

In order to find the parameter estimates for first-order model of GP3G, we calculated $(\mathbf{X'X})^{-1}$ (given in equation 4.3) and $\mathbf{X'y}$ using equation (4.1), then equation (4.2) is given as follows:

$$(X'Y) = \begin{bmatrix} 1519 \\ 27.3 \\ 28.4 \\ -92.8 \end{bmatrix} \qquad \hat{B} = (X'X)^{-1}X'Y_{GP3G} = \begin{bmatrix} 63.2917 \\ 1.9127 \\ 1.9876 \\ -6.5028 \end{bmatrix} \tag{4.6}$$

4.2.6 Parameter estimation for the second order model.

This section found the second order parameter estimates for GP1G, GP2G and GP3G. The method of least squares was also used to estimate the regression coefficients; the **X** below is the design matrix for *rose coco* bean.

$$
\begin{array}{cccccccccc}
x_0 & x_1 & x_2 & x_3 & x_1^2 & x_1^2 & x_3^2 & x_1x_2 & x_1x_3 & x_2x_3
\end{array}
$$

$$
X=\begin{bmatrix}
1 & 1.1072569 & 1.1072569 & 0 & 1.226018 & 1.226018 & 0 & 1.226018 & 0 & 0 \\
1 & -1.1072569 & 1.1072569 & 0 & 1.226018 & 1.226018 & 0 & -1.226018 & 0 & 0 \\
1 & 1.1072569 & -1.1072569 & 0 & 1.226018 & 1.226018 & 0 & -1.226018 & 0 & 0 \\
1 & -1.1072569 & -1.1072569 & 0 & 1.226018 & 1.226018 & 0 & 1.226018 & 0 & 0 \\
1 & 1.1072569 & 0 & 1.1072569 & 1.226018 & 0 & 1.226018 & 0 & 1.226018 & 0 \\
1 & -1.1072569 & 0 & 1.1072569 & 1.226018 & 0 & 1.226018 & 0 & -1.226018 & 0 \\
1 & 1.1072569 & 0 & -1.1072569 & 1.226018 & 0 & 1.226018 & 0 & -1.226018 & 0 \\
1 & -1.1072569 & 0 & -1.1072569 & 1.226018 & 0 & 1.226018 & 0 & 1.226018 & 0 \\
1 & 0 & 1.1072569 & 1.1072569 & 0 & 1.226018 & 1.226018 & 0 & 0 & 1.226018 \\
1 & 0 & -1.1072569 & 1.1072569 & 0 & 1.226018 & 1.226018 & 0 & 0 & -1.226018 \\
1 & 0 & 1.1072569 & -1.1072569 & 0 & 1.226018 & 1.226018 & 0 & 0 & -1.226018 \\
1 & 0 & -1.1072569 & -1.1072569 & 0 & 1.226018 & 1.226018 & 0 & 0 & 1.226018 \\
1 & 0.7829487 & 0 & 0 & 0.613009 & 0 & 0 & 0 & 0 & 0 \\
1 & -0.7829487 & 0 & 0 & 0.613009 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0.7829487 & 0 & 0 & 0.613009 & 0 & 0 & 0 \\
1 & 0 & 0 & -0.7829487 & 0 & 0 & 0.613009 & 0 & 0 & 0 \\
1 & 0 & 0.7829487 & 0 & 0 & 0.613009 & 0 & 0 & 0 & 0 \\
1 & 0 & -0.7829487 & 0 & 0 & 0.613009 & 0 & 0 & 0 & 0 \\
1 & 1.2735263 & 0 & 0 & 1.621869 & 0 & 0 & 0 & 0 & 0 \\
1 & -1.2735263 & 0 & 0 & 1.621869 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1.2735263 & 0 & 0 & 1.621869 & 0 & 0 & 0 \\
1 & 0 & 0 & -1.2735263 & 0 & 0 & 1.621869 & 0 & 0 & 0 \\
1 & 0 & 1.2735263 & 0 & 0 & 1.621869 & 0 & 0 & 0 & 0 \\
1 & 0 & -1.2735263 & 0 & 0 & 1.621869 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{4.7}
$$

$$
(X'X)=\begin{bmatrix}
24 & 0 & 0 & 0 & 14.2779 & 14.2779 & 14.2779 & 0 & 0 & 0 \\
0 & 14.2779 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 14.2779 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 14.2779 & 0 & 0 & 0 & 0 & 0 & 0 \\
14.2779 & 0 & 0 & 0 & 18.0374 & 6.0125 & 6.0125 & 0 & 0 & 0 \\
14.2779 & 0 & 0 & 0 & 6.0125 & 18.0374 & 6.0125 & 0 & 0 & 0 \\
14.2779 & 0 & 0 & 0 & 6.0125 & 6.0125 & 18.0374 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.0125 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.0125 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.0125
\end{bmatrix}
$$

$$
(X'X)^{-1}=\begin{bmatrix}
0.2735 & 0 & 0 & 0 & -0.1299 & -0.1299 & -0.1299 & 0 & 0 & 0 \\
0 & 0.0700 & 0 & & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0700 & & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0700 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.1299 & 0 & 0 & 0 & 0.1282 & 0.0451 & 0.0451 & 0 & 0 & 0 \\
-0.1299 & 0 & 0 & 0 & 0.0451 & 0.1282 & 0.0451 & 0 & 0 & 0 \\
-0.1299 & 0 & 0 & 0 & 0.0451 & 0.0451 & 0.1282 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1663 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1663 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1663
\end{bmatrix} \quad (4.8)
$$

4.2.6.1 The parameter estimation for GP1G

$$
(X'Y_{GP1G})=\begin{bmatrix} 122.8 \\ -72 \\ 90.6 \\ 16.5 \\ 762.3 \\ 710 \\ 748.4 \\ 15.9 \\ -34.3 \\ -15.9 \end{bmatrix} \qquad \text{hence} \qquad \hat{B}=(X'X)^{-1}X'Y_{GP1G}=\begin{bmatrix} 47.3911 \\ -5.0452 \\ 6.3461 \\ 1.1566 \\ 3.9485 \\ -0.3983 \\ 2.7962 \\ 2.6509 \\ -5.7095 \\ -2.6509 \end{bmatrix} \quad (4.9)
$$

4.2.6.2 The parameter estimation for GP2G

The equation (4.8) is obtained using the design matrix **X** given in equation (4.7) then GP2G is given as:-

$$(X'Y_{GP2G}) = \begin{bmatrix} 1097 \\ 20.6 \\ -10 \\ 39.3 \\ 620.8 \\ 692.1 \\ 718.7 \\ 36.8 \\ 17.2 \\ 6.1 \end{bmatrix} \quad \text{hence} \quad \hat{B} = (X'X)^{-1} X'Y_{GP2G} = \begin{bmatrix} 36.1296 \\ 1.4399 \\ -0.6980 \\ 2.7503 \\ 0.6806 \\ 6.6025 \\ 8.8179 \\ 6.1174 \\ 2.8548 \\ 1.0196 \end{bmatrix} \qquad (4.10)$$

4.2.6.3 The parameter estimation for GP3G

The equation (4.8) is obtained using the design matrix **X** given in equation (4.7) then GP3G is given as:-

$$(X'Y_{GP3G}) = \begin{bmatrix} 1519 \\ 27.3 \\ 28.4 \\ -92.8 \\ 894.6 \\ 920.2 \\ 985.1 \\ -65 \\ -1.2 \\ -28.2 \end{bmatrix} \quad \text{hence} \quad \hat{B} = (X'X)^{-1} X'Y_{GP3G} = \begin{bmatrix} 51.7514 \\ 1.9127 \\ 1.9876 \\ -6.5028 \\ 3.2471 \\ 5.3785 \\ 10.7726 \\ -10.8073 \\ -0.2039 \\ -4.6900 \end{bmatrix} \qquad (4.11)$$

4.3 Obtaining the generalized variance of the estimated linear parameters

This section deals with the residual variance, first order model variance-covariance and the second order model variance-covariance.

4.3.1 Residual variance

This section gives the residual variance for GP1G, GP2G and GP3G.

4.3.1.1 The residual variance for GP1G

Using table 4.4 the residuals are given as; $e = y_i - \hat{y} = y_i - \mathbf{X\beta}$.

Residuals =estimated errors= $e = y_i - \hat{y} = y_i - \mathbf{X\beta}$ = [6.5658 9.8932 -0.8807 2.4467 -7.3546 -7.5273 1.2068 1.0341 -3.3883 4.1651 -6.3270 1.2265 -2.8615 4.2383 7.9892 -6.1996 -4.1156 1.8217 2.6301 -7.2203 6.6009 5.5468 -2.8270 -6.6632]'

MSE=var(e)=(e'*e)/n-p= 665.8369/14=47.5598 (4.12)

4.3.1.2 The residual variance for GP2G

Using table 4.6 the residual variance for GP2G was calculated as below.

Residuals =estimated errors= $e = y_i - \hat{y} = y_i - \mathbf{X\beta}$ =[-4.3804 -5.1918 4.0740 3.2626 3.0855 7.2740 -5.8240 -1.6354 5.4422 -1.6034 7.0327 -0.0129 -3.6742 3.5805 -6.6884 2.6183 7.3694 6.2765 4.9328 -5.3998 -7.9337 -0.9286 -4.9492 -6.7269]'

MSE=var(e)=(e'*e)/n-p= 612.7118/14=43.7651 (4.13)

4.3.1.3 The residual variance for GP3G

Using table 4.8 residual variance for GP3G was calculated as below.

Residuals=estimated errors= $e = y_i - \hat{y} = y_i - \mathbf{X\beta}$ =[-6.3952 6.3406 -7.4937 5.2421 5.3926 -6.8717 8.4921 -3.7722 6.1965 5.0980 -2.7041 -3.8025 4.7605 -1.2443 -2.2638 8.5535 -5.6047 0.5076 -2.4536 0.4182 -5.9417 -2.5047 -0.0059 0.0565]'

Residual variance: MSE= $\widehat{\text{var}}(e)$ =(e'*e)/n-p= 595.4813 /14=42.5344 (4.14)

4.3.2 First order model variance-covariance

This section gives the first order variance for the parameter estimates and variance-covariance for the three groups.

4.3.2.1 The variance-covariance for GP1G

$$SSE = y'y - \hat{y}'\hat{y} = y'y - \hat{\beta}'X'X\hat{\beta} = y'y - \hat{\beta}'X'y = 64920 - 63790 = 1130$$

$$\hat{\sigma}^2 = MSE = \frac{SSE}{n-p} = \frac{1130}{24-4} = 56.5 \quad (4.15)$$

$$\text{var}(\hat{\beta}) = \hat{\sigma}^2[X'X]^{-1} = \begin{bmatrix} 2.3542 & 0 & 0 & 0 \\ 0 & 3.9572 & 0 & 0 \\ 0 & 0 & 3.9572 & 0 \\ 0 & 0 & 0 & 3.9572 \end{bmatrix} \quad (4.16)$$

The variance of all regression model parameter estimates is found as follows:

$$V\left(\hat{\beta}\right) = \sigma^2(\textit{diagonal elements of}\left(\mathbf{XX}\right)^{-1})$$

Therefore, $V(\hat{\beta}_0) = 2.3542$, $V(\beta_1) = V(\beta_2) = V(\beta_3) = 3.9572$

4.3.2.2 The variance-covariance for GP2G

$$SSE = y'y - \hat{y}'\hat{y} = y'y - \hat{\beta}'X'X\hat{\beta} = y'y - \hat{\beta}'X'y = 52001 - 50287 = 1714$$
$$\hat{\sigma}^2 = MSE = \frac{SSE}{n-p} = \frac{1714}{24-4} = 85.7 \tag{4.17}$$

$$\text{var}(\beta) = \hat{\sigma}^2[X'X]^{-1} = \begin{bmatrix} 3.5708 & 0 & 0 & 0 \\ 0 & 6.0023 & 0 & 0 \\ 0 & 0 & 6.0023 & 0 \\ 0 & 0 & 0 & 6.0023 \end{bmatrix} \tag{4.18}$$

The variance of all regression model parameter estimates is found as follows:

$$V\left(\hat{\beta}\right) = \sigma^2(\textit{diagonal elements of}\left(\mathbf{XX}\right)^{-1})$$

Therefore, $V(\hat{\beta}_0) = 3.5708$, $V(\beta_1) = V(\beta_2) = V(\beta_3) = 6.0023$.

4.3.2.3 The variance-covariance for GP3G

$$SSE = y'y - \hat{y}'\hat{y} = y'y - \hat{\beta}'X'X\hat{\beta} = y'y - \hat{\beta}'X'y = 99219 - 96852 = 2367$$
$$\hat{\sigma}^2 = MSE = \frac{SSE}{n-p} = \frac{2367}{24-4} = 118.35 \tag{4.19}$$

$$\text{var}(\beta) = \hat{\sigma}^2[X'X]^{-1} = \begin{bmatrix} 4.9312 & 0 & 0 & 0 \\ 0 & 8.2890 & 0 & 0 \\ 0 & 0 & 8.2890 & 0 \\ 0 & 0 & 0 & 8.2890 \end{bmatrix} \tag{4.20}$$

The variance of all regression model parameter estimates was found as follows:

$$V\left(\hat{\beta}\right) = \sigma^2(\textit{diagonal elements of}\left(\mathbf{XX}\right)^{-1})$$

Therefore, $V(\hat{\beta}_0) = 4.9312$, $V(\beta_1) = V(\beta_2) = V(\beta_3) = 8.2890$.

4.3.3 The second order model variance-covariance

This section deals with the second order standard error and variance-covariance for the three groups.

4.3.3.1 The standard error, variance-covariance for GP1G

$$SSE = y'y - \hat{y}'\hat{y} = y'y - \hat{\beta}'X'X\hat{\beta} = y'y - \hat{\beta}'X'y = 64920 - 64254 = 666$$

$$\hat{\sigma}^2 = \frac{SSE}{n-p} = \frac{666}{24-10} = 47.5714 \tag{4.21}$$

The covariance matrix of the least squares estimator depends on the variance of the disturbances and on the derivative matrix X.

$$\text{var}(\beta) = \hat{\sigma}^2[X'X]^{-1} =$$

$$\begin{bmatrix} 13.0102 & 0 & 0 & 0 & -6.1791 & -6.1791 & -6.1791 & 0 & 0 & 0 \\ 0 & 3.3318 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.3318 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.3318 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6.1791 & 0 & 0 & 0 & 6.0996 & 2.1435 & 2.1435 & 0 & 0 & 0 \\ -6.1791 & 0 & 0 & 0 & 2.1435 & 6.0996 & 2.1435 & 0 & 0 & 0 \\ -6.1791 & 0 & 0 & 0 & 2.1435 & 2.1435 & 6.0996 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.9121 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.9121 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.9121 \end{bmatrix} \tag{4.22}$$

The variance of all regression model parameter estimates wass found as follows:

$$V\left(\hat{\beta}\right) = \sigma^2 (diagonal\ elements\ of \left(\mathbf{XX}\right)^{-1})$$

Therefore, $V(\hat{\beta}_0) = 13.0102$, $V(\hat{\beta}_1) = V(\hat{\beta}_2) = V(\hat{\beta}_3) = 3.3318$

$$V(\hat{\beta}_{11}) = V(\hat{\beta}_{22}) = V(\hat{\beta}_{33}) = 6.0996, \; V(\hat{\beta}_{12}) = V(\hat{\beta}_{13}) = V(\hat{\beta}_{23}) = 7.9121 \quad (4.23)$$

4.3.3.2 The standard error, variance-covariance for GP2G

$$SSE = y'y - \hat{y}'\hat{y} = y'y - \hat{\beta}'X'X\hat{\beta} = y'y - \hat{\beta}'X'y = 52001 - 51388 = 613$$

$$\hat{\sigma}^2 = \frac{SSE}{n-p} = \frac{613}{24-10} = 43.7857 \quad (4.24)$$

$$\text{var}(\beta) = \hat{\sigma}^2[X'X]^{-1} =$$

$$\begin{bmatrix} 11.9748 & 0 & 0 & 0 & -5.6874 & -5.6874 & -5.6874 & 0 & 0 & 0 \\ 0 & 3.0667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.0667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.0667 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.6874 & 0 & 0 & 0 & 5.6142 & 1.9729 & 1.9729 & 0 & 0 & 0 \\ -5.6874 & 0 & 0 & 0 & 1.9729 & 5.6142 & 1.9729 & 0 & 0 & 0 \\ -5.6874 & 0 & 0 & 0 & 1.9729 & 1.9729 & 5.6142 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.2825 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.2825 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.2825 \end{bmatrix} \quad (4.25)$$

The variance of all regression model parameter estimates was found as follows:

$$V\left(\hat{\beta}\right) = \sigma^2(\textit{diagonal elements of}\left(\mathbf{XX}\right)^{-1})$$

Therefore, $V(\hat{\beta}_0) = 11.9748$, $V(\hat{\beta}_1) = V(\hat{\beta}_2) = V(\hat{\beta}_3) = 3.0667$,

$V(\hat{\beta}_{11}) = V(\hat{\beta}_{22}) = V(\hat{\beta}_{33}) = 5.6142$, $V(\hat{\beta}_{12}) = V(\hat{\beta}_{13}) = V(\hat{\beta}_{23}) = 7.2825$

4.3.3.3 The standard error, variance-covariance for GP3G

$$SSE = y'y - \hat{y}'\hat{y} = y'y - \hat{\beta}'X'X\hat{\beta} = y'y - \hat{\beta}'X'y = 99219 - 98624 = 595$$

$$\hat{\sigma}^2 = \frac{SSE}{n-p} = \frac{595}{24-10} = 42.5 \qquad (4.26)$$

$$\text{var}(\beta) = \hat{\sigma}^2[X'X]^{-1} =$$

$$\begin{bmatrix} 11.6232 & 0 & 0 & 0 & -5.5204 & -5.5204 & -5.5204 & 0 & 0 & 0 \\ 0 & 2.9766 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.9766 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.9766 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.5204 & 0 & 0 & 0 & 5.4493 & 1.9150 & 1.9150 & 0 & 0 & 0 \\ -5.5204 & 0 & 0 & 0 & 1.9150 & 5.4493 & 1.9150 & 0 & 0 & 0 \\ -5.5204 & 0 & 0 & 0 & 1.9150 & 1.9150 & 5.4493 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.0686 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.0686 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.0686 \end{bmatrix} \qquad (4.27)$$

The variance of all regression model parameter estimates was found as follows:

$$V\left(\hat{\beta}\right) = \sigma^2 (diagonal\ elements\ of \left(\mathbf{XX}\right)^{-1})$$

Therefore, $V(\hat{\beta}_0) = 11.6232$, $V(\hat{\beta}_1) = V(\hat{\beta}_2) = V(\hat{\beta}_3) = 2.9766$,

$V(\hat{\beta}_{11}) = V(\hat{\beta}_{22}) = V(\hat{\beta}_{33}) = 5.4493$, $V(\hat{\beta}_{12}) = V(\hat{\beta}_{13}) = V(\hat{\beta}_{23}) = 7.0686$

4.4 Fitting and testing the three models adequacies

This section fits first order and second order models for the three groups, tests the significance of parameter estimates, analyses the variance, lack of fit and coefficient of determinations.

4.4.1 The first-Order Model

The first-order model with three independent variables is expressed as:

$$\hat{y} = \beta o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3; \tag{4.28}$$

where:

$\hat{y}$ represents the predicted response where, in this research, is the yield amount of *rose coco* bean in grams.

β_0 is the mean response, that is the amount of *rose coco* beans when all the explanatory factors are zero.

β_1 is the parameter associated with the nitrogen fertilizer.

β_2 is the parameter associated with the phosphorus fertilizer.

β_3 is the parameter associated with the potassium fertilizer.

x_1 represents the observed yield on the *rose coco* bean under nitrogen at seven levels.

x_2 is the effect of phosphorus on *rose coco* beans, which has been controlled at seven levels.

x_3 is the effect of potassium on *rose coco* beans, which has been controlled at seven levels.

When there was a curvature in the response surface, then a higher degree polynomial was used. In any model, the levels of each factor are independent of the levels of other factors. The method of least square was used to estimate the parameters in the polynomials in which the response surface analysis was performed by using the fitted surface.

4.4.1.1 The first-order model fitting for GP1G

The first order regression equation for GP1G is given as in equation (4.4).

$$\hat{Y}_{GP1G} = 51.1667 - 5.0452X_1 + 6.3461X_2 + 1.1566X_3 \tag{4.29}$$

In GP1G the first order model was significant ($p=0.0057$) at 1%. The nitrogen ($p=0.0196$) and phosphorus ($p=0.0046$) were also significant factors, potassium fertilizer at 30grams is not contributing significantly to the *rose coco* yield. The coefficient of determination, $R^2=45.87\%$ is below 50% so the response is not well explained by the factors chosen though the combined linear factors are significant.

4.4.1.2 The first-order model fitting for GP2G

Therefore the first order regression equation for GP2G is given as equation (4.5);

$$\hat{Y}_{GP2G} = 45.7083 + 1.4399X_1 - 0.6980X_2 + 2.7503X_3 \quad (4.30)$$

In GP2G the first order model is insignificant (p=0.6462). The nitrogen, phosphorus and potassium are not significant factors. The R^2=7.78%, this is very low far much below, so the response is not well explained by the factors chosen in the first order model.

4.4.1.3 The first-order model fitting for GP3G

Therefore the first order regression equation for GP3G is given as

$$\hat{Y}_{GP3G} = 63.2917 + 1.9127X_1 + 1.9876X_2 - 6.5028X_3 \quad (4.31)$$

In GP3G the first order model is insignificant (p=0.1454). The nitrogen and phosphorus are insignificant factors, potassium (p=0.0352) is significant. The R^2=23.14%, this is below 50% so the response is not well explained by the factors chosen in the first order model confirming its insignificance.

4.4.2 The Analysis of variance of GP1G

The total sum of squares is given as;

$$\text{SST} = y'y - \frac{(\sum_{i=1}^{n} y_i)^2}{n} = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n} = y'y - n\bar{y}^2$$

$= 64920\text{-}24x51.1667^2 = 2087.2515$ (4.32)

and the sum of squares due to regression is give as

$$SSR = \hat{\beta}' X' y - n\bar{y}^2$$

$= 63790\text{-}62832.7485 = 957.2515$ (4.33)

The sum of squares due to error is

$$SSE = y' y - \hat{\beta}' X' y$$

$= 64920\text{-}63790 = 1130$ (4.34)

which verifies that

SST = SSR + SSE. (4.35)

Using the result of the equations (4.32), (4.33) and (4.34) the analysis of variance table generated from this model is as follows:

Table 4. 10: The Analysis of Variance table first order for GP1G.

