MODELLING THE GERMINATION OF *Melia volkensii* USING AN A-, D-, T-OPTIMAL FOUR FACTOR ROTATABLE CENTRAL COMPOSITE DESIGN

BY

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DECLARATION

DECLARATION BY THE CANDIDATE

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DEDICATION

This thesis is dedicated to my lovely wife Rosedoris, daughters Debralyn and Keyshiaryn and my cherished father Livingstone Anapapa Okango.

ABBREVIATIONS

ASALs	Arid and Semi-Arid Lands
CCD	Central Composite Design
GA_3	Gibberellic Acid
GOK	Government of Kenya
H_2O_2	Hydrogen Peroxide
H_2SO_4	Sulphuric Acid
KEFRI	Kenya Forestry Research Institute
KNO_3	Potassium Nitrate
NACOSTI	National Commission for Science, Technology and Innovation
RSM	Response Surface Methodology
SROAD	Slope Rotatability Over All Directions

ABSTRACT

Melia volkensii is a tree species endemic to the arid and semi-arid areas of Eastern Africa. Its natural range falls within areas which are characterized by dry bush land and wooded grassland. It is a fast growing species with the growth tremendously faster on farm than in the wild, suggesting remarkable potential gains through domestication. Despite the potential of the species, the tree is yet to be massively propagated for plantation establishment especially by farmers due to difficulties experienced in seed extraction, germination and propagation through cuttings when compared to other species. The objectives of the study were to construct an A-, D-, T- optimal four factor rotatable central composite design, to develop models for the germination of *Melia volkensii* and to determine optimal conditions for the germination of *Melia volkensii*. The experiments were conducted at Kenya Forestry Research Institute (KEFRI) laboratories in Muguga, Kiambu County. An A-, D-, T- optimal four factor rotatable central composite design was constructed from the general central composite design by determining the optimal weights satisfying the A-, D- and T- optimality criteria. Response surface methodology techniques were used to develop second order models for the germination of *Melia volkensii* as well as to analyze the associated response surfaces. The variables under investigation were soil pH, temperature, chemical concentration and length of time of seed pre-treatment. Comparisons were made on the use of four different chemicals for seed pre-treatment. These were Potassium Nitrate (KNO_3), Hydrogen Peroxide (H_2O_2), Gibberellic Acid (GA_3) and Sulphuric Acid (H_2SO_4). The experiment was performed by soaking 20 seeds of *Melia volkensii* in a chemical solution for a specified period of time. The seeds were then placed in a petri-dish containing soil of a particular pH. They were then placed in germination chambers of a defined temperature. The outcome was the number of seeds that germinated in a particular petri-dish. We established that in general, germination rates of *Melia volkensii* seeds were low. For the four chemicals used in the experiment the germination rates were found to be 31.67% for KNO_3, 39.08% for H_2O_2, 42.00% for GA_3 and 28.25% for H_2SO_4. The overall germination rate was found to be 35%. However when the conditions were favorable and set correctly germination rate was optimized at between 57% and 76%. Temperature and soil pH were found to be the most significant factors across the models. The optimum temperature ranged between 26.77^{o}C and 31.13^{o}C while the optimum soil pH was found to be between 3.95 and 5.52. To maximize germination rates, we recommend the soaking of *Melia volkensii* seeds for 8 hours in a 0.03% solution of GA_3 before planting them in soil of pH 5.5 at a constant temperature of 27^{o}C. The results of the study will profoundly contribute towards large scale adoption and availability of seedlings for *Melia volkensii* thereby transforming the landscape in the arid and semi-arid lands and in the long term changing their climate.

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CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

1.1.1 Central Composite Design

The central composite design (CCD) was developed by (Box & Wilson, 1951). It is commonly used for fitting second order models. These designs are mixtures of three building blocks: cubes, stars and center points. Consider the case where an experiment consists of m factors $x_1, x_2, \dots, x_m$. The cube portion is a 2^{m-h} fractional factorial design with h describing the size of the fraction of the full factorial to be used. If it is replicated n_c times, then it is a design for sample size $2^{m-h}n_c$. The star portion takes one observation at each of the vectors $\pm\alpha e_i$ for $i \leq m$, for some star radius α > 0 with e_i being the i-th Euclidean unit vector. For n_s replications, the star portion is a design for sample size $2mn_s$. The center point portion is the one point design in 0 being replicated n_0 times (Pukelsheim, 2006). The centre runs contain information about the curvature of the surface, if the curvature is significant, the star points allow for the experimenter to obtain an efficient estimation of the quadratic terms.Therefore the central composite design is for sample size

$$n = 2^{m-h}n_c + 2mn_s + n_o. \qquad (1.1)$$

(Box & Hunter, 1957) developed the notion of design rotatability. A rotatable design is one for which the variance has the same value at any two locations that are the same distance from the design center. Hence a design is said to be rotatable when the variance is a function only of the distance from the center of the design and not a function of the direction. Since a rotatable design provides equal accuracy of estimation of the response surface in all directions, it is well suited for locating unknown optimization.