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares	Variance ratio
Regression	3	957.2515	319.0838	5.65
Error	20	1130	56.5	
Total	23	2087.2515		

```
Sum of Squared Residuals          1129.79190
Predicted Residual SS (PRESS)     1672.86356
```

4.4.2.1 The test of significance in regression-GP1G

A good estimated regression model should explain the variation of the dependent variable in the sample. However, there are certain tests of hypotheses about the model parameters that could help the experimenter in measuring the effectiveness of the model. The first of all these tests requires the error term e_i's to be normally, identically and independently distributed with mean zero and variance σ^2. In order to check this assumption, the normal probability of residuals for the *rose coco* beans yield is graphed as shown in figure 4.4.

4.4.2.2 Normal probability plot of the residuals GP1G

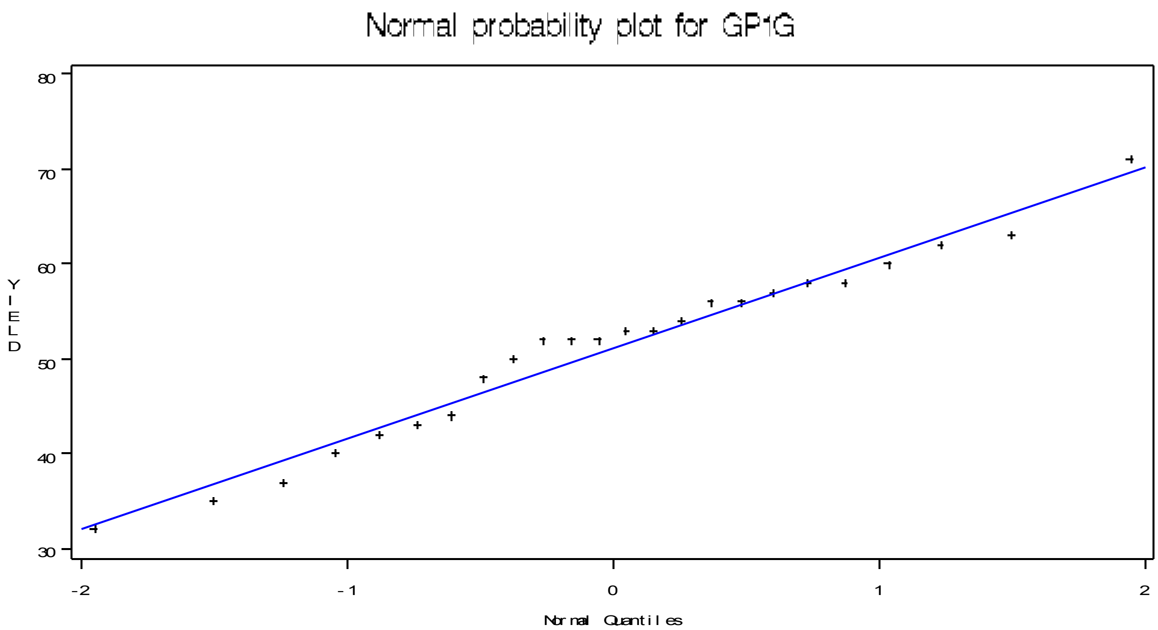


Figure 4. 4: The Normal Probability Plot of GP1G.

The residuals plot in figure 4.4 is approximately along a straight line, thus the normality assumption is satisfied. It is important to note that the error term is the difference between the observed value y_i and the corresponding fitted value $\hat{y}_i$, that is, $e_i = y_i - \hat{y}_i$ as shown in table 4.4. As a result of this assumption, observations y_i are also normally, identically and independently distributed.

4.4.2.3 The test of significance of the model

As a result of the normality assumption being satisfied, observations y_i are also normally, identically and independently distributed. Therefore, the test for the significance of the regression could be applied to determine the relationship between the dependent variable y and independent variables X_1, X_2, X_3. The hypotheses are;

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$, against

$H_1: \beta i \neq 0$ for at least one i.

From the analysis of variance table 4.10, Fc = 5.65. Comparing this value with the F-table value $F_{0.05, 3, 20} = 3.10$, we found out that there was a significant statistical evidence to reject the null hypothesis. It implies that at least one of the independent variables, nitrogen, phosphorus or potassium, contributes significantly to the model, therefore the first order model was adequate for GP1G. Further tests were carried out on the parameters β_1, β_2 and β_3 in order to identify the variable that significantly contributes to the model.

4.4.2.4 The test of significance of parameter estimates in first order model

The hypothesis test was used to gain a rough idea of the importance of the treatment effects. In order to determine whether given variables were justified to be included or excluded from the model, the test of hypotheses for the individual regression coefficients were undertaken as follows:

4.4.2.5 Test for β_1

Hypothesis

$H_0: \beta_1 = 0$, against

$H_1: \beta_1 \neq 0$.

The standard error for $\hat{\beta}_1$ (S.E $\hat{\beta}_1$), was found by use of the fact that Cov($\hat{\beta}$) = MSE(X'X)$^{-1}$. Thus, using the results in table 4.10 and the diagonal element of $(X'X)^{-1}$corresponding to this parameter estimate in equation (4.16), the standard error is given as,

S.E $\hat{\beta}_1$= $(3.9572)^{1/2}$ =1.98927

$$t = \frac{\hat{\beta}_i}{\sqrt{\text{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)} \tag{4.36}$$

t_c=-5.0452/1.9887=-2.5362

while $t_{0.025,20}$ = 2.086.

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we reject the null hypothesis, thus the parameter was significant and the predictor variable-nitrogen (X_1) was required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$ in the first order model.

4.4.2.6 Test for β_2

Hypothesis

H_0: $\beta_2 = 0$, against

H_1: $\beta_2 \neq 0$.

The standard error of the parameter estimate, $\hat{\beta}_2$ given by,

S.E $\hat{\beta}_2$ =$(3.9572)^{1/2}$ =1.98927

$$t = \frac{\hat{\beta}_i}{\sqrt{\text{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)} \tag{4.37}$$

t_c=6.3461/1.9887=3.1902

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we reject the null hypothesis, thus the parameter was significant and the predictor variable-phosphorus (X_2) was required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$ in the first order model.

4.4.2.7 Test for β_3

Hypothesis

$H_0: \beta_3 = 0$, against

$H_1: \beta_3 \neq 0$.

The standard error of the parameter estimate, $\hat{\beta}_3$ is given as;

S.E $\hat{\beta}_2 = (3.9572)^{1/2} = 1.98927$

$$t = \frac{\hat{\beta}_i}{\sqrt{\mathrm{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)} \tag{4.38}$$

$t_c = 1.1566/1.9887 = 0.5814$

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we do not reject the null hypothesis, thus the parameter was insignificant and the predictor variable-potassium (X_3) was not individually important in explaining the variation of the *rose coco* yield at $\alpha = 0.05$ in the first order model.

The coefficient of multiple determination is given as;

$$R^2 = 1 - \frac{SS_E}{SS_T} \quad or$$

$$R^2 = \frac{SSR}{SST} = \frac{957.2515}{2087.2515} = 0.4586 \tag{4.39}$$

which indicates that 45.86 % of the variation of the *rose coco* yield was accounted for by the model, which was below half (50%) to justify the correct relationship between the predictors and the response. The R^2 measures how well estimated model fits the data. When R^2 is closer to the 100%, the better the estimation of the regression equation fits the sample data. The first order model for GP1G $\hat{Y}_{GP1G} = 51.1667 - 5.0452X_1 + 6.3461X_2 + 1.1566X_3$ was significant ($p<0.05$). The N and P contribute significantly to the model. K was insignificant. So the model with significant terms only is $\hat{Y}_{GP1G} = 51.1667 - 5.0452X_1 + 6.3461X_2$

4.4.3 The analysis of variance of GP2G

The sum of squares due to total is given as

$SST = y'y - n\overline{y}^2$

$= 52001-24x45.708^2 = 1859.6897$ (4.40)

and the sum of squares due to regression was give as

$SSR = \hat{\beta}'X'y - n\overline{y}^2$

$= 50287-50141.3103=145.6897$ (4.41)

The sum of squares due to error is

$SSe = y'y - \hat{\beta}'X'y$

$= 52001-50287=1714$ (4.42)

which verifies that

SST = SSR + SSE. (4.43)

Using the result of the equations (4.40), (4.41) and (4.42) the analysis of variance table generated from this model is as follows:

Table 4. 11: The Analysis of Variance table first order for GP2G.

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares	Variance ratio
Regression	3	145.6897	48.5632	0.567
Error	20	1714	85.7	
Total	23	1859.6897		

4.4.3.1 The test of significance in regression-GP2G

In order to check this assumption, the normal probability of residuals for the *rose coco* beans yield is graph as shown in Figure 4.5.

4.4.3.2 Normal probability plot of the residuals GP2G

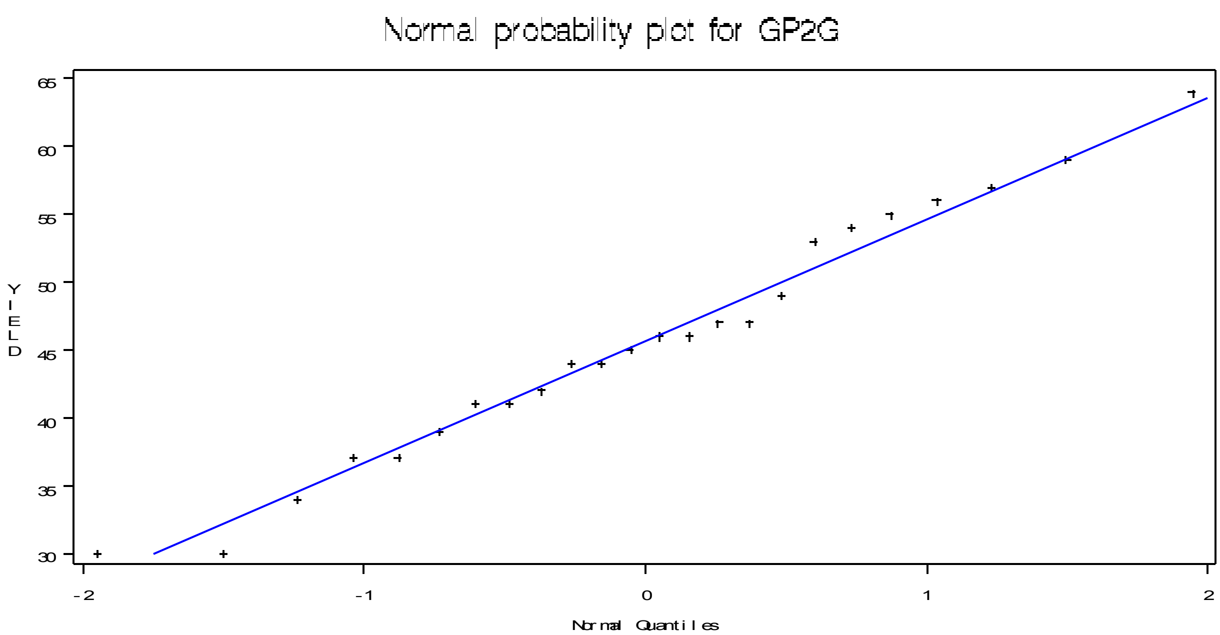


Figure 4. 5: The Normal Probability Plot of GP2G.

The residuals plot in figure 4.5 is approximately along a straight line, thus the normality assumption is satisfied.

4.4.3.3 The test of significance of the model

As a result of the normality assumption being satisfied, observations y_i are also normally, identically and independently distributed. Therefore, the test for the significance of the regression could be applied to determine if the relationship between the dependent variable y and independent variables X_1, X_2, X_3 exists. The hypotheses are,

H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, against

H_1: $\beta i \neq 0$ for at least one i.

From the analysis of variance table 4.11, Fc = 0.567. Comparing this value with the F-table value $F_{0.05,\ 3,\ 20} = 3.10$, we find that we do not reject the null hypothesis. It implies that the independent variables, nitrogen, phosphorus or potassium, contributes insignificantly to the model, therefore the first order model is inadequate for GP2G. We now do not carry out further tests on the parameters β_1, β_2 and β_3 in order to identify the variable that significantly contributes to the model.

The coefficient of multiple determination is given as;

$$R^2 = \frac{145.6897}{1859.6897} = 0.0783 \tag{4.44}$$

which indicates that 7.83 % of the variation of the *rose coco* yield was accounted for by the model, which was a very low value to justify the correct relationship between the predictors and the response. We test the first order model and it was insignificant (p=0.6462). The first order model was given by this equation:-

$$\hat{Y}_{GP2G} = 45.7083 + 1.4399X_1 - 0.6980X_2 + 2.7503X_3 \tag{4.45}$$

4.4.4 The analysis of variance of GP3G

The sum of squares due to total was given as

$$SST = y'y - n\bar{y}^2$$

$$= 99219\text{-}24x63.2917^2 \;\; =3078.8571 \qquad (4.46)$$

and the sum of squares due to regression was given as;

$$SSR = \hat{\beta}'X'y - n\bar{y}^2$$

$$= 96852\text{-}24x63.2917^2\text{=}711.8571 \qquad (4.47)$$

The sum of squares due to error is

$$SSE = y'y - \hat{\beta}'X'y$$

$$= 99219\text{-}96852\text{=}2367 \qquad (4.48)$$

which verifies that

SST = SSR + SSE. (4.49)

Using the result of the equations (4.46), (4.47) and (4.48) the analysis of variance table generated from this model was as follows:

Table 4. 12: The Analysis of Variance table first order for GP3G.

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares	Variance ratio
Regression	3	711.8571	237.2857	2.005
Error	20	2367	118.35	
Total	23	3078.8571		

4.4.4.1 The test for significance in regression-GP3G

In order to check this assumption, the normal probability of residuals for the *rose coco* beans yield was graph as shown in figure 4.6.

4.4.4.2 Normal probability plot of the residuals GP3G

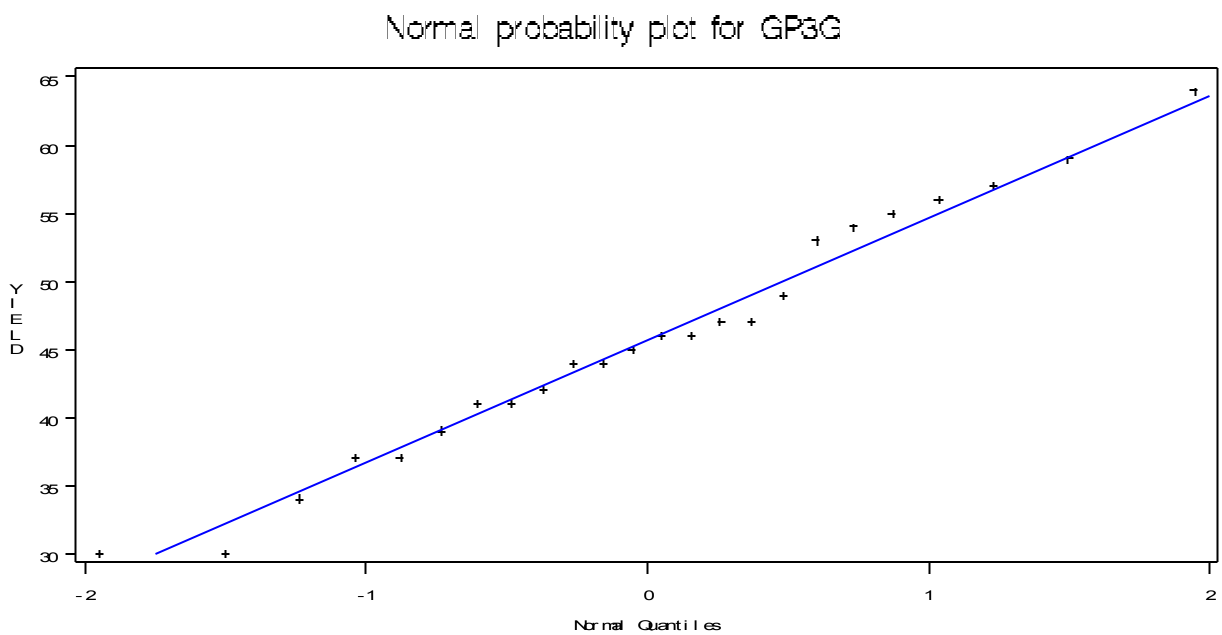


Figure 4. 6: The Normal probability plot for GP3G.

The residuals plot in figure 4.6 was approximately along a straight line, thus the normality assumption is satisfied.

4.4.4.3 The test of significance of the model

As a result of the normality assumption being satisfied, observations y_i are also normally, identically and independently distributed. Therefore, the test for the significance of the regression could be applied to determine if the relationship between the dependent variable y and independent variables X_1, X_2, X_3 exists. The hypotheses are,

$H_0: \beta_1 = \beta_2 = \beta_3 = 0$, against

$H_1: \beta i \neq 0$ for at least one i.

From the analysis of variance table 4.12, $F_c = 2.005$. Comparing this value with the F-table value $F_{0.05, 3, 20} = 3.10$, we found out that we do not reject the null hypothesis. It implies that the independent variables, nitrogen, phosphorus or potassium, contributes insignificantly to the model, therefore the first order model is inadequate for GP3G.

Therefore, there was no need to go further tests on the parameters β_1, β_2 and β_3 in order to identify the variable that significantly contribute to the model.

The coefficient of multiple determination was given as;

$$R^2 = \frac{711.8571}{3078.8571} = 0.2312 \tag{4.50}$$

which indicates that 23.12 % of the variation of the *rose coco* yield was accounted for by the first order model, which was a rather low value to justify the correct relationship between the predictors and the response. If we test the first order model it was insignificant (p=0.1454). The first order model was given by this equation:-

$$\hat{Y}_{GP3G} = 63.2917 + 1.9127X_1 + 1.9876X_2 - 6.5028X_3 \tag{4.51}$$

4.4.5 Design for fitting the second-order model-GP1G

This section fits the second order model, test the significance of parameter estimates in second order model and lack of fit for the three replicates of GP1G.

4.4.5.1 Fitting of the second-order model

The second-order polynomial model for three factors was used to fit the data given below:

$$\hat{Y} = \beta_0 + \sum_{i=1}^{3} \beta_i x_i + \sum_{i=1}^{3} \beta_{ii} x_i^2 + \sum_{i=1}^{2} \sum_{j=i+1}^{3} \beta_{ij} x_i x_j \tag{4.52}$$

where $\hat{Y}$ is the predicted response used as a dependent variable; x_i are the coded levels of independent variables; β_0 intercept; β_i the linear terms coefficients; β_{ii} the quadratic terms coefficients and β_{ij} the interaction terms coefficients.

We explore the second order model fit to verify and explore the curvature if it exists. The second-order model includes all the terms in the first-order model plus all quadratic terms and all cross product terms. The model was expressed as with three independent variables:

$$Y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x^2{}_1 + \beta_{22} x^2{}_2 + \beta_{33} x^2{}_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + e \tag{4.53}$$

The fitted second order model in terms of coded factors for GP1G in equation (4.9) is given as:

$$\begin{aligned}\hat{Y}_{GP1G} = {} & 47.3911 - 5.0452X_1 + 6.3461X_2 + 1.1566X_3 + 3.9485X_1^2 - 0.3983X_2^2 + 2.7962X_3^2 \\ & + 2.6509X_1X_2 - 5.7095X_1X_3 - 2.6509X_2X_3\end{aligned} \tag{4.54}$$

4.4.5.2 The test of significance of parameter estimates in second order model

We undertook the test of hypotheses for the individual regression coefficients as follows to see its significance on the model:

4.4.5.3 Test for β_i

Hypothesis

H_0: $\beta_i = 0$, for i=1,2,3 against

H_1 : $\beta_i \neq 0$.

The standard error for $\hat{\beta}_i$, S.E $\hat{\beta}_i$, was found by using the

Cov($\hat{\beta}$) = MSE$(X'X)^{-1}$. Thus, using the equation (4.22) the diagonal element of $\hat{\sigma}^2 (X'X)^{-1}$ the standard error is given as, S.E $\hat{\beta}_i$ =$(3.3318)^{1/2}$ =1.8253

$$t = \frac{\hat{\beta}_i}{\sqrt{\text{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)}$$

t_n=-5.0452/1.8253=-2.7640

t_p=6.3461/1.8253=3.4767 (4.55)

t_k=1.1566/1.8253=0.6336

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we reject the null hypothesis of nitrogen and phosphorus, thus the parameters are significant and the predictor variables-nitrogen and phosphorus are required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$. The potassium fertilizer was insignificant.

4.4.5.4 Test for β_{ii}

Hypothesis

H_0: $\beta_{ii} = 0$, against

H_1: $\beta_{ii} \neq 0$.

The standard error of the parameter estimate, $\hat{\beta}_{ii}$ given as;

S.E $\hat{\beta}_{ii}$= $(6.0996)^{1/2}$ =2.4697

$$t = \frac{\hat{\beta}_i}{\sqrt{\text{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)}$$

t_n^2=3.9485/2.4697=1.5988

t_p^2=-0.3983/2.4697=-0.1613 (4.56)

t_k^2=2.7962/2.4697=1.1322

Since $-t_{\alpha/2} < t_c < t_{\alpha/2}$, we accept the null hypothesis, thus the parameters were insignificant and the predictor quadratic variables of nitrogen, phosphorus and potassium were not required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$.

4.4.5.5 Test for β_{ij}

Hypothesis

H_0: $\beta_{ij} = 0$, against

H_1: $\beta_{ij} \neq 0$.

The standard error of the parameter estimate, $\hat{\beta}_{ij}$ given as;

S.E $\hat{\beta}_{ij}$=$(7.9121)^{1/2}$ =2.8128

$$t_c = \frac{\hat{\beta}_i}{\sqrt{\text{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)}$$

t_{np}=2.6509/2.8128=0.9424

t_{nk}=-5.7095/2.8128=-2.0298 (4.57)

t_{pk}=-2.6509/2.8128=-0.9424

Since $-t_{\alpha/2} < t_c < t_{\alpha/2}$, we accept the null hypothesis, thus the parameters were insignificant and the predictor interactions variables of nitrogen and phosphorus, nitrogen and potassium, phosphorus and potassium were not required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$.

Table 4. 13: The ANOVA results on the three replicate of *rose coco* beans-GP1G.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
FO(N, P, K)	3	2816	938.57	1.3969	0.2522
TWI(N, P, K)	3	842	280.50	0.4175	0.7411
PQ(N, P, K)	3	925	308.30	0.4589	0.7120
Residuals	62	41656	671.88		
Lack of fit	14	1962	140.13	0.1694	0.9996
Pure error	48	39695	826.97		

Table 4.13 comes from the three replicates in GP1G to show the subdivision of residuals into lack of fit and pure error.

Table 4. 14: The ANOVA results in the fertilizer concentration on *rose coco* beans-GP1G

Regression	DF	Type I Sum of Squares	Mean Sq	R-Square	F Value	Pr > F
First Order	3	957.541428	319.18	0.4587	6.71	0.0049
Pure Quadratic	3	183.455066	61.15	0.0879	1.29	0.3178
Two-Factor Interaction	3	280.500000	93.50	0.1344	1.97	0.1655
Residuals	14	665.84	47.56			
Lack of fit	14	665.84	47.56			
Pure error	0	0.00				
Total Model	9	1421.496494		0.6810	3.32	0.0219

Coefficient of variation (CV)=13.48, coefficient determination (R^2)=0.6810, Adjusted R-squared =0.4759 correlation coefficient (r)=0.8252, root MSE=6.896359, response mean=51.166667, PRESS=2488.850781

Table 4.14 shows the results of the averaged data of the three replicates in GP1G that is why the pure error is zero because pure error emanates from replicated values (center points). The statistical testing of the model was done by the Fisher's statistical test for analysis of variance (ANOVA) and the results as shown in table 4.14 indicates that there was no significant effect of the quadratic components and cross products. However, the total second order model was significant at 0.05 (p=0.0219). The analysis of variance (F-test) showed that the first model fits well with the experimental data. The Coefficient of variation (C.V.) is a measure expressing the standard deviation as a percentage of the mean (Thomas and Nelson, 1996). It indicates the degree of precision with which the treatments are compared. The higher the value of the coefficient of variation (CV), the lower the reliability of the experiment, lower values of CV indicated a very high degree of precision and a good deal of reliability of the experimental values. Here, a lower value of CV (13.48) indicates the greater reliability of the experiment. The goodness of fit of the model was checked by the coefficient of determination (R^2) and correlation coefficient (R). The closer value of R (correlation coefficient) to 1, the better the correlation between the experimental and predicted values. Here, the value of R (0.8252) for equation (4.54) being close to 1 indicated a close agreement between the experimental results and the theoretical values predicted by the model equation. The R^2 of quadratic equation was 68.10% with mean response of 51.1667 grams of *rose coco* beans, implying that R^2 in GP1G of second order model explain 68.10% of the variation in the model than R^2=45.87% of the first order model. The model was adequate to express the actual relationship between the response and significant variables, with a satisfactory coefficient of determination (R^2=0.6810), which indicated 68.10% of the variability in the response could be explained by the second-order polynomial predictive equation (4.54), which means that the model was unable to explain only 31.9% of the total variations, quadratic model was insignificant having N and P significant factors.

Table 4. 15: The estimated effects & coefficients of the empirical model for GP1G

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	47.391125	3.606523	13.14	<.0001***
N	1	-5.045204	1.825105	-2.76	0.0152*
P	1	6.346081	1.825105	3.48	0.0037**
K	1	1.156615	1.825105	0.63	0.5365
N*N	1	3.948538	2.469426	1.60	0.1321
P*N	1	2.650859	2.812504	0.94	0.3619
P*P	1	-0.398323	2.469426	-0.16	0.8742
K*N	1	-5.709542	2.812504	-2.03	0.0618●
K*P	1	-2.650859	2.812504	-0.94	0.3619
K*K	1	2.796168	2.469426	1.13	0.2765

Signif. codes:'***'significant at 0.001,'**'significant at 0.01,'*'significant at 0.05, '●'significant at 0.1,' ' significant at 1.

Table 4.15 lists the regression coefficients and the corresponding p-values for the second-order polynomial model given as in equation (4.54) as:

$$\hat{Y}_{GP1G} = 47.3911 - 5.0452X_1 + 6.3461X_2 + 1.1566X_3 + 3.9485X_1^2 - 0.3983X_2^2 + 2.7962X_3^2 + 2.6509X_1X_2 - 5.7095X_1X_3 - 2.6509X_2X_3$$

The $\hat{Y}_{GP1G}$ is the predicted response for *rose coco* beans in group 1. The results presented in Table 4.15 suggest that linear effects of nitrogen and phosphorus were primary determining factors on the *rose coco* beans yield as these had the largest coefficients. The interaction effect N*K was a secondary determining factor and those other terms of the model showed no significant effect on the yield. The positive coefficient, P, enhances the yield the most. However, all the other terms N and N*K had negative coefficients. The X_1 represent Nitrogen (N), X_2 represent Phosphorus (P), X_3 represents Potassium (K).

The significance of each coefficient was determined by the student's t-test and p-value, which are in table 4.15. The larger the magnitude of t-test and smaller p-value, the more significant the corresponding coefficients. It can be seen in table 4.15 that the regression coefficients of the linear terms of nitrogen and phosphorus N(0.0152) and P(0.0037), respectively) had significant effects on the yield (p-value <0.05) and the interaction terms in K*N (potassium and nitrogen-(0.0618)) is marginally significant. Among these, P was significant at the 1% significance level, while N was significant at the 5% level.

Table 4. 16: The Analysis of Variance Table for N,P,K in 2nd order model- GP1G.

Factor	DF	Sum of Squares	Mean Square	F Value	Pr > F
N	4	723.277182	180.819295	3.80	0.0270
P	4	660.747662	165.186916	3.47	0.0360
K	4	318.328520	79.582130	1.67	0.2118

We saw that in GP1G ANOVA nitrogen and phosphorus are significant factors at $p<0.05$. Potassium was not significant at $\alpha=0.05$ level of significance.

4.4.5.6 The lack-of-fit test for the three replicates GP1G

Lack-of-fit tests were performed on the fitted models. The lack-of-fit test compares the variation around the model with pure variation within replicated observations using the results of table 4.1. The lack of fit measures the adequacy of the quadratic response surface model.

Table 4. 17: The lack of fit test for second order model-GP1G.

Residual	DF	Sum of Squares	Mean Square	F Value	Pr > F
Lack of Fit	14	2073.344319	148.096023	0.19	0.9993
Pure Error	48	37619	783.722222		
Total Error	62	39692	640.193726		

The lack of fit of the model was insignificant indicating that the second order model fits the data adequately. This is when the three replicates in group one (GP1G) is used.

4.4.6 Design for fitting the second-order model-GP2G

A second-order polynomial model for three factors was used to fit the data of GP2G using the model in equation (4.52). We explore the second order model fit to verify and explore the curvature if it exists. The second-order model includes all the terms in the first-order model plus all quadratic terms and all cross product terms. The model was expressed as with three independent variables as given by equation (4.52).

4.4.6.1 The test of significance of parameter estimates in second order model

We undertook the test of hypotheses for the individual regression coefficients as follows to see its significance on the model:

4.4.6.2 Test for β_i

Hypothesis

H_0: $\beta i = 0$, for i=1,2,3 against

H_1: $\beta i \neq 0$.

The standard error for $\hat{\beta}_i$, S.E $\hat{\beta}_i$, is found by use of the fact that

Cov($\hat{\beta}$) = MSE$(X'X)^{-1}$. Thus, using the equation (4.25) the diagonal element of $\hat{\sigma}^2 (X'X)^{-1}$ the standard error is given as, S.E $\hat{\beta}_i$ = $(3.0667)^{1/2}$ =1.7512

$$t = \frac{\hat{\beta}_i}{\sqrt{\mathrm{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)} \qquad (4.58)$$

t_n=1.4399/1.7512=0.8222

t_p=-0.6980/1.7512=-0.3986 (4.59)

t_k=2.7503/1.7512=1.5705

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we do not reject the null hypothesis, thus the parameters are insignificant and the predictor variables-nitrogen, phosphorus and potassium are not required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$.

4.4.6.3 Test for β_{ii}

Hypothesis

H_0: $\beta_{ii} = 0$, against

H_1: $\beta_{ii} \neq 0$.

The standard error of the parameter estimate, $\hat{\beta}_{ii}$ is using equation (4.25),

S.E $\hat{\beta}_{ii}$ = $(5.6142)^{1/2}$ =2.3694

$$t = \frac{\hat{\beta}_i}{\sqrt{\mathrm{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)}$$

t_n^2=0.6806/2.3694=0.2872

t_p^2=6.6025/2.3694=2.7865 (4.60)

t_k^2=8.8179/2.3694=3.7215

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we reject the null hypothesis of quadratic of P and K, thus the parameter are significant and the predictor quadratic variables of phosphorus and potassium are required in explaining the variation of the rose coco yield at $\alpha = 0.05$. The quadratic of nitrogen was insignificant.

4.4.6.4 Test for β_{ij}

Hypothesis

$H_0: \beta_{ij} = 0$, against

$H_1: \beta_{ij} \neq 0$.

The standard error for the parameter estimate, $\hat{\beta}_{ij}$ is using equation (4.25),

S. E $\hat{\beta}_{ij}$ = $(7.2825)^{1/2}$ =2.6986

$$t = \frac{\hat{\beta}_i}{\sqrt{\mathrm{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)}$$

t_{np}=6.1174/2.6986=2.2669

t_{nk}=2.8548/2.6986=1.0579 (4.61)

t_{pk}=1.0196/2.6986=0.3778

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we reject the null hypothesis of N*P, thus the parameter was significant and the predictor variable-N*P was important in explaining the variation of the rose coco yield at $\alpha = 0.05$ others are insignificant.

Table 4. 18: The ANOVA results on the three replicate of *rose coco* beans-GP2G.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
FO(N, P, K)	3	85.5	28.49	0.1041	0.95740
TWI(N, P, K)	3	717.8	239.28	0.8739	0.45953
PQ(N, P, K)	3	2354.8	784.92	2.8669	0.04364
Residuals	62	16975.1	273.79		
Lack of fit	14	1962.4	140.17	0.4482	0.94877
Pure error	48	15012.7	312.76		

The table 4.18 comes from the three replicates in GP2G to show the subdivision of residuals into lack of fit and pure error.

Table 4. 19: The ANOVA results in the fertilizer concentration on *rose coco* beans-GP2G.

Regression	DF	Type I Sum of Squares	mean sq	R-Square	F Value	Pr > F
First Order	3	144.555104	48.185	0.0778	1.10	0.3815
Pure Quadratic	3	821.441245	273.814	0.4419	6.26	0.0065
Two-Factor Interaction	3	280.250000	93.417	0.1508	2.13	0.1417
Residuals	14	612.71	43.765			
Lack of fit	14	612.71	43.765			
Pure error	0	0.00				
Total Model	9	1246.246349		0.6704	3.16	0.0264

Coefficient of variation (CV)=14.47, coefficient determination (R^2)=0.6704, Adjusted R-squared =0.4585, correlation coefficient (r)=0.8188, root MSE=6.615523, response mean=45.708333, PRESS=2091.1898012

Table 4.19 is the results of the averaged data of the three replicates in GP2G. In the table, the combined quadratic terms are significant at 1% with a p-value of 0.0065 also the total model was significant at 5% with a p value of 0.0264. We tested the first order model it was insignificant (p=0.3815). Linear and cross product terms were not significant, which corresponds to the fact that the quadratic model for GP2G was appropriate. Therefore, the analysis of variance (F-test) showed that the second order model fits well with the experimental data in GP2G. The R^2 of quadratic equation was 67.04% with mean response of 45.7083 grams of *rose coco* beans, implying that R^2 in GP2G of second order model explain 67.04% of the variation in the model than R^2=7. 83% of the first order model. The model was adequate to express the actual relationship between the response and significant variables, with a satisfactory coefficient of determination (R^2=0.6704), which indicated 67.04% of the variability in the response could be explained by the second-order polynomial predictive equation (4.62), meaning that the model was unable to explain 32.96% of the total variations. The value of R (0.8188) for equation (4.62) being close to 1 indicated a close agreement between the experimental results and the theoretical values predicted by the model equation.

Table 4. 20: The estimated effects & coefficients of the empirical model for GP2G.

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	36.129616	3.459657	10.44	<.0001***
N	1	1.439859	1.750782	0.82	0.4246
P	1	-0.697954	1.750782	-0.40	0.6962
K	1	2.750284	1.750782	1.57	0.1385
N*N	1	0.680614	2.368865	0.29	0.7781
P*N	1	6.117366	2.697972	2.27	0.0397*
P*P	1	6.602530	2.368865	2.79	0.0145*
K*N	1	2.854771	2.697972	1.06	0.3079
K*P	1	1.019561	2.697972	0.38	0.7112
K*K	1	8.817910	2.368865	3.72	0.0023**

Signif. codes:'***'significant at 0.001,'**'significant at 0.01,'*'significant at 0.05, '•'significant at 0.1,' ' significant at 1.

Table 4.20 lists the regression coefficients and the corresponding p-values for the second-order polynomial model given as in equation 4.62 as:

$$\hat{Y}_{GP2G} = 36.1296 + 1.4399X_1 - 0.6980X_2 + 2.7503X_3 + 0.6806X_1^2 + 6.6025X_2^2 + 8.8179X_3^2 + 6.1174X_1X_2 + 2.8548X_1X_3 + 1.0196X_2X_3 \quad (4.62)$$

The $\hat{Y}_{GP2G}$ is the predicted response for *rose coco* beans in group 2. The results presented in Table 4.20 suggest that quadratic effects of phosphorus (p=0.0145) and potassium (p=0.0023), the interaction effect N*P (p=0.0397) are significant factors on the rose coco bean, among these, K^2 is significant at the 1% significance level, while P^2 and N*P are significant at the 5% level, those other terms of the model showed no significant effect on the yield. The positive coefficient of P^2 (quadratic phosphorus) K^2 (quadratic potassium), N*P enhance the yield since they are the largest coefficients in the model equation (4.62). The X_1 represent Nitrogen (N), X_2 represent Phosphorus (P), X_3 represents Potassium (K). Table 4.20 suggests that quadratic effects of phosphorus (P^2), potassium (K^2) and interaction N*P were the determining significant factors on the *rose coco* bean yield as these had the more or less the same coefficients and those other terms of the model showed no significant effect on the yield. The P^2, K^2 and N*P showed the positive coefficient, meaning all the three (quadratic phosphorus, quadratic potassium and nitrogen and phosphorus combined) enhance the *rose coco* yield.

4.4.6.5 The lack of fit for the three replicates in GP2G

Lack-of-fit tests were performed on the fitted models. The lack-of-fit test compares the variation around the model with pure variation within replicated observations using the results of table 4.5.

Table 4. 21: The lack of fit test for GP2G.

Residual	DF	Sum of Squares	Mean Square	F Value	Pr > F
Lack of Fit	14	1838.135953	131.295425	0.50	0.9236
Pure Error	48	12692	264.416667		
Total Error	62	14530	234.357031		

The lack of fit of the model was insignificant, so the second order model was adequate for the experiments of GP2G.

Table 4. 22: The Analysis of Variance Table for N,P,K in 2nd order model- GP2G.

Factor	DF	Sum of Squares	Mean Square	F Value	Pr > F
N	4	307.213714	76.803429	1.75	0.1941
P	4	578.197324	144.549331	3.30	0.0420
K	4	769.676867	192.419217	4.40	0.0165

In table 4.22, the phosphorus and potassium were significant factors at 5% level of significance. Nitrogen was not significant at the 5 % level of significance.

4.4.7 Design for fitting the second-order model GP3G

The second-order polynomial model for three factors was used to fit the data of GP3G using the model equation (4.52). The model was expressed as with three independent variables as given by equation (4.53).

4.4.7.1 The test of significance of parameter estimates in second order model

The hypothesis test was used to gain a rough idea of the importance of the treatment effects. In order to determine whether given variables are justified to be included or excluded from the model, we undertook the test of hypotheses for the individual regression coefficients as follows:

4.4.7.2 Test for β_i

Hypothesis

$H_0: \beta i = 0$, for i=1,2,3 against

$H_1: \beta i \neq 0$.

The standard error for $\hat{\beta}_i$, S.E $\hat{\beta}_i$, was found by use of the fact that

Cov($\hat{\beta}$) = MSE$(X'X)^{-1}$. Thus, using the equation (4.27) the diagonal element of $\hat{\sigma}^2 (X'X)^{-1}$ the standard error was given as, S.E $\hat{\beta}_i$ = S.E $\hat{\beta}_i$ =$(2.9766)^{1/2}$ =1.7253

$$t = \frac{\hat{\beta}_i}{\sqrt{\operatorname{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)} \qquad (4.63)$$

t_n=1.9127/1.7253=1.1086

t_p=1.9876/1.7253=1.1520 (4.64)

t_k=-6.5028/1.7253=-3.7691

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we do not reject the null hypothesis of N and P, thus the parameters were insignificant and the predictor variables-nitrogen and phosphorus was not required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$. We reject the null hypothesis of K, thus the parameter was significant and the predictor variables-potassium was required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$.

4.4.7.3 Test for β_{ii}

Hypothesis

$H_0: \beta_{ii} = 0$, against

$H_1: \beta_{ii} \neq 0$.

The standard error of the parameter estimate, $\hat{\beta}_{ii}$ using equation (4.27),

S. E $\hat{\beta}_{ii}$=$(5.4493)^{1/2}$ =2.3344

$$t = \frac{\hat{\beta}_i}{\sqrt{\operatorname{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)}$$

t_n^2=3.2471/2.3344=0.2872

t_p^2=5.3785/2.3344=2.7865 (4.65)

t_k^2=10.7726/2.3344=3.7215

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we reject the null hypothesis of quadratic of P and K, thus the parameters were significant and the predictor quadratic variables of phosphorus and potassium were required in explaining the variation of the *rose coco* yield at $\alpha = 0.05$. The quadratic effect of nitrogen was insignificant.

4.4.7.4 Test for β_{ij}

Hypothesis

H_0: $\beta_{ij} = 0$, against

H_1: $\beta_{ij} \neq 0$.

The standard error of the parameter estimate, $\hat{\beta}_{ij}$ using equation (4.27),

S. E $\hat{\beta}_{ij}$ =$(7.0686)^{1/2}$ =2.6587

$$t = \frac{\hat{\beta}_i}{\sqrt{\text{var}(\hat{\beta}_i)}} \sim t_{(\alpha, n-p)}$$

t_{np}=-10.8073/2.6587=-4.0649

t_{nk}=-0.2039/2.6587=-0.0767 (4.66)

t_{pk}=-4.6900/2.6587=-1.7640

Since $-t_{\alpha/2} < tc < t_{\alpha/2}$, we reject the null hypothesis of NP, thus the parameter was significant and the predictor variable-N*P was important in explaining the variation of the *rose coco* yield at α = 0.05, others were insignificant.

Table 4. 23: The ANOVA results on the three replicate of rose coco beans-GP3G.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
FO(N, P, K)	3	1724.2	574.75	1.1641	0.33070
TWI(N, P, K)	3	3648.8	1216.28	2.4634	0.07072
PQ(N, P, K)	3	1875.1	625.05	1.2659	0.29388
Residuals	62	30612.4	493.75		
Lack of fit	14	2925.7	208.98	0.3623	0.97935
Pure error	48	27686.7	576.81		

The table 4.23 comes from the three replicates in GP3G to show the subdivision of residuals into lack of fit and pure error.

Table 4. 24: The ANOVA results in the fertilizer concentration on rose coco bean-GP3G.

Type I Sum

Regression	DF	of Squares	Mean Sq	R-Square	F Value	Pr > F
First Order	3	712.400678	237.467	0.2314	5.58	0.0099
Pure Quadratic	3	936.326222	312.109	0.3041	7.34	0.0034
Two-Factor Interaction	3	834.750000	278.250	0.2711	6.54	0.0054
Residuals	14	595.48	42.534			
Lack of fit	14	595.48	42.534			
Pure error	0	0.00				
Total Model	9	2483.476900		0.8066	6.49	0.0011

Coefficient of variation (CV)= 10.3044, coefficient determination (R^2)= 0.8066, Adjusted R-squared =0.6823, correlation coefficient (r)=0.8981, root MSE=6.521839, response mean=63.291667, PRESS=2645.6089489

Table 4.24 is the results of the averaged data of the three replicates in GP3G. The statistical testing of the model was done by the Fisher's statistical test for analysis of variance (ANOVA) and the results are shown in table 4.24. In the table 4.24, all the linear (p=0.0099), quadratic (p=0.0034) and cross product (p=0.0054) terms were significant at 1%; therefore the total model was significant with p values of 0.0011.The second order model for GP3G was highly significant. The analysis of variance (F-test) showed that the second model fits well with the experimental data. The goodness of fit of the model can be checked by the determination coefficient (R^2) and correlation coefficient (r). The determination coefficient (R^2) implies that the sample variation of 80.66% with mean response of 63.2917gms of *rose coco* bean production was attributed to the independent variables, and about 19.34% of the total variation couldn't be explained by the model, implying that R^2 in GP3G of second order model explain 80.66% of the variation in the model than R^2=23.12% of first order model in equation (4.50). The closer the value of R (correlation coefficient) to one, the better the correlation between the experimental and predicted values. Here, the value of R (0.8981) for equation (4.67) being close to 1 indicated a close agreement between the experimental results and the theoretical values predicted by the model equation.

Table 4. 25: The estimated effects & coefficients of the empirical model for GP3G.

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	51.751435	3.410664	15.17	<.0001***
N	1	1.912726	1.725989	1.11	0.2865
P	1	1.987578	1.725989	1.15	0.2688
K	1	-6.502796	1.725989	-3.77	0.0021**
N*N	1	3.247054	2.335319	1.39	0.1861
P*N	1	-10.807347	2.659765	-4.06	0.0012**
P*P	1	5.378536	2.335319	2.30	0.0371*
K*N	1	-0.203912	2.659765	-0.08	0.9400
K*P	1	-4.689981	2.659765	-1.76	0.0997•
K*K	1	10.772613	2.335319	4.61	0.0004***

Signif. codes:'***'significant at 0.001,'**'significant at 0.01,'*'significant at 0.05, '•'significant at 0.1,' ' significant at 1.