1.1.2 Response Surface Methodology

Response surface methodology (RSM) is concerned with the modelling of one or more responses to the settings of several explanatory variables. The nature of the function relating the responses to the variables is assumed to be unknown and the function or surface is modelled empirically using a first- or a second- order polynomial model. The broad aims of RSM are to investigate the nature of the response surface over a region of interest and to identify operating conditions associated with maximum or minimum responses. RSM is generally conducted in three phases, as emphasized in (Myers & Montgomery, 2002). Phase 0 involves the screening of explanatory variables to identify those which have a significant effect on the responses, phase 1 is concerned with the location of optimum operating conditions by conducting a sequence of suitable experiments and phase 2 involves the fitting of an appropriate empirical model, usually a second-order polynomial model, in order to examine the nature of the response surface in the vicinity of the optimum.

1.1.3 *Melia volkensii*

Melia volkensii is deciduous, open crowned and laxly branched. Mature trees range between 6 m and 20 m tall. Trees with 25 cm diameter are common. The bark is grey, fairly smooth, furrowing with age. Leaves are light, bright green, bipinnate with (sub)opposite leaflets, 3-7 per pinna, up to 35 cm long, and are densely hairy when young. The leaflets are oval to lanceolate, tapering to the apex. The margins are entire or serrated, becoming almost glabrous when mature. Dimensions range between 4 cm and

7.5 cm long. Flowers are small, white and fragrant, in loose sprays. Male and female flowers are on the same tree (andromonoecious). Inflorescence is congested, up to 12 cm long, axillary and on older branchlets. Petals are tetra- to pentamerous, white, free and may curl backwards; stamens are the same number as the petals, sometimes twice as many, and united into a tube. The fruit is drupe-like and oval; colour changes from green to pale grey as the fruit matures. Fruit size is normally 4 cm long with a very thick, bony endocarp. Because of the divided leaves, the generic name is derived from the Greek melia (the ash). (Maundu & Tengnas, 2005).

Melia volkensii is primarily a tree of semi-arid and arid climates with a wide range of diverse climate (Kidundo, 1997). Many trees *Melia volkensii* are found in Taita , Kitui, lower Embu, and lower Meru counties. In the driest climates it grows near the base of hills where runoff increases ground water supplies. Some trees survive near the rock hills of Takaba in Mandera county, but are endangered now because of increased building activities. It likes deeper, well-drained sandy soils that are not alkaline (salty) or too acidic. The tree does not do well on black cotton soils. The timber is quite durable and easy to work with. Many in the sub-humid and semi-arid regions use it for housing construction, poles, mortar and pestles, general purpose wood, and fuel. According to (Muok *et al*, 2010) *Melia volkensii* can be propagated through three methods; seed, wildlings and vegetatively by use of stem and root cuttings.

The current emphasis of afforestation in arid and semi-arid lands is based on the planting of high value trees and shrubs. Such trees species provide the farmers with valuable products and services to meet their basic needs in terms of food, shelter and clothing. For the past three decades, research work has attempted to identify such tree and shrub species, both indigenous and exotic. In most trials, *Melia volkensii* has outperformed most of the other dry land species (Kidundo, 1997).

Melia volkensii is one of the most prized and important multipurpose tree species producing a range of useful products such as high quality timber, fodder, fuel wood and other environmental services. In areas where it naturally occurs it is common to find isolated trees left intact in croplands (Stewart & Blomley, 1994). Over exploitation of *Melia volkensii* for timber without replanting has resulted in rapid depletion of its natural supply. Attempts to propagate the species have proved to be both difficult and expensive mainly due to seed dormancy and seed extraction process. Currently there are no methods developed or reported on the application of either tissue culture or rejuvenated leafy stem cuttings for the propagation of *Melia volkensii*. The current available methods of propagation using seed or root and stem cuttings are problematic and hence not amenable to mass production of planting materials. The methods are labor intensive, difficult to optimize and have not been able to meet the growing demand for planting materials.