Table 4.25 lists the regression coefficients and the corresponding p-values for the second-order polynomial model given as in eq. 4.67 as:

$$\hat{Y}_{GP3G} = 51.7514 + 1.9127X_1 + 1.9876X_2 - 6.5028X_3 + 3.2471X_1^2 + 5.3785X_2^2 + 10.7726X_3^2 - 10.8073X_1X_2 - 0.2039X_1X_3 - 4.6900X_2X_3 \quad (4.67)$$

The $\hat{Y}_{GP3G}$ is the predicted response for *rose coco* beans in group 3. The regression coefficients of the linear term for potassium (p=0.0021) have significant effects on the yield (p-value <0.05), the quadratic P^2 (p=0.0371) and K^2 (p=0.0004) have significant effects on the yield and the interaction terms in N*P (nitrogen and phosphorus, p=0.0012) is significant. Among these, K, K^2 (quadratic potassium), N*P was significant at the 1% significance level, while P^2 (quadratic phosphorus) was significant at the 5% level, those other terms of the model showed no significant effect on the yield. The positive coefficients of P^2, K^2 enhance the yield since they are the largest coefficients in the model equation (4.67). The largest negative coefficient of K and NP minimizes the yield of rose coco at their respective fertilizer input. The X_1 represents Nitrogen (N), X_2 represents Phosphorus (P), X_3 represents Potassium (K).

This suggests that Potassium (K), quadratic effects of phosphorus (P^2), quadratic effects of potassium (K^2) and interaction N*P were the determining significant factors on the *rose coco* beans yield as these had the largest coefficients and those other terms of the model showed no significant effect on the yield.

4.4.7.5 The lack of Fit for the three replicates in GP3G

Lack-of-fit tests were performed on the fitted models. The lack-of-fit test compares the variation around the model with pure variation within replicated observations using the results of table 4.9.

Table 4. 26: The lack of fit test for GP3G.

Residual	DF	Sum of Squares	Mean Square	F Value	Pr > F
Lack of Fit	14	1786.444300	127.603164	0.25	0.9968
Pure Error	48	24694	514.458333		
Total Error	62	26480	427.103940		

The lack of fit of the model was insignificant, so the second order model was adequate for the experiments of GP3G.

Table 4. 27: The Analysis of Variance Table for N,P,K in 2nd order model- GP3G.

Factor	DF	Sum of Squares	Mean Square	F Value	Pr > F
N	4	836.965298	209.241325	4.92	0.0109
P	4	1116.523540	279.130885	6.56	0.0034
K	4	1641.346675	410.336669	9.65	0.0006

We saw in the table 4.27 that all the three N, P, K are significant factors at 5% level of significance.

4.5 Finding the settings of the experimental factors that produces the optimal response

This section gives the three dimensional surfaces and contour plots, canonical analysis, stationary points and the ridge analysis.

4.5.1 Three dimensional surfaces and contour plots

This section illustrates the three dimensional surfaces and contour plots for the three groups.

4.5.1.1 Three dimensional surfaces and contour plots of GP1G

Graphical visualization of contour plots helps in understanding the second-order response surface. Specifically, three dimensional surface plots and their accompanying contour plots help characterize the shape of the surface and through this we are able to approximately locate the optimum response. Using the fit of the second-order model we illustrate quadratic response surfaces such as minimum, maximum, ridge, and saddle point in the case that an optimum exits, then this point is a stationary point.

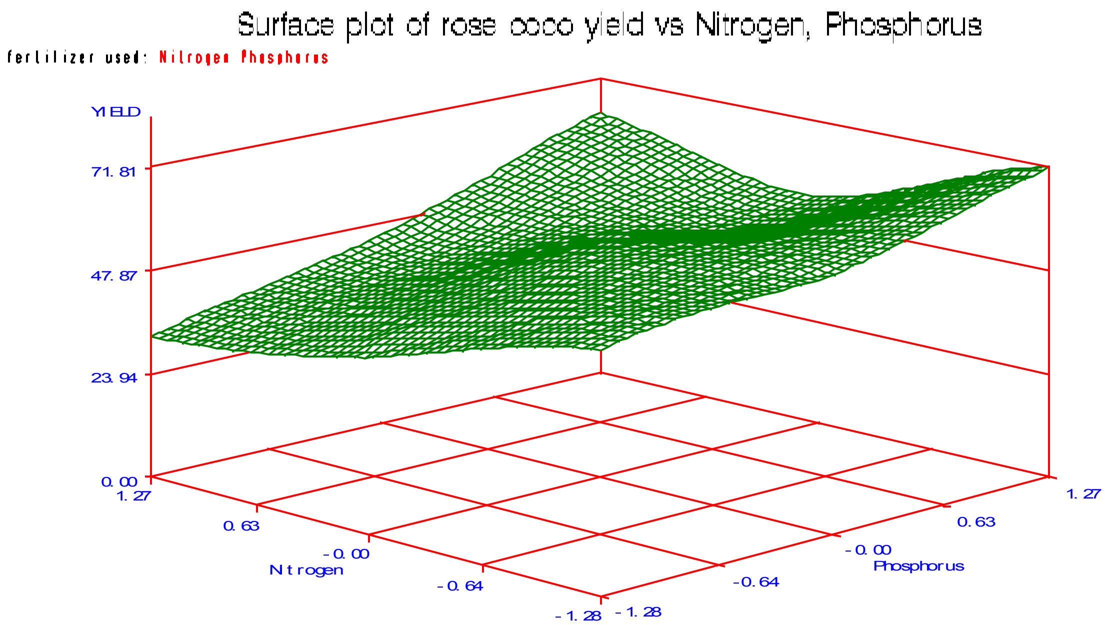


Figure 4.7 a: The response surface plots for the treatments of nitrogen and phosphorus fertilizer concentrations in GP1G.

The N and P are significant, maximum yield of 71.81grams was achieved with lower levels of N and higher P, also high N and high P still gave higher yield of around 60grams.

The contour plot is a two-dimensional and the third design variable must be held constant to construct the graph. The contour plot for nitrogen and phosphorus when potassium is held constant was given as:

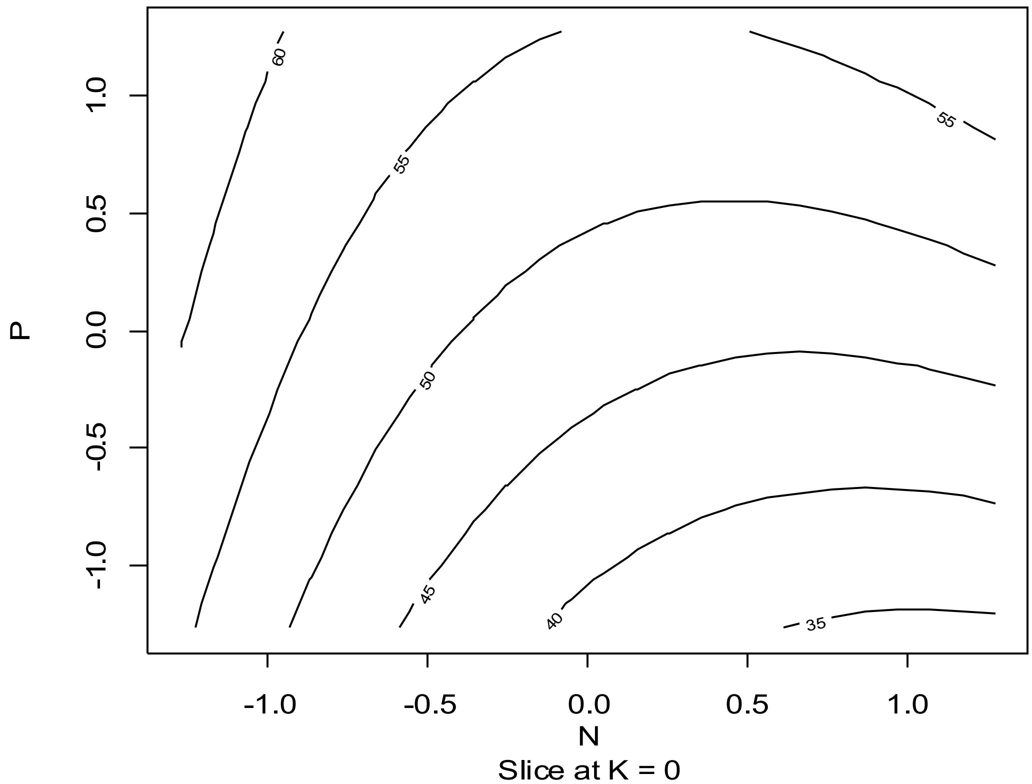


Figure 4.7a 1: The contour plot for nitrogen and phosphorus fertilizers in GP1G of 4.7a.

In the figure 4.7a 1 the maximum yield of 60gram per *rose coco* yield was obtained by reducing the nitrogen input from the center of 10grams and increasing the phosphorus to the maximum scale from the centre point of 20grams.

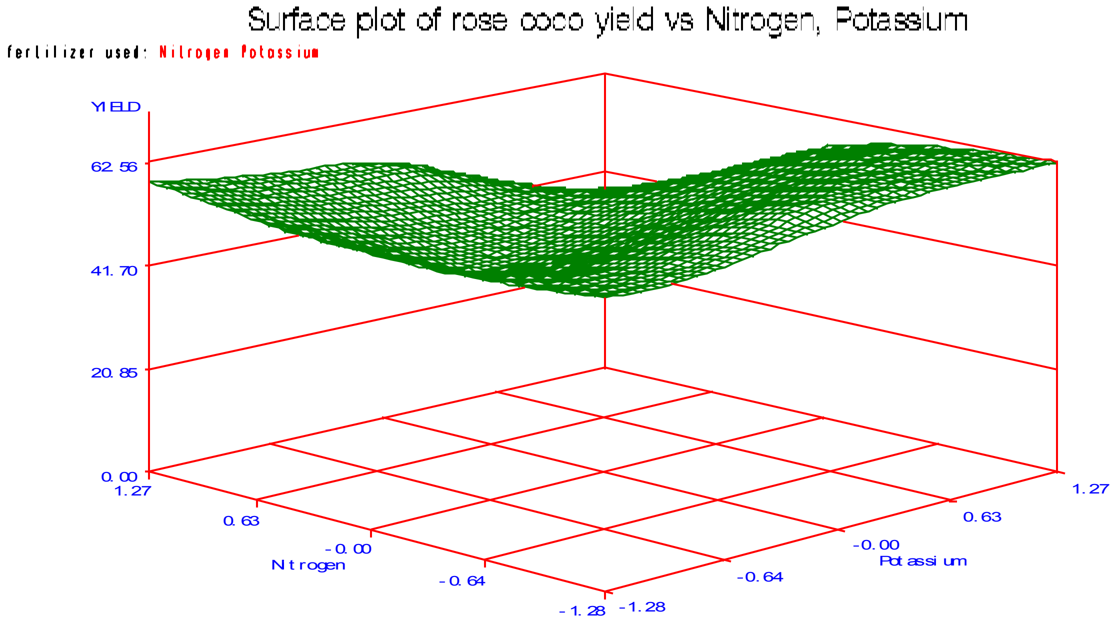


Figure 4.7 b: The response surface plots for the treatments of nitrogen and potassium fertilizer concentrations in GP1G.

In figure 4.7b, high yield of 62.56grams was noticed when we have low levels of nitrogen and high level of potassium, as well high yield with high levels of nitrogen and low levels of potassium.

The contour plot for nitrogen and potassium when phosphorus was held constant is given as:

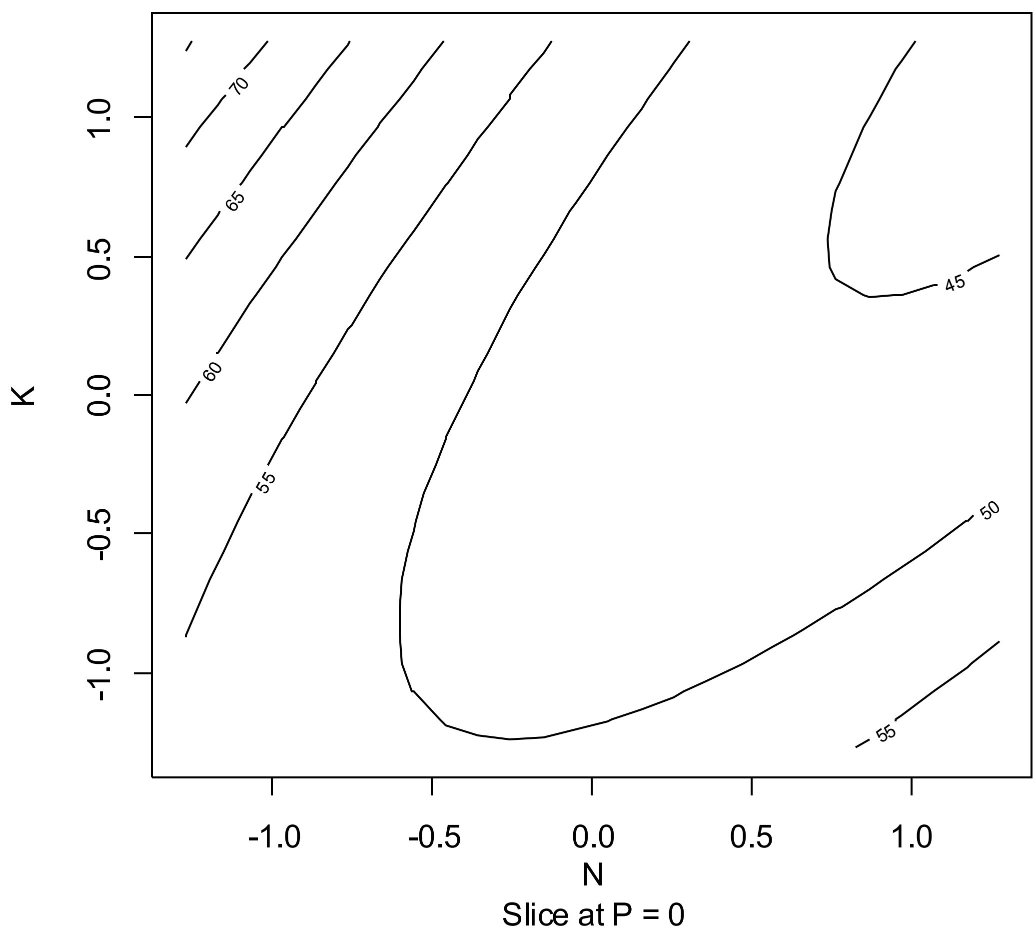


Figure 4.7a 2: The contour plot of nitrogen and potassium fertilizers in GP1G of 4.7b.

In the figure 4.7a 2 the maximum yield of 70grams was obtained by reducing the nitrogen fertilizer input to the lowest scale from the centre point 10grams and increasing the potassium to the maximum scale from the centre point of 30grams.

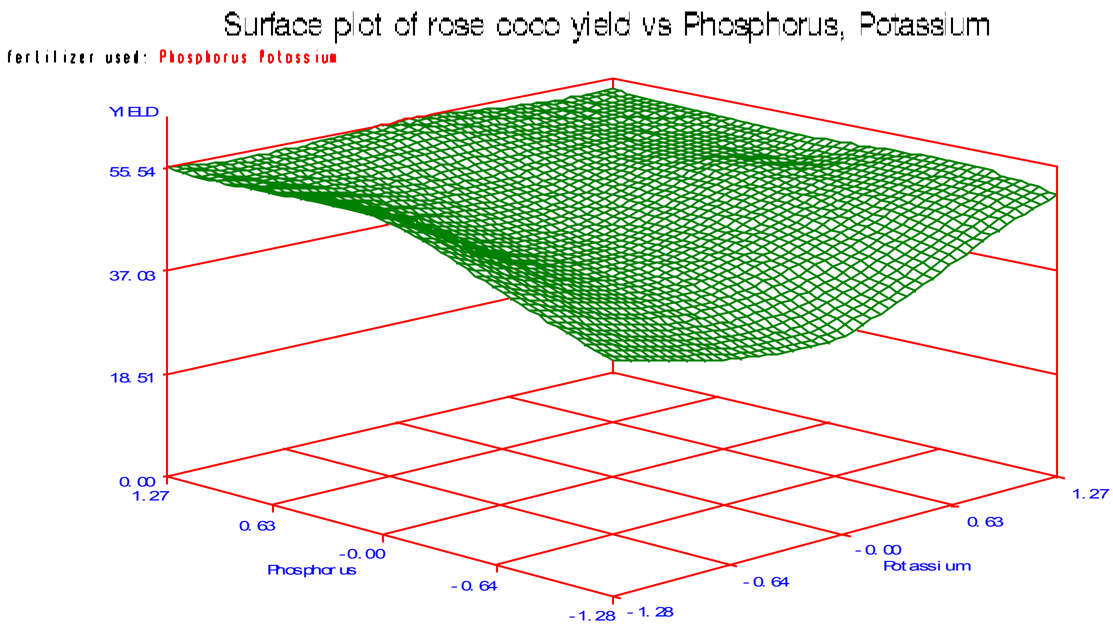


Figure 4.7 c: The response surface plots for the treatments of phosphorus and potassium fertilizer concentrations in GP1G.

In the figure 4.7c, we saw high yield of 55.54grams under high levels of phosphorus from the center point of 20grams and under low levels of potassium fertilizer from the center point 30grams, we saw also high yield with high level of phosphorus with high level of potassium.

The contour plot for phosphorus and potassium when nitrogen was held constant is given as:

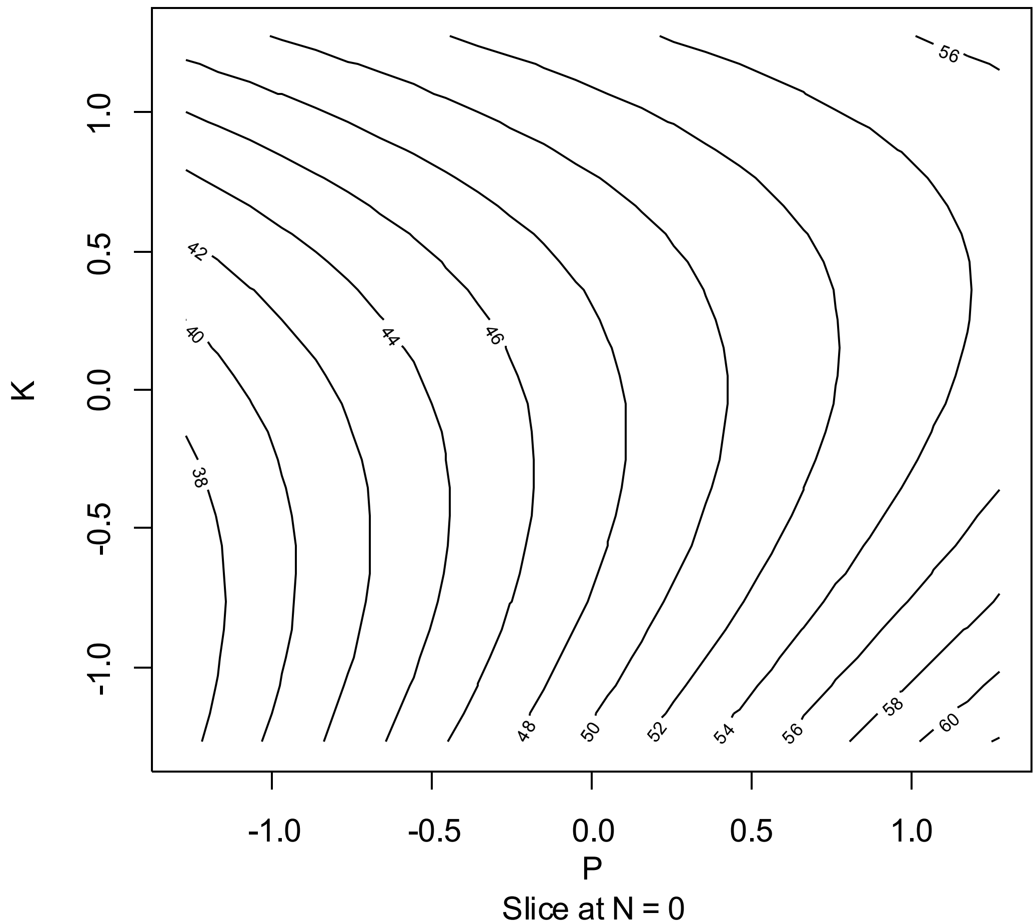


Figure 4.7a 3: The contour plot of phosphorus and potassium fertilizers in GP1G of 4.7c.

In the figure 4.7a 3 the maximum yield of 60grams is obtained by increasing the phosphorus to the maximum scale from the center point 20grams and reducing the potassium to the lowest scale from the centre point of 30grams.

4.5.1.2 Three dimensional surfaces and contour plots of GP2G

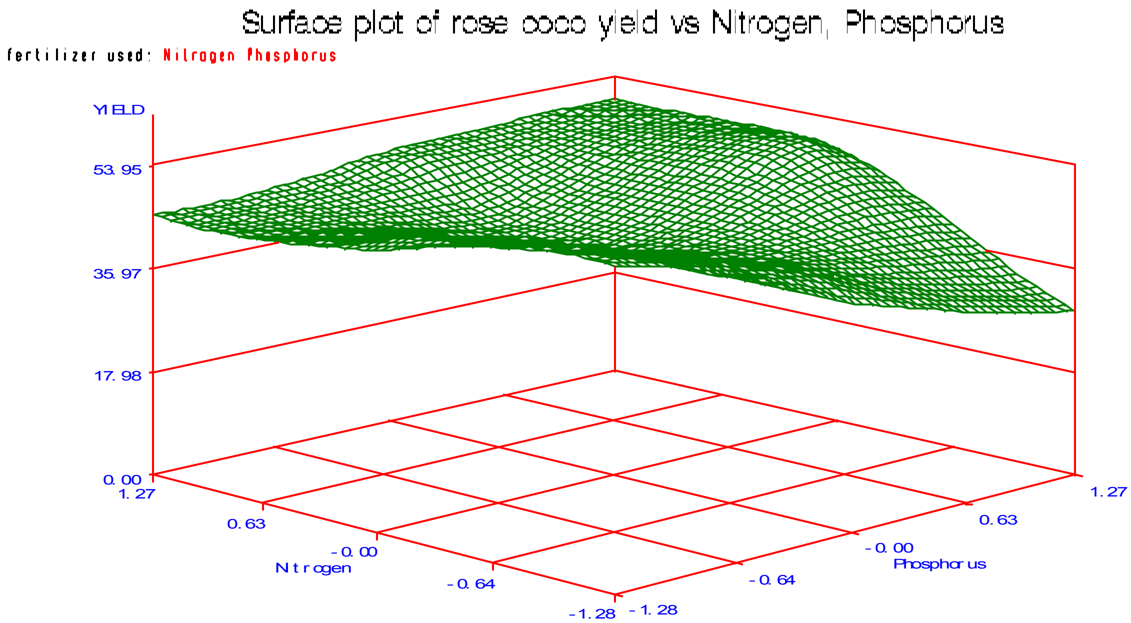


Figure 4.8 a: The response surface plots for the treatments of nitrogen and phosphorus fertilizer concentrations in GP2G.

In the figure 4.8a, maximum yield of 53.95grams is achieved with lower levels of N and lower levels of P, also a yield of about 52grams with high N and high P.

The contour plot is a two-dimensional and the third design variable must be held constant to construct the graph. The contour plot for nitrogen and phosphorus when potassium was held constant is given as:

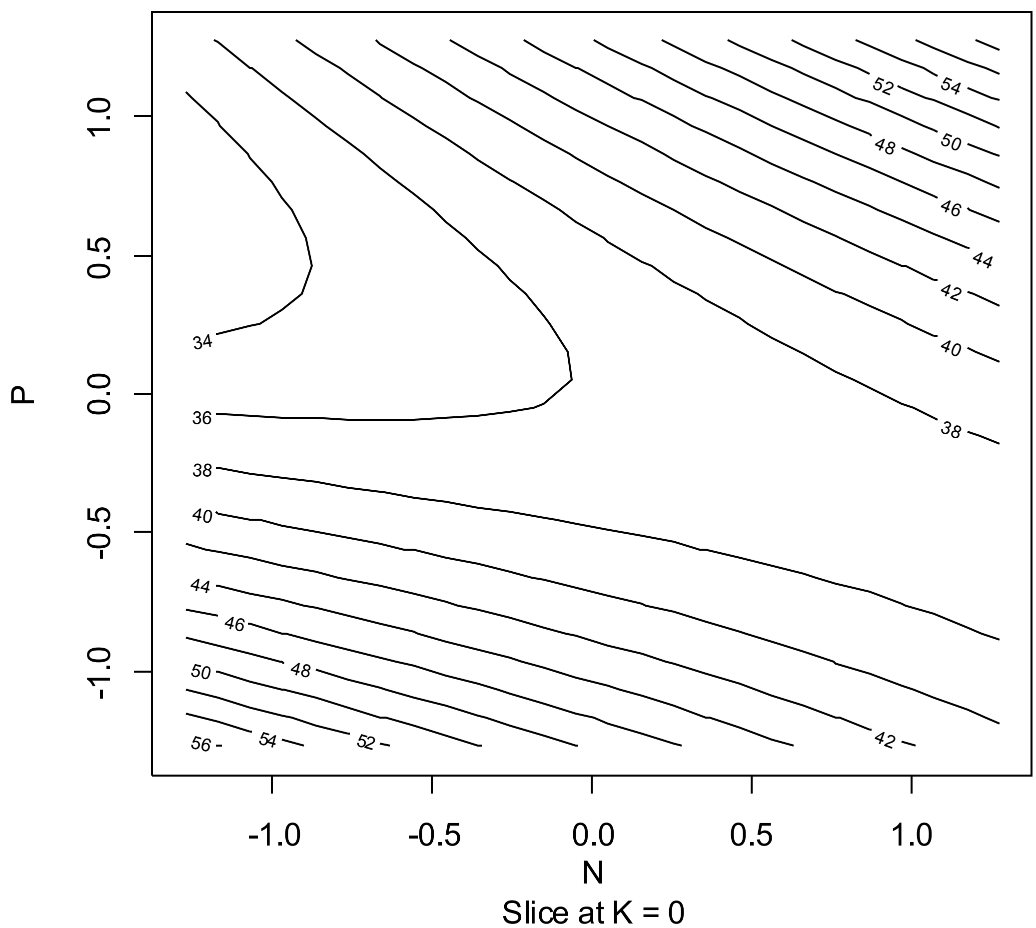


Figure 4.8 b 1: The contour plot of nitrogen and phosphorus fertilizers in GP2G of 4.8a.

In the figure 4.8 b 1 the maximum yield of 56grams per rose coco plant was obtained by lowering the nitrogen input from the center of 20grams and reducing the phosphorus to the lower scale from the centre point of 30grams, also a yield of 58grams is obtained by getting nitrogen and phosphorus at maximum scale.

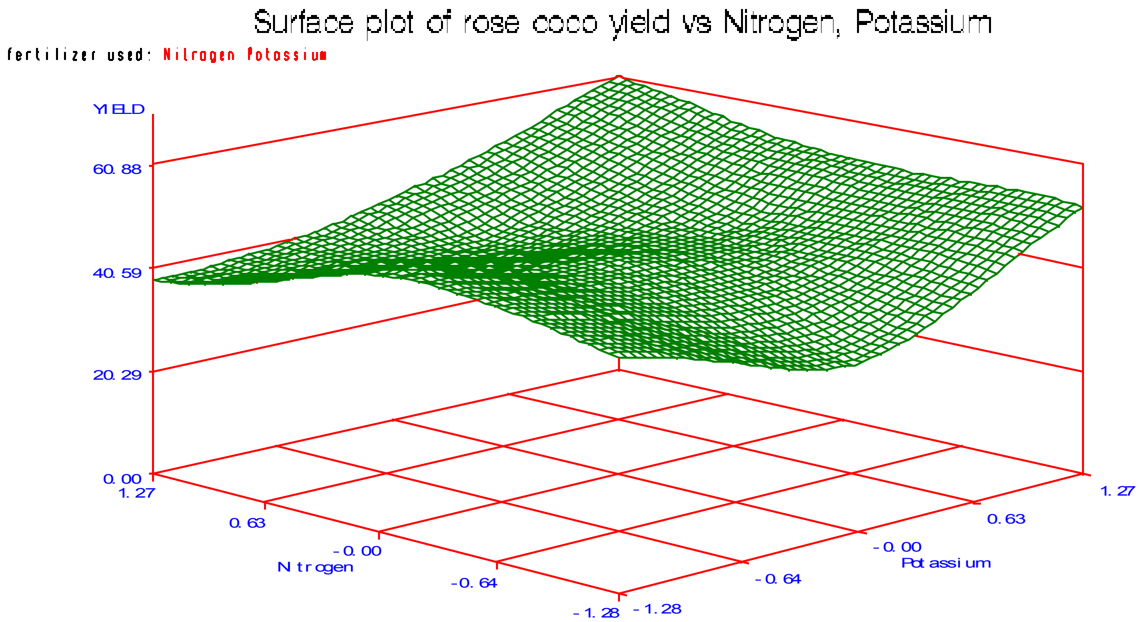


Figure 4.8 b: The response surface plots for the treatments of nitrogen and potassium fertilizer concentrations in GP2G.

In figure 4.8b, high yield of 60.88grams was noticed when we have high levels of nitrogen from the center point of 20grams and high level of potassium from the center point of 40grams.

The contour plot for nitrogen and potassium when phosphorus was held constant is given as:

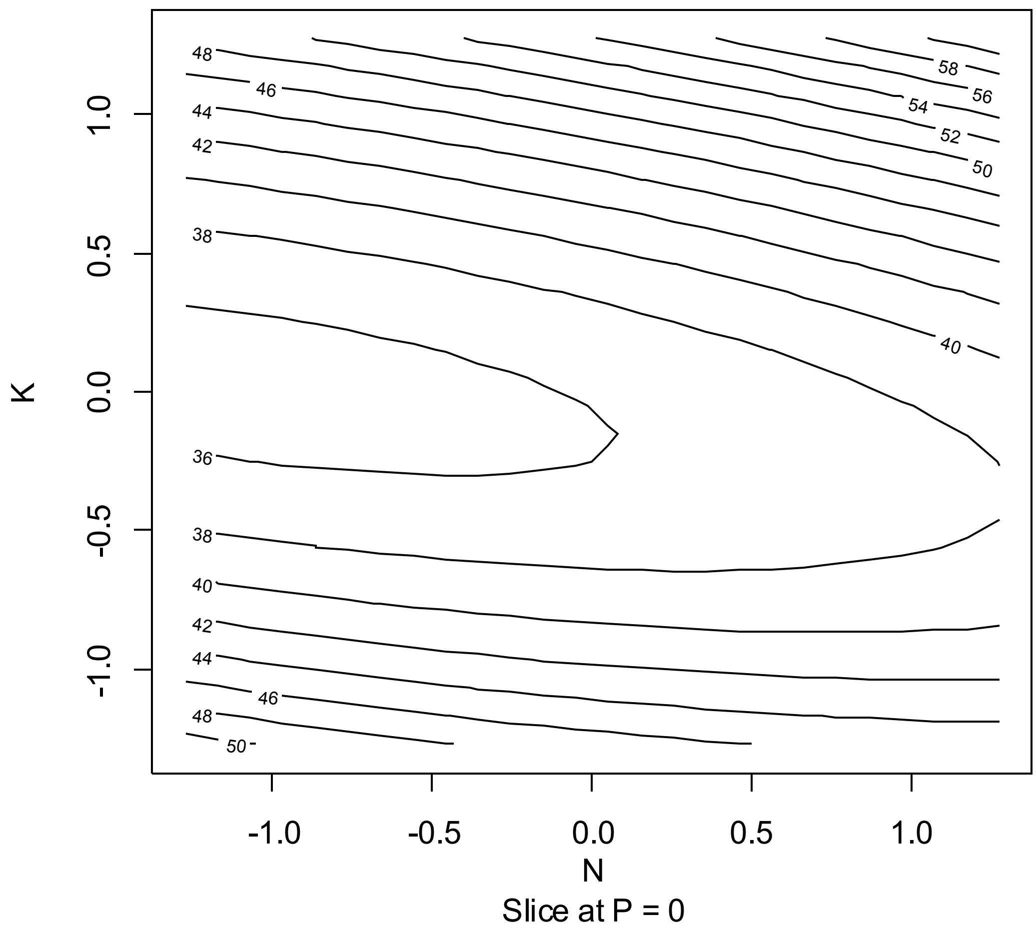


Figure 4.8 b 2: The contour plot of nitrogen and potassium fertilizers in GP2G of 4.8b.

In the figure 4.8 b 2 the maximum yield of 60grams was obtained by increasing the nitrogen fertilizer input to the highest scale from the centre point 20grams and increasing the potassium to the maximum scale from the centre point of 40grams.

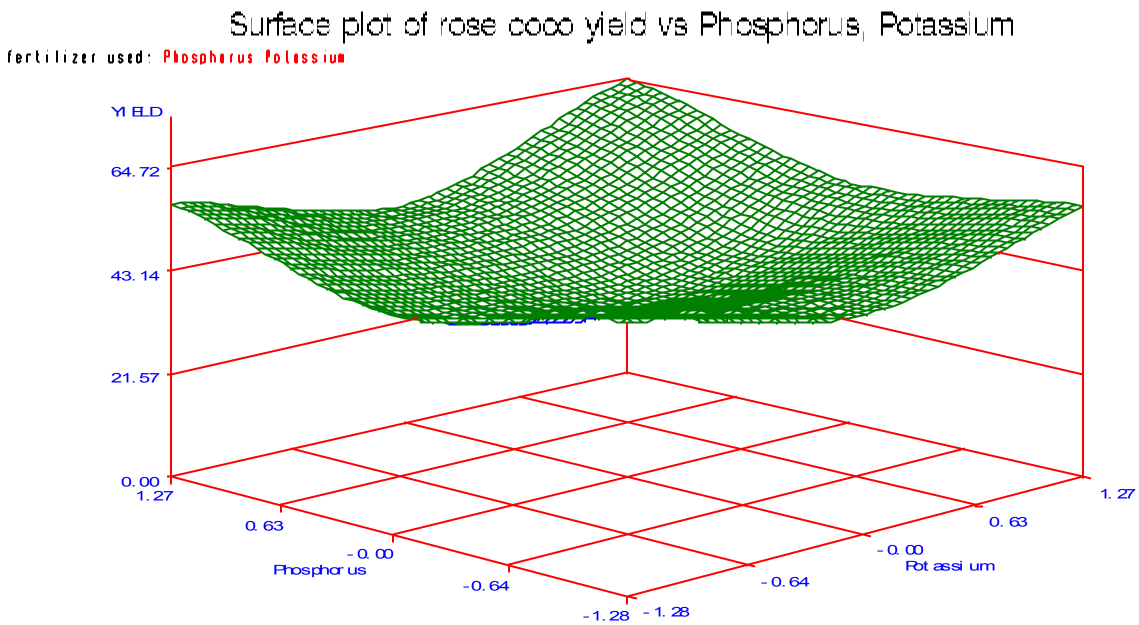


Figure 4.8 c: The response surface plots for the treatments of phosphorus and potassium fertilizer concentrations in GP2G.

In the figure 4.8c maximum yield of 64.72grams of *rose coco* beans was achieved by high levels from the centre point of 30grams of phosphorus and higher levels from the centre point of 40grams of potassium fertilizers.

The contour plot for phosphorus and potassium when nitrogen was held constant is given as:

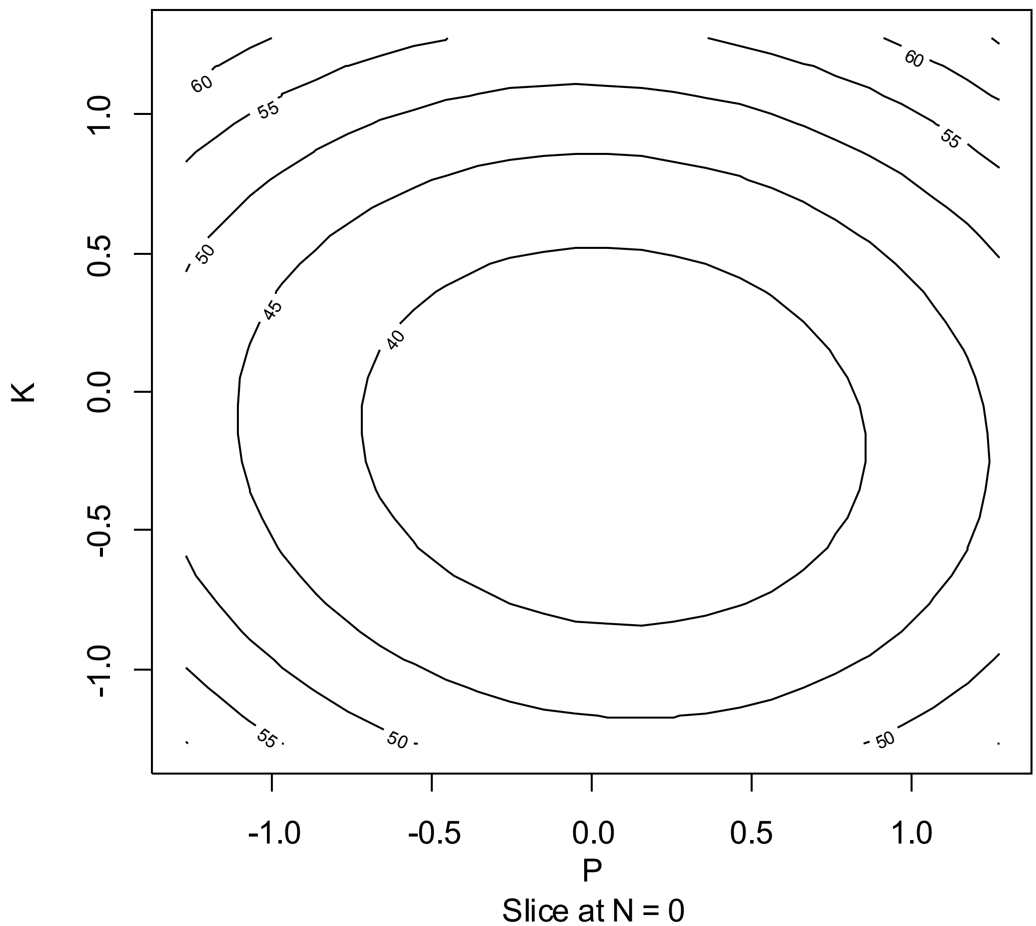


Figure 4.8 b 3: The contour plot of phosphorus and potassium fertilizers in GP2G of 4.8c.

In the figure 4.8 b 3, the maximum yield of 62grams was obtained by increasing the phosphorus to the maximum scale from the center point 30grams and increasing the potassium to the highest scale from the centre point of 40grams.

4.5.1.3 Three dimensional surfaces and contour plots of GP3G

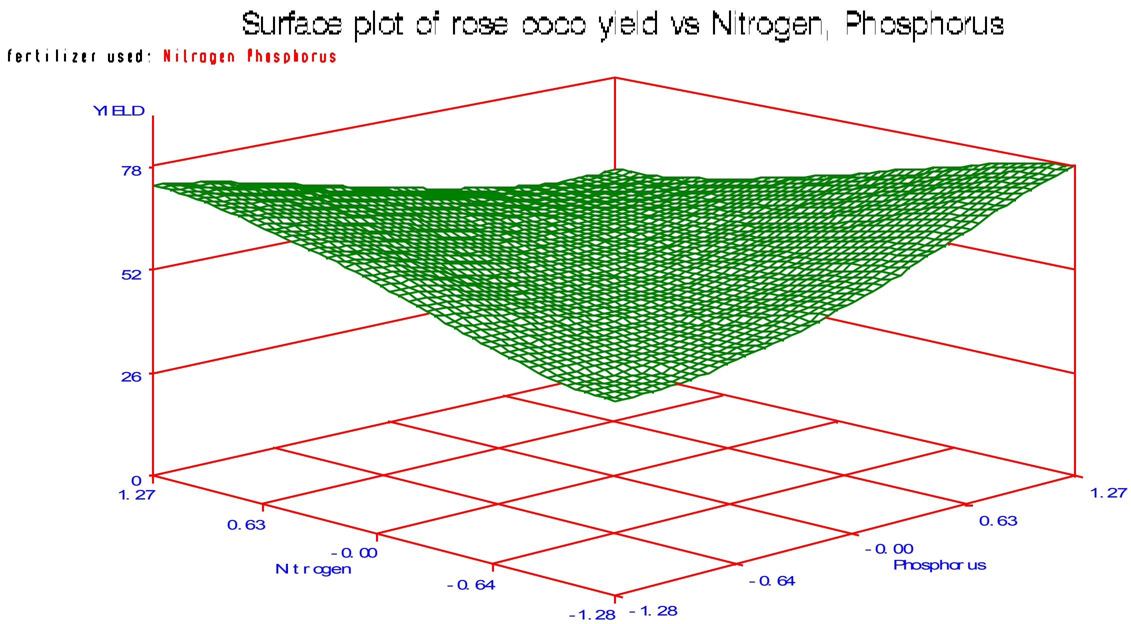


Figure 4.9 a: The response surface plots for the treatments of nitrogen and phosphorus fertilizer concentrations in GP3G.

In the figure 4.9a, maximum yield of 78grams was achieved with lower levels of nitrogen and higher levels of phosphorus, also a yield of about 71grams with high nitrogen and low phosphorus.

The contour plot is a two-dimensional and the third design variable must be held constant to construct the graph. The contour plot for nitrogen and phosphorus when potassium was held constant is given as:

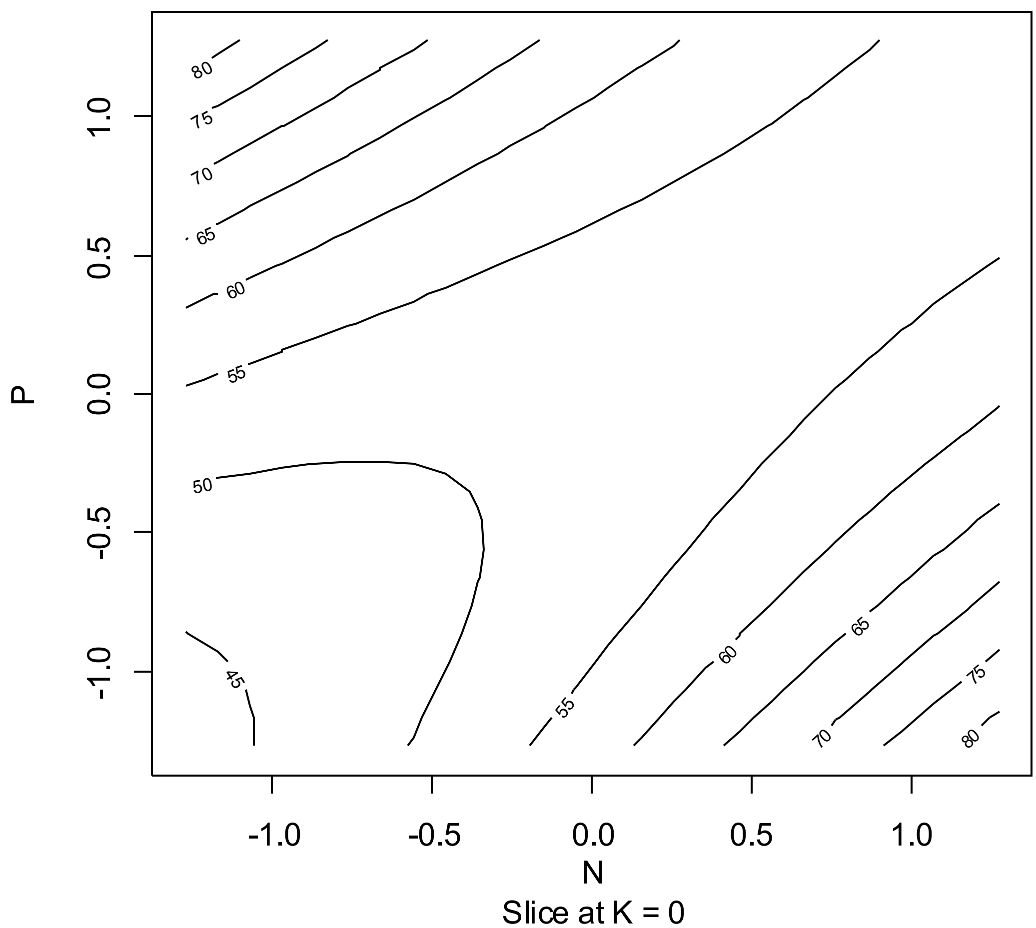


Figure 4.9 c 1: The contour plot of nitrogen and phosphorus fertilizers in GP3G of 4.9a.

In the figure 4.9 c 1 the maximum yield of 80gram per rose coco plant was obtained by lowering the nitrogen input from the center of 30grams and increasing the phosphorus to the higher scale from the centre point of 40grams, the same maximum yield of 80grams was obtained by higher nitrogen and lower phosphorus.

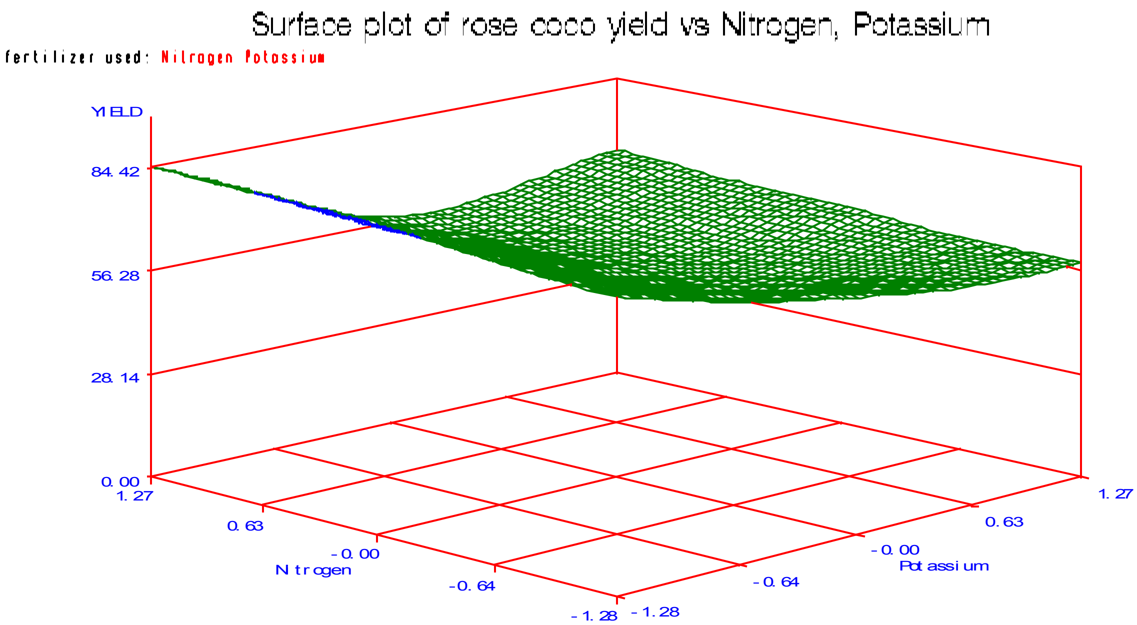


Figure 4.9 b: The response surface plots for the treatments of nitrogen and potassium fertilizer concentrations in GP3G.

In figure 4.9b , high yield of 84.42grams was obtained when we have high level of nitrogen from the center point of 30grams and low level of potassium from the center point of 50grams.

The contour plot for nitrogen and potassium when phosphorus was held constant is given as:

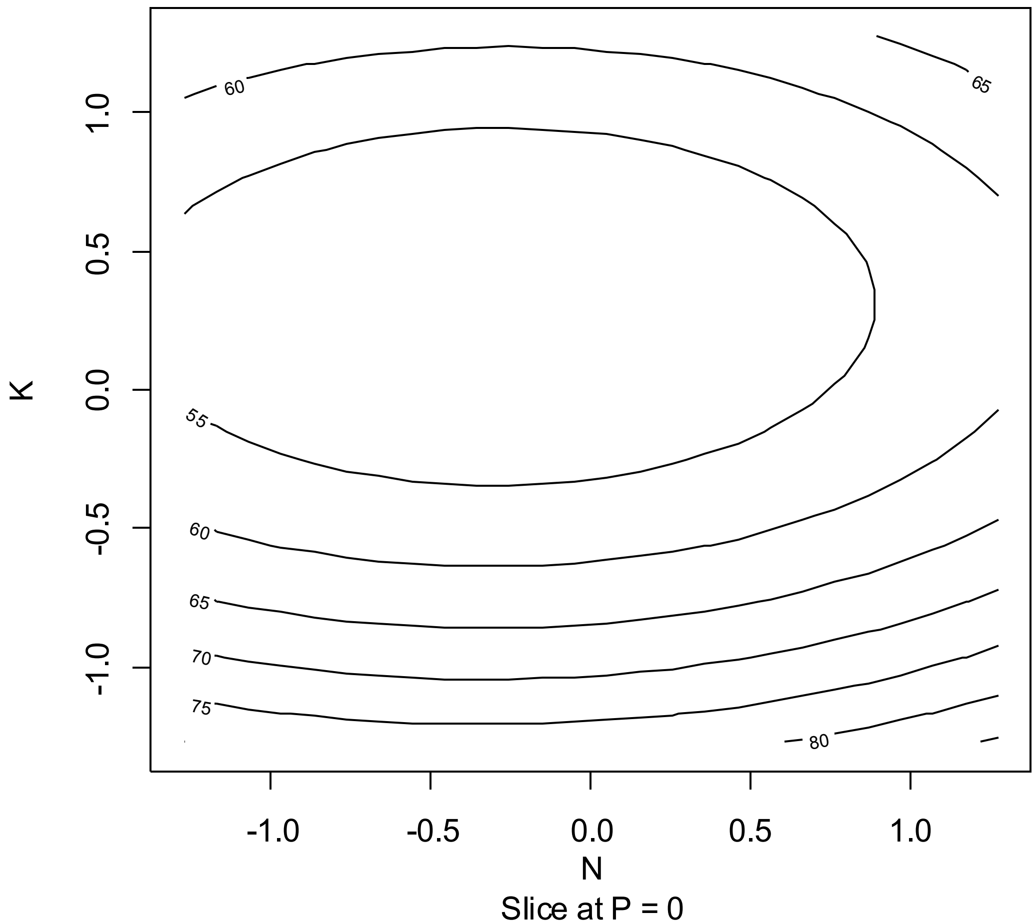


Figure 4.9 c 2: The contour plot of nitrogen and potassium fertilizers in GP3G of 4.9b.

In the figure 4.9 c 2, the maximum yield of 85grams was obtained by increasing the nitrogen fertilizer input to the highest scale from the centre point 30grams and reducing the potassium to the lowest scale from the centre point of 50grams.

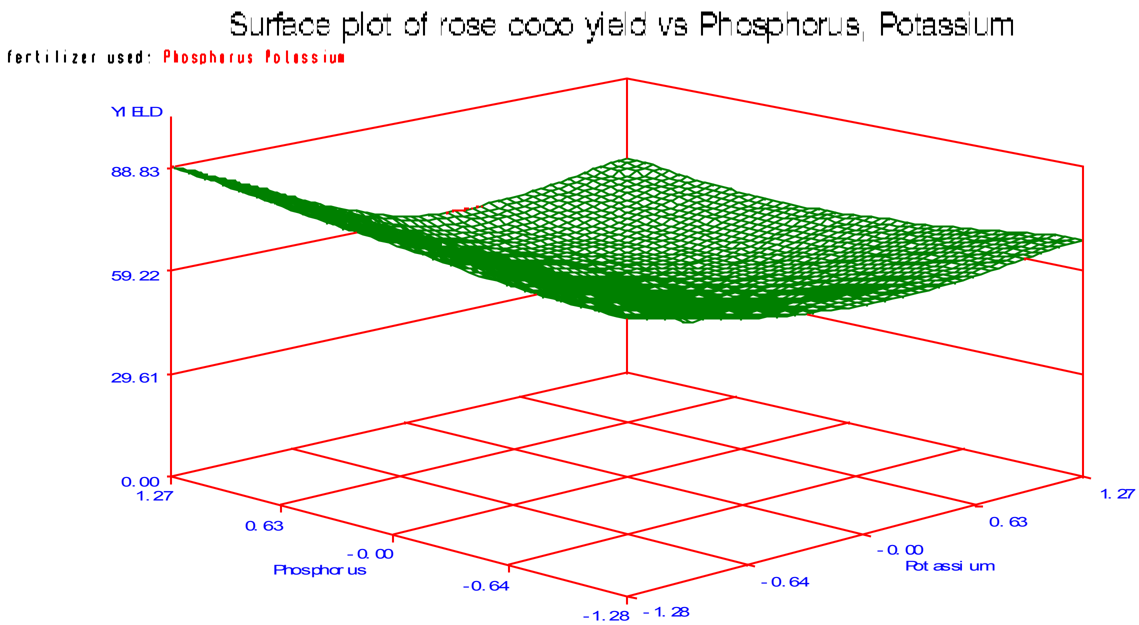


Figure 4.9 c: The response surface plots for the treatments of phosphorus and potassium fertilizer concentrations in GP3G.

In the figure 4.9c maximum yield of 88.83grams of *rose coco* beans was achieved by high levels of phosphorus from the centre point of 40grams and low level of potassium fertilizers from the centre point of 50grams.

The contour plot for phosphorus and potassium when nitrogen was held constant is given as:

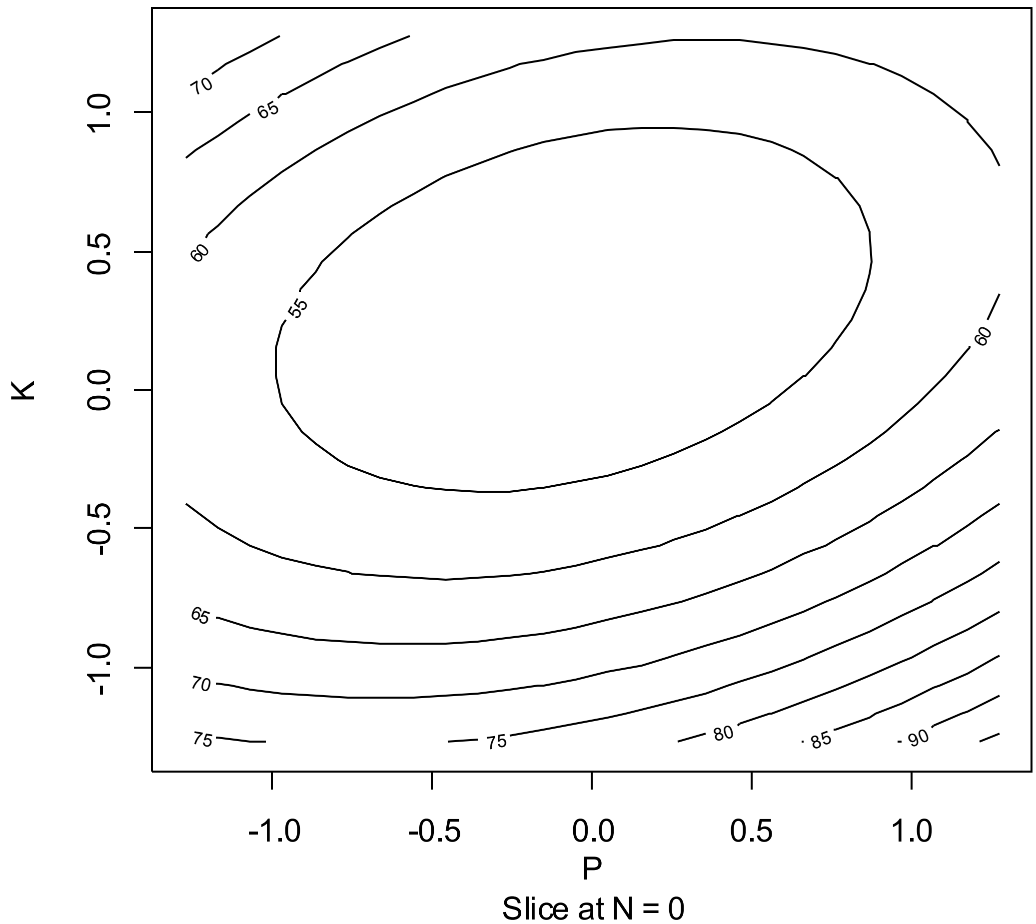


Figure 4.9 c 3: The contour plot of phosphorus and potassium fertilizers in GP3G 4.9c.

In the figure 4.9 c 3, the maximum yield of 95grams was obtained by increasing the phosphorus to the maximum scale from the center point 40grams and reducing the potassium to the lowest scale from the centre point of 50grams.

4.5.2 The canonical analysis

This section gives the canonical analysis for the three groups.

4.5.2.1 The canonical analysis of GP1G

We determined the setting for optimal and efficient production of *rose coco* beans using canonical analysis. Canonical analysis is used to predict the shape of the curve generated by the model. The canonical analysis in table 4.28 and figure 4.7a, 4.7b and 4.7c of the GP1G response surface indicates that the predicted response surfaces are saddle points. The coded eigenvalues are λ_1=6.770341, λ_2=0.483456 and λ_3=-0.907415. The eigenvalue of 6.770341 shows that the valley orientation of the saddle was more curved, eigenvalue 0.4834566 shows that the valley orientation of the saddle was less curved, the less hilly orientation was with an eigenvalue of – 0.967415. The coefficients of the associated eigenvectors show that the valley was more aligned with nitrogen, a less valley with potassium and the hill with phosphorus. Because the canonical analysis resulted in a saddle point, the estimated surface does not have a unique optimum. The surface was more sensitive to the changes of nitrogen compared to fertilizers of K. The results from these saddle point indicated that under natural values 10.978780gms of N, 20.991461gms of P and 33.358334gms of K fertilizer were needed to achieve the saddle point bean yield of 54.88gms per *rose coco* plant.

Table 4. 28: The Canonical Analysis of Response Surface, Eigenvectors and Eigenvalues-GP1G.

Canonical Analysis of Response Surface

Factor	Critical Value
N	1.957550
P	3.304884
K	3.358325

Predicted value at stationary point: 54.881682

Eigenvalues	Eigenvectors N	P	K
6.770341	0.744195	0.251973	-0.618614
0.483456	0.658879	-0.124703	0.741841
-0.907415	-0.109781	0.959666	0.258823

Stationary point is a saddle point

4.5.2.2 The canonical analysis of GP2G

Table 4. 29: The Canonical Analysis of Response Surface, Eigenvectors and Eigenvalues-GP2G.

Canonical Analysis of Response Surface

Factor	Critical Value
N	0.769867
P	-0.283395
K	-0.264186

Predicted value at stationary point: 36.419471

Eigenvalues	Eigenvectors N	P	K
9.570182	0.288816	0.443430	0.848502
7.278125	0.265859	0.814266	-0.516033
-0.747253	0.919731	-0.374620	-0.117284

Stationary point is a saddle point

The canonical analysis table 4.29 and figure 4.8a, 4.8b and 4.8c of response surfaces indicates that the predicted response surface was shaped a saddle point. The coded eigenvalues are λ_1=9.570182, λ_2=7.278125 and λ_3=-0.747253. The eigenvalue of 9.570182 shows that the valley orientation of the saddle was more curved, eigenvalue 7.278125 shows that the valley orientation of the saddle was less curved, the less hilly orientation was with an eigenvalue of – 0.747253. The coefficients of the associated eigenvectors show that the valley was more aligned with potassium, a less valley with phosphorus and the hill with nitrogen. Because the canonical analysis resulted in a saddle point, the estimated surface does not have a unique optimum. The results from these indicated that under natural values 20.384936gms of N, 29.914981gms of P and 39.735813gms of K fertilizer were needed to achieve the saddle point of *rose coco* bean yield of 36.42gms per plant. The positive signs of the eigenvalues for P and K fertilizers indicated the directions of upward curvature. The largest eigenvalue (in absolute) for the N fertilizer, means that the N fertilizer was more pronounced and the curvature of the response surface was in the associated direction. So in GP1G and GP2G nitrogen shows mixed reaction of upward curvature and downward curvature respectively.

4.5.2.3 The canonical analysis of GP3G

Table 4. 30: The Canonical Analysis of Response Surface, Eigenvectors and Eigenvalues-GP3G.

Canonical Analysis of Response Surface

Factor	Critical Value
N	0.444188
P	0.436329
K	0.401005

Predicted value at stationary point: 51.306032

	Eigenvectors		
Eigenvalues	N	P	K
12.224838	0.298898	-0.511793	0.805437
8.574872	-0.586456	0.567316	0.578120
-1.401508	0.752814	0.645152	0.130574

Stationary point is a saddle point

The canonical analysis table 4.30 and figure 4.9a, 4.9b and 4.9c of response surface indicates that the predicted response surface was shaped a saddle point. The coded eigenvalues are λ_1=12.224838, λ_2=8.574872 and λ_3=-1.401508. The eigenvalue of 12.224838 shows that the valley orientation of the saddle was more curved, eigenvalue 8.574872 shows that the valley orientation of the saddle was less curved, the less hilly orientation was with an eigenvalue of 1.401508. The coefficients of the associated eigenvectors show that the valley in both of the first two was more aligned with potassium and the hilly with nitrogen. Because the canonical analysis resulted in a saddle point, the estimated surface does not have a unique optimum. The surface was more sensitive to the changes in amount of K & P, compared to fertilizers of N. The results from these indicated that under natural values 30.222094gms of N, 40.130899gms of P and 50.401005gms of K fertilizer were needed to achieve the saddle point of *rose coco* bean yield of 51.31gms per plant.

4.5.3 The stationary points

This section gives the stationary points for the three groups.

4.5.3.1 The stationary point-GP1G

The stationary point can be found by using matrix algebra. The fitted second-order model in matrix form is:

$$\hat{y} = \hat{\beta}_0 + x'\mathbf{b} + x'\mathbf{B}x$$

The derivative of $\hat{y}$ with respect to the elements of the vector x is

$$\frac{\partial \hat{y}}{\partial x} = \mathbf{b} + 2\mathbf{B}x$$

Therefore, setting the derivative vector to 0 yields the stationary point:

$$x_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$$

This could be a maximum, minimum, or a saddle point of the fitted surface. The eigenvalues (call them λs here) and eigenvectors of **B** are the key to characterizing the shape. The $\mathbf{x_s}$ is a point of maximum if all λ's are negative, the point of minimum if all λ's are positive and saddle point if λ's are of mixed sign.

$$\text{Where } \mathbf{b} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \hat{\beta}_{11} & \frac{1}{2}\hat{\beta}_{12} & \frac{1}{2}\hat{\beta}_{13} \\ & \hat{\beta}_{22} & \frac{1}{2}\hat{\beta}_{23} \\ sym & & \hat{\beta}_{33} \end{bmatrix} \tag{4.68}$$

b is a (3x1) vector of the first-order regression coefficients and **B** is a (3x3) symmetric matrix whose main diagonal elements are the quadratic coefficients and whose off-diagonal elements are one-half the mixed quadratic coefficients (Montgomery, 2005). The estimated response value at the stationary point is

$$\hat{y}_s = \hat{\beta}_0 + \frac{1}{2} x_s' \mathbf{b} \tag{4.69}$$

Therefore, locating stationary points for the yield-GP1G

$$\mathbf{b} = \begin{bmatrix} -5.0452 \\ 6.3461 \\ 1.1566 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3.9485 & 1.32545 & -2.85475 \\ 1.32545 & -0.3983 & -1.32545 \\ -2.85475 & -1.32545 & 2.7962 \end{bmatrix} \quad \mathbf{B}^{-1} = \begin{bmatrix} 0.9663 & -0.0261 & 0.9742 \\ -0.0313 & -0.9733 & -0.4932 \\ 0.9717 & -0.4880 & 1.1184 \end{bmatrix} \tag{4.70}$$

The stationary point using the equation

$x_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$ is

$$x_s = \begin{bmatrix} 1.9571 \\ 3.2946 \\ 3.3530 \end{bmatrix} \tag{4.71}$$

We can find the natural values of nitrogen, phosphorus and potassium using the stationary points from table 4.28.

$$1.9571 = \frac{\psi_{1u} - 10}{0.5}, \quad \psi_{1u} = 10.97855, \quad 3.2946 = \frac{\psi_{2u} - 20}{0.3}, \quad \psi_{2u} = 20.98838 \tag{4.72}$$

$$3.353 = \frac{\psi_{3u} - 30}{1}, \ \psi_{3u} = 33.353$$

Using the equation $\hat{y} = \hat{\beta}_0 + x'\mathbf{b} + x'\mathbf{B}x$, we found out that estimated maximum response yield of *rose coco* bean at the stationary point was

$\hat{Y} = 54.88$ grams

Thus, it could be concluded that this level of main factors setting resulted in saddle optimum yield for the *rose coco* bean for the given amount of predictor variables. The maximum yield of *rose coco* bean is obtained when under this combination of fertilizers in GP1G

$$-0.850004 = \frac{\psi_{1u} - 10}{0.5}, \ \psi_{1u} = 9.574998, \ 0.157611 = \frac{\psi_{2u} - 20}{0.3}, \ \psi_{2u} = 20.0472833 \quad (4.73)$$

$$0.502645 = \frac{\psi_{3u} - 30}{1}, \ \psi_{3u} = 30.502645$$

Therefore, we can obtain a maximum yield of 58.68gms per *rose coco* bean with a combination of 9.57gms nitrogen, 20.05gms phosphorus and 30.50gms potassium fertilizers.

4.5.3.2 The stationary point-GP2G

Locating stationary points for the yield-GP2G was given as;

$$\mathbf{b} = \begin{bmatrix} 1.4399 \\ -0.6980 \\ 2.7503 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.6806 & 3.0587 & 1.4274 \\ 3.0587 & 6.6025 & 0.5098 \\ 1.4274 & 0.5098 & 8.8179 \end{bmatrix} \quad \mathbf{B}^{-1} = \begin{bmatrix} -1.1135 & 0.5042 & 0.1511 \\ 0.5042 & -0.0762 & -0.0772 \\ 0.1511 & -0.0772 & 0.0934 \end{bmatrix} \quad (4.74)$$

The stationary point using the equation

$x_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$ is

$$x_s = \begin{bmatrix} 0.7699 \\ -0.2834 \\ -0.2642 \end{bmatrix} \quad (4.75)$$

This was a saddle point since it showed mixed signs of λ's.

We could find the natural values of nitrogen, phosphorus and potassium fertilizers using the stationary point from table 4.29.

$$0.7699 = \frac{\psi_{2u} - 20}{0.5} \; , \; \psi_{1u} = 20.38495 \; , \; -0.2834 = \frac{\psi_{2u} - 30}{0.3} \; , \; \psi_{2u} = 29.91498$$

$$-0.2642 = \frac{\psi_{3u} - 40}{1} \; , \; \psi_{3u} = 39.7358 \qquad (4.76)$$

Using the equation $\hat{y} = \hat{\beta}_0 + x'\mathbf{b} + x'\mathbf{B}x$, we can find that estimated maximum response *yield* of *rose coco* bean at the stationary point was

$\hat{Y} = 36.4gm$

Thus, it could be concluded that this level of main factors setting results in best optimum yield for the *rose coco* beans. The maximum yield of *rose coco* beans was obtained when under this combination of fertilizers in GP2G

$$0.270935 = \frac{\psi_{1u} - 20}{0.5} \; , \; \psi_{1u} = 20.1354675 \; , \; 0.226583 = \frac{\psi_{2u} - 30}{0.3} \; , \; \psi_{2u} = 30.0679749$$

$$0.935550 = \frac{\psi_{3u} - 40}{1} \; , \; \psi_{3u} = 40.935550 \qquad (4.77)$$

Therefore, we can obtain a maximum yield of 48.36gms per *rose coco* beans with a combination 20.14gms nitrogen, 30.07gms phosphorus and 40.94gms potassium fertilizers.

4.5.3.3 The stationary point-GP3G

Locating stationary points for the yield-GP3G

$$\mathbf{b} = \begin{bmatrix} 1.9127 \\ 1.9876 \\ -6.5028 \end{bmatrix} \; \mathbf{B} = \begin{bmatrix} 3.2471 & -5.40365 & -0.10195 \\ -5.40365 & 5.3785 & -2.3450 \\ -.10195 & -2.3450 & 10.7726 \end{bmatrix} \quad \mathbf{B}^{-1} = \begin{bmatrix} -0.3570 & -0.3979 & -0.0900 \\ -0.3979 & -0.2380 & -0.0556 \\ -0.0900 & -0.0556 & 0.0799 \end{bmatrix}$$

(4.78)

The stationary point using the equation

$x_s = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b}$ is

$$x_s = \begin{bmatrix} 0.4442 \\ 0.4363 \\ 0.4010 \end{bmatrix} \quad (4.79)$$

We could find the natural values of nitrogen, phosphorus and potassium using the stationary points from table 4.30.

$$0.4442 = \frac{\psi_{2u} - 30}{0.5}, \ \psi_{1u} = 30.2221, \ 0.4363 = \frac{\psi_{2u} - 40}{0.3}, \ \psi_{2u} = 40.13089 \quad (4.80)$$

$$0.4010 = \frac{\psi_{3u} - 50}{1}, \ \psi_{3u} = 50.4010$$

Using the equation $\hat{y} = \hat{\beta}_0 + x'\mathbf{b} + x'\mathbf{B}x$, we can find that estimated maximum response yield of *rose coco* beans at the stationary point was

$\hat{Y} = 51.31 grams$

Thus, it can be concluded that this level of main factors setting results in best optimum yield for the *rose coco* bean.

The maximum yield of *rose coco* bean was obtained when under this combination of fertilizers in GP3G

$$-0.077342 = \frac{\psi_{1u} - 30}{0.5}, \ \psi_{1u} = 29.961329, \ 0.365469 = \frac{\psi_{2u} - 40}{0.3}, \ \psi_{2u} = 40.1096407$$

$$-0.927605 = \frac{\psi_{3u} - 50}{1}, \ \psi_{3u} = 49.072395 \quad (4.81)$$

Therefore, we obtained a maximum yield of 70.25gms per *rose coco* bean with a combination of 29.96gms nitrogen, 40.11gms phosphorus and 49.07gms potassium fertilizers.

4.5.4 The ridge analysis

A RIDGE statement computes the ridge of optimum response. The ridge starts at a given point x_0, and the point on the ridge at radius R from x_0 is the collection of factor settings that optimizes the predicted response at this radius. The ridge analysis can be used as a tool to help interpret an existing response surface or to indicate the direction in which further experimentation should be performed. The default starting point, x_0, has each coordinate equal to the point midway between the highest and lowest values of the factor in the design. The default radii at which the ridge is computed are 0, 0.1, ..., 0.9, 1. If the ridge analysis is based on the response surface fit to coded values for the factor variables, then this result in a ridge that starts at the point with a coded zero value for each coordinate and extends toward, but not beyond, the edge of the range of experimentation. Alternatively, both the center point of the ridge and the radii at which it is to be computed can be specified. The starting point should be well inside the range of experimentation. The coded radii give the distances from the ridge starting point at which to compute the optimal (SAS User Guide 2013). Ridge analysis was performed to determine the critical levels of the design variables that produce the maximum response.

4.5.4.1 The ridge analysis –GP1G

The ridge analysis was performed to determine the critical levels of the design variables that produce the maximum and minimum response.

Table 4. 31: The Estimated Ridge of Minimum Response for Variable YIELD-GP1G.

Radius	Estimated Response	Standard Error	Factor Values N	P	K
0.0	47.391125	3.606523	0	0	0
0.1	46.573422	3.594075	0.057896	-0.080662	-0.011904
0.2	45.753978	3.557499	0.109534	-0.166041	-0.020805
0.3	44.928378	3.499176	0.156344	-0.254479	-0.028228
0.4	44.093828	3.423340	0.199358	-0.345000	-0.035092
0.5	43.248466	3.336481	0.239315	-0.437000	-0.041945
0.6	42.390993	3.247865	0.276746	-0.530094	-0.049111
0.7	41.520461	3.170067	0.312049	-0.624021	-0.056779
0.8	40.636149	3.119186	0.345525	-0.718596	-0.065057
0.9	39.737496	3.114224	0.377413	-0.813685	-0.073998
1.0	38.824051	3.175047	0.407901	-0.909188	-0.083625

Table 4. 32: The Estimated Ridge of Maximum Response for Variable YIELD-GP1G.

Radius	Estimated Response	Standard Error	Factor Values N	P	K
0.0	47.391125	3.606523	0	0	0
0.1	48.214348	3.594075	-0.065957	0.073143	0.017314
0.2	49.055109	3.557499	-0.141593	0.134455	0.043284
0.3	49.931933	3.499176	-0.226741	0.179101	0.080692
0.4	50.868599	3.423340	-0.318150	0.205031	0.129395
0.5	51.888533	3.336481	-0.411298	0.214920	0.186128
0.6	53.009694	3.247865	-0.503250	0.213572	0.247237
0.7	54.243860	3.170067	-0.592995	0.205018	0.310362
0.8	55.598337	3.119186	-0.680503	0.191884	0.374295
0.9	57.077682	3.114224	-0.766061	0.175760	0.438473
1.0	58.684816	3.175047	-0.850004	0.157611	0.502645

4.5.4.2 The ridge analysis-GP2G

The ridge analysis was performed to determine the critical levels of the design variables that produce the maximum and minimum response.

Table 4. 33: The Estimated Ridge of Minimum Response for Variable YIELD-GP2G.

Radius	Estimated Response	Standard Error	Factor Values N	P	K
0.0	36.129616	3.459657	0	0	0
0.1	35.873343	3.447716	-0.065440	0.034219	-0.067429
0.2	35.682881	3.412629	-0.160534	0.079533	-0.088901
0.3	35.501614	3.356681	-0.258543	0.122718	-0.089978
0.4	35.313422	3.283933	-0.354944	0.163818	-0.084724
0.5	35.113822	3.200611	-0.450083	0.203742	-0.076907
0.6	34.901149	3.115604	-0.544396	0.242973	-0.067807
0.7	34.674645	3.040974	-0.638159	0.281766	-0.057969
0.8	34.433913	2.992165	-0.731544	0.320266	-0.047669
0.9	34.178724	2.987405	-0.824658	0.358560	-0.037059
1.0	33.908940	3.045751	-0.917572	0.396704	-0.026231

Table 4. 34: The Estimated Ridge of Maximum Response for Variable YIELD-GP2G.

Radius	Estimated Response	Standard Error	Factor Values N	P	K

0.0	36.129616	3.459657	0	0	0
0.1	36.527378	3.447716	0.034989	-0.011132	0.093015
0.2	37.097337	3.412629	0.061455	-0.006476	0.190214
0.3	37.848386	3.356681	0.086399	0.008282	0.287170
0.4	38.784290	3.283933	0.111471	0.029901	0.382988
0.5	39.907098	3.200611	0.137023	0.056351	0.477545
0.6	41.218069	3.115604	0.163074	0.086296	0.570929
0.7	42.718028	3.040974	0.189561	0.118833	0.663284
0.8	44.407541	2.992165	0.216410	0.153337	0.754755
0.9	46.287011	2.987405	0.243552	0.189361	0.845473
1.0	48.356732	3.045751	0.270935	0.226583	0.935550

4.5.4.3 The ridge analysis-GP3G

The ridge analysis was performed to determine the critical levels of the design variables that produce the maximum and minimum response.

Table 4. 35: The Estimated Ridge of Minimum Response for Variable YIELD-GP3G.

Radius	Estimated Response	Standard Error	Factor Values N	P	K
0.0	51.751435	3.410664	0	0	0
0.1	51.142673	3.398892	-0.038479	-0.031963	0.086589
0.2	50.696721	3.364302	-0.106285	-0.080759	0.148935
0.3	50.337171	3.309146	-0.193912	-0.147243	0.175264
0.4	49.998756	3.237429	-0.282585	-0.218021	0.180589
0.5	49.653355	3.155286	-0.368113	-0.288100	0.177458
0.6	49.290454	3.071483	-0.451052	-0.357014	0.170567
0.7	48.905533	2.997910	-0.532192	-0.424991	0.161723
0.8	48.496347	2.949793	-0.612079	-0.492276	0.151737
0.9	48.061651	2.945100	-0.691071	-0.559047	0.141024
1.0	47.600701	3.002620	-0.769404	-0.625432	0.129819

Table 4. 36: The Estimated Ridge of Maximum Response for Variable YIELD-GP3G.

Radius	Estimated Response	Standard Error	Factor Values N	P	K
0.0	51.751435	3.410664	0	0	0
0.1	52.563118	3.398892	0.019147	0.027378	-0.094254
0.2	53.593652	3.364302	0.026607	0.055878	-0.190184
0.3	54.850777	3.309146	0.026256	0.087039	-0.285893
0.4	56.339123	3.237429	0.020297	0.121001	-0.380719
0.5	58.061791	3.155286	0.010145	0.157476	-0.474445
0.6	60.020980	3.071483	-0.003227	0.196086	-0.567045
0.7	62.218302	2.997910	-0.019119	0.236473	-0.658571
0.8	64.654972	2.949793	-0.037012	0.278327	-0.749109
0.9	67.331915	2.945100	-0.056519	0.321394	-0.838756
1.0	70.249853	3.002620	-0.077342	0.365469	-0.927605

In general the ridge analysis estimated maximum response for the variable yield was 58.68gms, 48.36gms and 70.25gms for GP1G, GP2G and GP3G respectively at nitrogen of 9.57gms, phosphorus of 20.05gms and potassium of 30.50gms for GP1G, nitrogen of 20.14gms, phosphorus of 30.07gms and potassium of 40.94gms for GP2G and nitrogen of 29.96gms, phosphorus of 40.11gms and potassium of 49.07gms for GP3G.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter presents conclusions and recommendations in line with the four specific objectives.

5.2 Estimation of the linear parameters in an existing A-optimum and D-efficient calculus optimum value second order rotatable design

The study was able to estimate the parameter coefficient successfully for the three groups. Below is the table 5.1 showing the parameter coefficients obtained and their p-values.

Table 5. 1: Regression coefficients of predicted second order polynomial models for the response of GP1G, GP2G and GP3G.

Coefficients	GP1G (p-value)	GP2G (p-value)	GP3G (p-value)
β_0 intercept	47.391125 (0.0001)	36.129616 (0.0001)	51.751435 (0.0001)
Linear			
β_1 (N)	-5.045204 (0.0152)	1.439859 (0.4246)	1.912726 (0.2865)
β_2 (P)	6.346081 (0.0035)	-0.697954 (0.6962)	1.987578 (0.2688)
β_3 (K)	1.156615 (0.5365)	2.750284 (0.1385)	-6.502796 (0.0021)
Quadratic			
β_{11} (N^2)	3.948538	0.680614	3.247054

	(0.1321)	(0.7781)	(0.1861)
β_{22} (P^2)	-0.398323 (0.8742)	6.602530 (0.0145)	5.378536 (0.0371)
β_{33} (K^2)	2.796168 (0.2765)	8.817910 (0.0023)	10.772613 (0.0004)
Cross product			
β_{12} (N*P)	2.650859 (0.3619)	6.117366 (0.0397)	-10.807347 (0.0012)
β_{13} (N*K)	-5.709542 (0.0618)	2.854771 (0.3079)	-0.203912 (0.9400)
β_{23} (P*K)	-2.650859 (0.3619)	1.019561 (0.7112)	-4.689981 (0.0997)
R^2	68.10%	67.04%	80.66%
Adjusted R^2	47.59%	45.85%	68.23%
Sum of squared residuals	666	613	595
Coefficient of variation	13.4782	14.4733	10.3044
PRESS	2488	2091	2645
p -the probability	0.0219	0.0264	0.0011

N=nitrogen fertilizer; P=phosphorus fertilizer; K=potassium fertilizer

We saw in the three groups, that is, GP1G, GP2G and GP3G there was a large difference between ordinary residual and PRESS residual meaning the second order models fitted the data well.

5.3 Obtaining the generalized variance of the estimated linear parameters

In the table 5.2 the variance under first order models for the three groups (GP1G, GP2G, GP3G) are increasing, but for the second order models they are decreasing implying that second order models fitted the data appropriately.

Table 5. 2: The eigenvalues, predicted *rose coco* yield at the stationary points, and uncoded critical values of fertilizer levels.

	Fertilizer	Eigenvalue (λ)	Natural critical value	Variance, σ^2.First order model	Variance, σ^2.Second order model	Predicted rose coco yield at stationary point	Estimated Ridge of Maximum Response for rose coco YIELD
GP1G	N	6.770341	10.978780	56.5	47.5714	54.88gms	58.78gms
	P	0.483456	20.991461				
	K	-0.907415	33.358334				
GP2G	N	9.570182	20.384936	85.7	43.7857	36.42gms	48.36gms
	P	7.278125	29.914981				
	K	-0.747253	39.735813				
GP3G	N	12.224838	30.222094	118.35	42.5	51.31gms	70.25gms
	P	8.574872	40.130899				
	K	-1.401508	50.401005				

5.4 Fitting and testing the three model adequacies

The analysis of variance (ANOVA) of response surface for *rose coco* yield showed that the twenty four second order rotatable designs was adequate due to satisfactory levels of coefficient of determinations, R^2 (0.68, 0.67, 0.81) for the three cases GP1G, GP2G and GP3G respectively and coefficient of variations (CV, 13.48, 14.47 and 10.30 respectively). Generally, high values of CV indicate that experimental design developed was inadequate. In addition, linear, quadratic and cross product terms were all found to be significant at 1% for the GP3G. On the other hand, quadratic was highly significant for GP2G whereas GP1G was significant only on the linear terms. However, for the three groups total model was significant at 5% ((p = 0.0219, 0.0264 and 0.0011, respectively). The results showed GP2G and GP3G fitted the second order models well for the *rose coco* yield using the three fertilizer treatments, nitrogen (N), phosphorus (P) and potassium (K).

Table 5. 3: The analysis of variance of the factors and the critical values of the *rose coco* yield response.

Independent variables[a]	Analysis of variance					Critical values	
	d.f	Sum of squares	Mean square	*F*-value	*P*-value	Coded	Uncoded
GP1G							
N	4	723.277182	180.819295	3.80	0.0270	1.957550	10.978780
P	4	660.747662	165.186916	3.47	0.0360	3.304884	20.991461
K	4	318.328520	79.582130	1.67	0.2118	3.358325	33.358334
GP2G							
N	4	307.213714	76.803429	1.75	0.1941	0.769867	20.384936
P	4	578.197324	144.549331	3.30	0.0420	-0.283395	29.914981
K	4	769.676867	192.419217	4.40	0.0165	-0.264186	39.735813

GP3G							
N	4	836.965298	209.241325	4.92	0.0109	0.444188	30.222094
P	4	1116.523540	279.130885	6.56	0.0034	0.436329	40.130899
K	4	1641.346675	410.336669	9.65	0.0006	0.401005	50.401005

[a]N=nitrogen fertilizer; P=phosphorus fertilizer; K=potassium fertilizer

In GP3G the coefficient of determination, R^2 for the fitted model was 80.03%, which means that 80.03% of the total variability of the system was explained by the chosen factors N, P, K meaning investigation further need to be done on other pertinent factor components of fertilizers required by *the rose coco plant* for the coefficient of determination to increase to say 99.9%.

5.5 Finding the settings of the experimental factors that produce the optimal response

Graphics and visualization techniques are some of the best tools for understanding response surfaces, for which this research work utilized in the expounding of the nature and shape of the response surface generated from the fitted first order and second order models for the three groups. We visualize mountain and valleys of the second order response surface of *rose coco* beans for GP1G, GP2G and GP3G each at two combinations of fertilizers N, P, K. The curvature showed in GP2G and GP3G validated the second order model because it fitted the data well. Contour plots showing the contours of the surface, that is, curves of N, P, K pairs that have the same response value were generated which depicts the pattern and nature of the combination of the predictive factors, nitrogen, phosphorus and potassium fertilizer concentration of the *rose coco* beans yield.

The eigenvalues were used to determine whether the solution gave a maximum, minimum or saddle point on the response curve. The 3-dimensional response surface plots were a good way to visualize the fertilizer interaction. The canonical analysis of response surface for all groups, that is, GP1G, GP2G and GP3G indicated that the stationary points were saddle points, implying that in the experimental region there were no maximum or minimum points. The study uses the ridge analysis as an alternative solution to overcome the saddle point problem. Ridge analyses were performed (table 5.1) to determine the critical levels of the design variables that could produce a maximum response.

Table 5. 4: Canonical analysis based on coded and actual values, eigenvalues and eigenvectors for the three groups.

		Canonical Analysis				
	Coded values		Eigenvectors			Remark
		Eigenvalues	N	P	K	
GP1G	Coded values	6.770341	0.744195	0.251973	-0.618614	Saddle point
		0.483456	0.658879	-0.124703	0.741841	
		-0.907415	-0.109781	0.959666	0.258823	
	Uncoded/natural values	21.655306	0.869293	0.353108	-0.345898	
		0.740548	0.400775	-0.093898	0.911352	
		-8.231376	-0.289326	0.930859	0.223142	
GP2G	Coded values	9.570182	0.288816	0.443430	0.848502	Saddle point
		7.278125	0.265859	0.814266	-0.516033	
		-0.747253	0.919731	-0.374620	-0.117284	
	Uncoded/natural values	78.906109	0.259596	0.965120	0.033973	
		9.186905	0.175457	-0.081729	0.981089	
		-3.191124	0.949644	-0.248726	-0.190554	
GP3G	Coded values	12.224838	0.298898	-0.511793	0.805437	Saddle point
		8.574872	-0.586456	0.567316	0.578120	
		-1.401508	0.752814	0.645152	0.130574	
	Uncoded/natural	79.989081	-0.471069	0.876678	-0.097616	

	values	10.972726	-0.236146	-0.018711	0.971537	
		-7.439426	0.849899	0.480713	0.215839	

Table 5.4 displays the eigenvalues and eigenvectors. The canonical analysis indicates that the directions of principal orientation for the predicted response surface are along the axes associated with the three factors. In GP1G uncoded values, the largest eigenvalue (21.655306) corresponds to the eigenvector (0.869293, 0.353108, -0.345898), the largest component of which (0.869293) is associated with N; similarly, the second-largest eigenvalue in absolute (-8.231376) is associated with P. The third eigenvalue (0.740548), associated with K. The coded form of the canonical analysis indicates that the estimated response surface is at a saddle point, in uncoded terms, the model predicts that the yield of *rose coco* saddles when N=10.978780 grams, P=20.991461 grams, and K=33.358334 grams of fertilizers. In GP2G uncoded values, the largest eigenvalue (78.906109) corresponds to the eigenvector (0.259596, 0.965120, 0.033973), the largest component of which (0.965120) is associated with P; similarly, the second-largest eigenvalue (9.186905) is associated with K. The third eigenvalue (-3.191124), associated with N. The coded form of the canonical analysis indicates that the estimated response surface is at a saddle point, in uncoded terms, the model predicts that the yield of *rose coco* saddles when N=20.384936grams, P=29.914981grams, and K=39.735813grams. In GP3G uncoded values, the largest eigenvalue (79.989081) corresponds to the eigenvector (-0.471069, 0.876678, -0.097616), the largest component of which (0.876678) is associated with P; similarly, the second-largest eigenvalue (10.972726) is associated with K. The third eigenvalue (-7.439426), associated with N. The coded form of the canonical analysis indicates that the estimated response surface is at a saddle point, in uncoded terms, the model predicts that the yield of *rose coco* saddles when N=30.222094grams, P=40.130899grams, and

K=50.401005grams. In this canonical analysis, we saw that in GP1G the curvature is upward, was much more sensitive to nitrogen at 10gms for both coded and uncoded, in GP2G the upward curvature, that is, valley orientation was inclined to potassium at 40gms for coded but inclined to phosphorus for uncoded and in the GP3G was a valley orientation inclines towards potassium but inclined to phosphorus for uncoded. On the hilly or downward curvature is in GP1G, GP2G and GP3G are inclined to phosphorus at 20grams, nitrogen at 20grams and nitrogen at 30grams respectively. The optimal values all the three variables (nitrogen, phosphorus and potassium) for optimal production of *rose coco* beans predicted in GP1G (54.88grams) by the model were: nitrogen fertilizers at 10.98grams; phosphorus fertilizer at 20.99gms; potassium fertilizer at 33.36grams. The production of 36.4grams was achieved in GP2G with 20.38grams nitrogen fertilizer, 29.91gram phosphorus fertilizer and 39.74grams potassium fertilizer. In GP3G the production of 51.3grams was achieved with 30.22grams nitrogen, 40.13grams phosphorus and 50.40grams potassium fertilizers, this means that we have to be cautious with the specific amount of each fertilizer applied to *rose coco* plant because it might be affecting the effectiveness of the other fertilizers variable component as shown by the mean yield for the three groups after steadily increasing the fertilizers linearly.

The stationary points of all the three groups were the saddle points, indicating that optimum conditions for GP1G, GP2G and GP3G did not exist in the experimental range. It was found in the current study that three factors, nitrogen concentration, phosphorus concentration and potassium concentration, were important fertilizers for the *rose coco* beans yield.

Since analysis of the surface response revealed that the stationary point for all the three groups was saddled points. Hence, the study used ridge analysis to offer an alternative solution for the saddle problem. Under the three combinations of the fertilizers, nitrogen 9.57grams, phosphorus

20.05grams and potassium 30.50grams, 58.68grams of *rose coco* yield was obtained in GP1G, nitrogen 20.14grams, phosphorus 30.07grams and potassium 40.94grams, 48.36grams of *rose coco* yield was obtained in GP2G and nitrogen 29.96grams, phosphorus 40.11grams and potassium is 49.07grams, 70.25grams of *rose coco* yield was obtained in GP3G. The results from this study show that a twenty four-points second order rotatable design was one of the suitable methods to optimize the best operating conditions in multi-factor operating environment for the purpose of obtaining maximum *rose coco* beans yield. The study has demonstrated the applicability of the twenty four points, second order rotatable design for the optimal and efficient production of *rose coco* beans. Among the three tests that are GP1G, GP2G and GP3G, we saw that GP3G is more desirable for further researches and acted as a starting point for farmers to plant *rose coco* beans so as to achieved the optimal/maximum potential yield. We therefore recommend the use of GP3G since it gave the above board the required coefficient of determination, R^2 of 80.66%, and the maximum yield of 70.25grams was achieved. With the use of N, P and K fertilizers in this research study design, if adopted the farmers have a high potentiality to increase their yields or even triple the *rose coco* yield in the study area. Furthermore, this research can be extended to the area where beans are grown and a preliminary research of this kind should be conducted to ascertain the initial starting point for the inorganic fertilizers. The ministry of agriculture in Kenya should focus more researches of this kind, such that production using inorganic fertilizers (N, P, K) concentration matches the crop and the soil requirements of the rose coco beans in the growing areas for maximum yield to be realized. The research needs to be tried in the open field with the found optimal fertilizer combinations to see its performance. The yield of *rose coco* beans decreases as we increase the phosphorus fertilizer, more investigations need to be done to ascertain the amount of nitrogen and potassium fertilizer needed for the effectiveness of the phosphorus fertilizer. The investigation also needs to be done if the certain amount of each component of the fertilizer affects the effectiveness of each other on the yield of *rose coco* beans.

5.6 Recommendation for further research

The randomization of the initial fertilizer application is advisable because in this research; a linear increment of the fertilizers was used in the order of N, P then K, therefore, thorough repetitive screening of all the fertilizer components needs to be done to ascertain the right initial amount of fertilizers that could achieve maximum yield. The optimal output of *rose coco* beans needs to be obtained when the free/letter parameter (f) assumes a unit value, then a comparison can be done with the current study of calculus optimum value and determine which one gives the best estimates among the two.

REFERENCES

Africa Agriculture (2008). *Kenya Seeks Increased Production of Beans* – September 7, 2008. http://africanagriculture.blogspot.com/2008/09/kenya-seeks-increasedproduction-of.html

Arjumand banu.S.S, Ananth Nag.B and Puttaiah, E.T (2013). *Effectiveness of Farm yard Manure, Poultry Manure and Fertilizer –NPK on the Growth Parameters of French Bean (Phaseolus vulgaris)*. Shimoga, Karnataka

Atkinson, A.C., & Donev, A.N.(1992). *Optimum experimental designs*. Oxford University Press, Oxford Clarendon.

Bildirici, N. and Yilmaz N. (2005). *The effect of different nitrogen and phosphorus doses and bacteria inoculation (Rhizobium phaseoli) on the yield and yield components of field bean (Phaseolus vulgaris L.)*. J Agron 4(3):207-215.

Baudoin P., Vanderborght T., Kimani P., Mwang'ombe A. (2001). *Crop production in tropical Africa. Directorate general for international cooperation. Ministry of foreign affairs, external trade and international Cooperation.* Kluwer Academic Press, Begium.

Box, M. J., and Draper, N. R.(1974). *Minimum point second order designs*. *Technometrics*, 16(4), 613_616. See also letters to Editor, 17(2), p. 282.

Buruchara R., (2007). *Background information on Common Beans (Phaseolus vulgaris L) in Biotechnology, Breeding & Seed Systems for African Crops*. Available online (http://www.africancrops.net/rockefeller/crops/beans/index.htm). Accessed on 01/09/2012.

Carley, K.M., Natalia, K.Y., Jeff, R.,(2004). *Response Surface Methodology*. Carnegie Mellon University, School of Computer Science, Institute for Software Research International, CASOS Technical Report CMU-ISRI-04-136.

Chen H., Yuancai C., Huaiyu Z., and Shiyu F. (2010). *Enhanced Biodegradation of pulping effluents by a statistical experimental design using microbial consortia.* Bioresources(3) 1581-1594.

Cobley, L.S. and W.M. Steele (1976). *An introduction to the botany of tropical crops.* Longman group Limited, London

Craig, J.A. (1978). *D-Optimal Design Method: Final Report and User's Manual*, USAF Contract F33615-78-C-3011, FZM-6777, General Dynamics, Forty Worth Division.

Ensminger, A. H., Ensminger, M. E., Konlande, J. E., & Robson, J. R. K. (1994). *Food & nutrition encyclopaedia (2^{nd} Ed.).* Boca Raton, FL: CRC Press.

Fayza. A. Faheed & Zeinab Abd-El Fattah, (2008). *Effect of Chlorella Vulgaris as Bio-*

fertilizer on growth parameters. J. Agric. Soc. Sci , 4:165-169.

GoK, (2006). *Statistical Abstract. Government of Kenya*. Nairobi, Kenya.

Karanja N.K., Mutua G.K., Kimenju J.W., (2007). *Evaluating the effect of Rhizobium and Bacillus Biinoculants on nodulation and nematode control in Phaseolus vulgaris.* Springer, Netherlands, pp 863-868.

Katungi E., Farrow A., Mutuoki T., Gebeyehu S., Karanja D., Alamayehu F., Sperling L., Beebe S., J.C. Rubyogo J.C. and Buruchara R. (2010). *Improving common bean productivity: An Analysis of socioeconomic factors in Ethiopia and Eastern Kenya. Baseline Report Tropical legumes II*. Centro International de Agricultural Tropical – CIAT. Cali, Colombia.

Khamis, A., et al. (2006). *The use of response surface analysis in obtaining maximum profit in oil palm industry*. Songklanakarin J. Sci. Technol., 28(2) : 417-429

Khuri, A. I., and Cornell, J. A. (1996). *Response Surfaces: Designs and Analysis*. Marcel Dekker, New York.

Kiefer, J. (1958). *On the nonrandomized optimality and randomized nonoptimality of symmetrical designs*. Ann. Math. Stat., vol.29 pp. 675-699.

Kiefer, J. and Wolfowitz, J. (1959). *Optimum Designs in Regression Problem*. *Annals of Mathematical Statistics* 30, pp. 271-294.

Kimenju J.W., Muiru D.M., Karanja N.K., Nyongesa M.W., Miano D.W., Mutua G.K., (2004). *Assessing the role of organic amendments in management of root knot nematodes on common beans, Phaseolus vulgaris L.* Tropical Microbiology and Biotechnology 3(1), 14-23.

Koske, J.K., Mutiso J.M., & Kosgei, M.K.(2008). *A specific optimum second order rotatable design of twenty four points with a practical example*. East African Journal of pure and applied science, school of science, Moi University, Eldoret Kenya.

Koech, F. (2016). *Relative efficiency and DT- optimality criteria for the six specific second order rotatable designs.* Unpublished M. Phil. Thesis, Moi University.

Kwabiah A.B., Stoskopf N.C., Palm C.A., Voroney R.P., Rao M.R., Gacheru E., (2003). *Phosphorus availability and maize response to organic and inorganic fertilizer inputs in a short term study in Western Kenya.* Agricultural Ecosystems and Environment 95,49-59.

Leonard, D. (1986). *Soil, Crop, and Fertilizer Use: A Field Manual for Development Workers. Under contract with Peace Corps. 4th edition revised and expanded*. United State Peace Corps. Information collection and exchange. Reprint R0008.

Leterme, P. and C. Munoz (2002). *Factors influencing pulse consumption in Latin America*. British Journal Nutrition, S252-S255.

Little, T.M., and Hills, F.J. (1978). *Agricultural experimental design and analysis*. New York: John Wiley.

Marschner, H. (2012). *Marschner's mineral nutrition of higher plants*. Academic Press, London. 651 pp.

Mitchell, TJ. (1974). *'An Algorithm for the Construction of D-optimal Experimental Designs,'. Technometrics,* 16, 203-210.

Montgomery D. C.(1991). *Design and Analysis of Experiments*, John Wiley and Sons, N.J.

Montgomery, D.C. (1997). *Design and Analysis of Experiments, Fourth Edition*. New York, John Wiley and Sons.

Montgomery D. C. (2005). *Design and Analysis of Experiments response surface methods and design*. John Willey and Sons Inc. New Jersey, .

Myers, R.H. (1971). *Response Surface Methodology*, Boston: Allyn and Baron Reprint 1976, Ann Arbor, MI, Edwards Bros.

Myers, R. H., Khuri, A. I., and Cassidy, M. M. (1989). *Response surface methodology*. 1966-1988. Technometrics 31:137-157.

Myers, R. H., and Montgomery, D. C. (2002). *Response Surface Methodology: Process and Product Optimization Using Designed Experiments 2nd ed*. J. Wiley, New York.

Myers, R. H, Montgomery D.C, Adweson-Cook C.M.(2008). *Response surface methodology: Process and product optimization using designed experiments [M]*. New York: John Wiley and Sons, Inc.

Myers, RH Montgomery DC, and Anderson-Cook CM (2009) Response Surface Methodology (3rd ed.), Wiley.

Mutiso, J. M. (1998). *Second and Third Order Specific and Sequential Rotatable Designs in K Dimensions*. D.Phil. Thesis, Moi University.

Oehlert, G. W. (2000). *Design and analysis of experiments: Response surface design*. New York. W.H. Freeman and Company,

Pachico, D. (1993). *The demand for bean technology*. In Rachier, G.O. ; P.M.Kimani., R, Juma and L. Ongadi. Bean research activities at KARI Kakamega.

Pukelsheim, F.(1993). *Optimal Design of Experiments*. John Wiley.

SAS Institute (2013): *SAS User's Guide, SAS Institute Inc*. Cary, NC, USA.

Simpson P, Hamer R, Jo Chan-Hee, Huang BE, Goel R, Siegel E, Dennis R, and Bogle M (2004). *Assessing Model Fit and Finding a Fit Model (Paper 214-29). Proceedings of the Twenty-Ninth Annual SASUsers Group International Conference,* Cary, NC: SAS Institute, Inc.

Unal, R., Lepsch, R. A., Engelund, W. and Stanley, D. O. (1996). "*Approximation Model Building and Multidisciplinary Design Optimization Using Response Surface Methods*," *6thAIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization,* Bellevue, WA, AIAA, Inc., Vol. 1, pp. 592-597.

Wasonga C.J., Sigunga D.O.,Musandu A.O.,(2008). *Phosphorus requirements by maize varieties in different soil types of Western Kenya*. African Crop Science Journal 16(2), 161-173.

Wortmann C.S, Kirkby R.A, Eledu C.A, and Allen D.J. (1998). *Atlas of common bean (Phaseolus vulgaris L.) production in Africa*. CIAT, Cali, Colombia; pp.133.

APPENDIX 1

1.1 The codes for generating the response surfaces using SAS software

GP1G

```
DATA RP;
INPUT N P K YIELD;
title 'Surface plot of rose coco yield vs Nitrogen, Phosphorus';
CARDS;
1.1072569 1.1072569 0 63
-1.1072569 1.1072569 0 71
1.1072569 -1.1072569 0 35
-1.1072569 -1.1072569 0 56
1.1072569 0 1.1072569 37
-1.1072569 0 1.1072569 62
1.1072569 0 -1.1072569 57
-1.1072569 0 -1.1072569 54
0 1.1072569 1.1072569 52
0 -1.1072569 1.1072569 52
0 1.1072569 -1.1072569 53
0 -1.1072569 -1.1072569 40
0.7829487 0 0 43
-0.7829487 0 0 58
0 0 0.7829487 58
0 0 -0.7829487 42
0 0.7829487 0 48
0 -0.7829487 0 44
1.2735263 0 0 50
-1.2735263 0 0 53
0 0 1.2735263 60
0 0 -1.2735263 56
0 1.2735263 0 52
0 -1.2735263 0 32
;
proc g3grid data=rp out=out1;
grid N*P=yield / join spline
smooth=.1
axis1=-1.28 to 1.28 by 0.05
axis2=-1.28 to 1.28 by 0.05;
run; quit;

proc g3grid data=rp out=rpgrid;
grid N*P=yield/join spline
smooth=.1
axis1=-1.28 to 1.28 by 0.1
axis2=-1.28 to 1.28 by 0.1;
run;

proc g3d data=out1;
note j=N f=K "fertilizer used: "
j=n c=red "Nitrogen "
j=n c=red "Phosphorus ";
plot N*P=Yield/ctext=blue cbottom=red
ctop=green rotate=45 zmin=0 zmax=1.0 xticknum=5 yticknum=5 style=3 grid;
label N='Nitrogen' P='Phosphorus' K='Potassium';
run; quit;
```

GP1G

```
DATA RP;
INPUT N P K YIELD;
title 'Surface plot of rose coco yield vs Phosphorus, Potassium';
CARDS;
1.1072569 1.1072569 0 63
-1.1072569 1.1072569 0 71
1.1072569 -1.1072569 0 35
-1.1072569 -1.1072569 0 56
1.1072569 0 1.1072569 37
-1.1072569 0 1.1072569 62
1.1072569 0 -1.1072569 57
-1.1072569 0 -1.1072569 54
0 1.1072569 1.1072569 52
0 -1.1072569 1.1072569 52
0 1.1072569 -1.1072569 53
0 -1.1072569 -1.1072569 40
0.7829487 0 0 43
-0.7829487 0 0 58
0 0 0.7829487 58
0 0 -0.7829487 42
0 0.7829487 0 48
0 -0.7829487 0 44
1.2735263 0 0 50
-1.2735263 0 0 53
0 0 1.2735263 60
0 0 -1.2735263 56
0 1.2735263 0 52
0 -1.2735263 0 32
;
proc g3grid data=rp out=out1;
grid P*K=yield / join spline
smooth=.1
axis1=-1.28 to 1.28 by 0.05
axis2=-1.28 to 1.28 by 0.05;
run; quit;

proc g3grid data=rp out=rpgrid;
grid P*K=yield/join spline
smooth=.1
axis1=-1.28 to 1.28 by 0.1
axis2=-1.28 to 1.28 by 0.1;
run;

proc g3d data=out1;
note j=P f=N "fertilizer used: "
j=n c=red "Phosphorus "
j=n c=red "Potassium ";
plot P*K=Yield/ctext=blue cbottom=red
ctop=green rotate=45 zmin=0 zmax=1.0 xticknum=5 yticknum=5 style=3 grid;
label N='Nitrogen' P='Phosphorus' K='Potassium';
run; quit;
```

GP1G

```
DATA RP;
INPUT N P K YIELD;
title 'Surface plot of rose coco yield vs Phosphorus, Potassium';
CARDS;
1.1072569 1.1072569 0 63
-1.1072569 1.1072569 0 71
1.1072569 -1.1072569 0 35
-1.1072569 -1.1072569 0 56
1.1072569 0 1.1072569 37
-1.1072569 0 1.1072569 62
1.1072569 0 -1.1072569 57
-1.1072569 0 -1.1072569 54
0 1.1072569 1.1072569 52
0 -1.1072569 1.1072569 52
0 1.1072569 -1.1072569 53
0 -1.1072569 -1.1072569 40
0.7829487 0 0 43
-0.7829487 0 0 58
0 0 0.7829487 58
0 0 -0.7829487 42
0 0.7829487 0 48
0 -0.7829487 0 44
1.2735263 0 0 50
-1.2735263 0 0 53
0 0 1.2735263 60
0 0 -1.2735263 56
0 1.2735263 0 52
0 -1.2735263 0 32
;
proc g3grid data=rp out=out1;
grid P*K=yield / join spline
smooth=.1
axis1=-1.28 to 1.28 by 0.01
axis2=-1.28 to 1.28 by 0.01;
run; quit;

proc g3grid data=rp out=rpgrid;
grid P*K=yield/join spline
smooth=.1
axis1=-1.28 to 1.28 by 0.1
axis2=-1.28 to 1.28 by 0.1;
run;

proc g3d data=out1;
note j=P f=N "fertilizer used: "
j=n c=red "Phosphorus "
j=n c=red "Potassium ";
plot P*K=Yield/ctext=blue cbottom=red
ctop=green rotate=45 zmin=0 zmax=1.0 xticknum=5 yticknum=5 style=3 grid;
label N='Nitrogen' P='Phosphorus' K='Potassium';
run; quit;
```

1.2 The codes for generating the contour plots using R software

GP3G

x1 (or N)	x2 (or P)	x3 (or K)	y1	y2	y3
1.1072569	1.1072569	0	63	49	47
-1.1072569	1.1072569	0	71	30	82
1.1072569	-1.1072569	0	35	44	68
-1.1072569	-1.1072569	0	56	55	50
1.1072569	0	1.1072569	37	59	69
-1.1072569	0	1.1072569	62	53	53
1.1072569	0	-1.1072569	57	37	87
-1.1072569	0	-1.1072569	54	45	70
0	1.1072569	1.1072569	52	64	67
0	-1.1072569	1.1072569	52	56	73
0	1.1072569	-1.1072569	53	57	84
0	-1.1072569	-1.1072569	40	54	67
0.7829487	0	0	43	34	60
-0.7829487	0	0	58	39	51
0	0	0.7829487	58	37	51
0	0	-0.7829487	42	42	72
0	0.7829487	0	48	47	51
0	-0.7829487	0	44	47	54
1.2735263	0	0	50	44	57
-1.2735263	0	0	53	30	55
0	0	1.2735263	60	46	55
0	0	-1.2735263	56	46	75
0	1.2735263	0	52	41	63
0	-1.2735263	0	32	41	58

```
data<-read.csv("D:\\mut\\PhD moi Research\\Research Ph.D Thesis\\yieldrosecoco.csv")
attach(data)
names(data)

library(rsm)

model1<-rsm(y1~SO(N,P,K),data=data)
summary(model1)

model2<-rsm(y2~SO(N,P,K),data=data)
summary(model2)

model3<-rsm(y3~SO(N,P,K),data=data)
summary(model3)

par(mfrow=c(1,3))
persp(model1,~N+P+K)

par(mfrow=c(1,3))
persp(model2,~N+P+K)
```

```
par(mfrow=c(1,3))
persp(model3,~N+P+K)

par(mfrow=c(1,3))
contour(model1,~N+P+K)

par(mfrow=c(1,3))
contour(model2,~N+P+K)

par(mfrow=c(1,3))
contour(model3,~N+P+K)

par(mfrow=c(1,1))
contour(model3,~N+P)
```

1.3 The codes for generating the Ridge Analysis using SAS software

GP3G

```
DATA RP;
INPUT N P K YIELD;
CARDS;
1.1072569 1.1072569 0 47
-1.1072569 1.1072569 0 82
1.1072569 -1.1072569 0 68
-1.1072569 -1.1072569 0 50
1.1072569 0 1.1072569 69
-1.1072569 0 1.1072569 53
1.1072569 0 -1.1072569 87
-1.1072569 0 -1.1072569 70
0 1.1072569 1.1072569 67
0 -1.1072569 1.1072569 73
0 1.1072569 -1.1072569 84
0 -1.1072569 -1.1072569 67
0.7829487 0 0 60
-0.7829487 0 0 51
0 0 0.7829487 51
0 0 -0.7829487 72
0 0.7829487 0 51
0 -0.7829487 0 54
1.2735263 0 0 57
-1.2735263 0 0 55
0 0 1.2735263 55
0 0 -1.2735263 75
0 1.2735263 0 63
0 -1.2735263 0 58
;
PROC RSREG;
MODEL YIELD = N P K/LACKFIT NOCODE PRESS;
ridge max min;
title 'Rose coco beans analysis';
RUN;
```

1.4 Greenhouse photos

Constructed greenhouse for rose coco beans

Mr. Elias assisting to fix drip line pipes

Rose coco beans three weeks after planting

Rose coco beans four week after planting

Rose coco beans six weeks after planting

The rose coco beans starting to flower

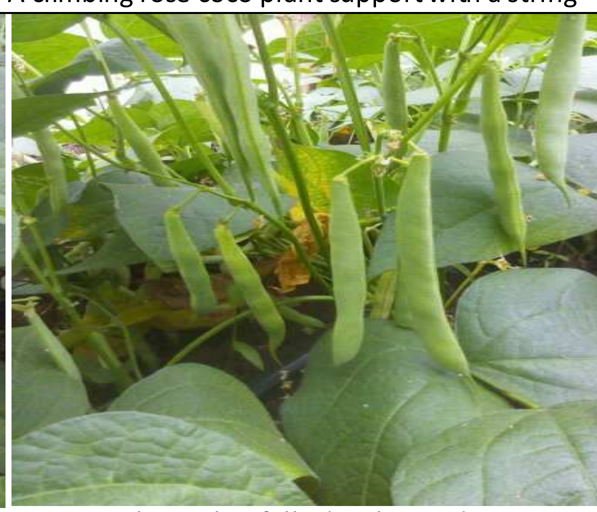
The rose coco beans in flowering stages

The rose coco beans in flowering stages

The rose coco beans have some developed pods

A climbing rose coco plant support with a string

Rose coco beans have fully develop pods



Rose coco beans has fully develop pods


Some rose coco beans starts to ripe


Some rose coco beans starts to dry up


Harvested beans in white labeled polythene bags


Showing each rose coco bean harvested per plant


Rose coco beans on polythene bags


Dried rose coco beans