The Kenya Vision 2030 is based on three pillars: the economic pillar, the social pillar, and the political pillar. The social pillar aims to achieve a just and cohesive society enjoying equitable social development in a clean and secure environment. Under the environment sub pillar, the vision targets to increase forest cover from less than three per cent to ten per cent (GOK, 2007). This will help sustain water catchments for hydropower, agriculture, human consumption, wildlife and tourism. Other benefits include prevention of soil erosion, increasing of biodiversity, sequestration of carbon and to provide timber for local people. *Melia volkensii* is among the tree species that can considerably contribute towards the achievement of this goal in the vision.

1.2 Statement of the Problem

Seed germination for *Melia volkensii* is very difficult to achieve (Maundu & Tengnas, 2005). A majority of farmers who have this species on their farm rely on natural regeneration of seedlings. However reliance on natural regeneration is a major constraint to the expansion of use of this species as the low germination rate obtained by farmers mean that it is only an option where trees are already abundant (Stewart & Blomley, 1994). The use of alternative propagation methods may result in problems because trees originating from root cuttings are reported to be unstable (Stewart & Blomley, 1994). Instability problems due to shallow rooting, have also been identified in rubber plantations established from cuttings (Carron & Enjarlic, 1983), and studies of other tree species indicate that both propagation method and transplanting can have long term effects on root architecture which could alter the ways that trees compete with adjacent crops (Bell *et al*., 1993; Brutsch *et al*., 1977; Halter & Chanway, 1993; Khurana *et al*., 1997; Riedacker & Belgrand, 1983).

There was therefore need to investigate and provide conditions that optimize seed germination rates for *Melia volkensii.* The study investigated optimum conditions that are favorable for the germination of *Melia volkensii.* Four factors were identified for consideration. These were temperature, soil pH, chemical concentration and length of time of seed pre-treatment. Comparison was made on the use of four different chemicals for seed pre-treatment. The chemicals included Sulphuric Acid (H_2SO_4), Gibberellic Acid (GA_3), Hydrogen Peroxide (H_2O_2) and Potassium Nitrate (KNO_3).

1.3 Justification of the Study

Kenya's arid and semi-arid lands (ASALs) represent 80% of total land area. The natural resources of ASALs are being degraded rapidly. The crisis has been aggravated over the last three decades by repeated drought and inappropriate land use practices as a result of rapid population increase of people and livestock. This has resulted in clearing of forests for agricultural production, settlement and cutting of trees for charcoal production for both home and commercial purposes.

Afforestation in ASALs has been emphasized to ensure a sustainable management system, which will contribute towards poverty alleviation.

One of the highly valued multipurpose trees in ASALs, which has been recommended for planting in Kenya, is *Melia volkensii*. Therefore to increase availability of seedlings for wide scale growing of the tree species, it was imperative to study conditions that are likely to increase the germination rate of *Melia volkensii*.

1.4 Research Objectives

The general objective of the study was to model the germination of *Melia volkensii* using an A-, D-, T- optimal four factor rotatable central composite design. The specific objectives were:

1. To construct an A-, D-, T- optimal four factor rotatable central composite design.
2. To develop models for the germination of *Melia volkensii*.
3. To determine optimal conditions for the germination of *Melia volkensii*.

CHAPTER TWO

LITERATURE REVIEW

2.1 Central Composite Design

The CCD has been widely studied since it was introduced by (Box & Wilson, 1951). The design is probably the most commonly used one in the construction of second order (quadratic) model for the response variable without needing to use a complete three-level factorial experiment. (Box & Draper, 1963) suggested several criteria which can be used in the selection of design. (Myers, 1976) suggested optimal CCDs under several design criteria. (Myers & Montgomery, 2002) discussed the efficiency of experimental designs, and compared the CCD with other designs under D-, A- and E -optimality criteria.

Rotatable designs generate information about the response surface equally in all directions and are therefore useful when no or little prior knowledge is available about the nature of the response surface. This concept was introduced by (Box & Hunter, 1957).

(Draper, 1960c) provided a method of constructing second-order rotatable designs in k-dimensions from the second-order rotatable designs in (k-1)-dimensions. (Herzberg, 1967) provided an alternative method which always works and for which the results of the experiment according to the (k-1)-dimensional design need not be discarded. Many third order rotatable designs have been described in (Gardiner *et al.*, 1959; Draper, 1960a, 1960b, 1961; Thaker & Das, 1961; Herzberg, 1964; Tyagi, 1964 and Nigam, 1967). (Mutiso & Koske, 2005) developed some third order rotatable designs in five dimensions whereas (Koske *et al.*, 2011) constructed a third order rotatable design in 5-

dimensions with 320 points through balanced incomplete block designs. Third order rotatable designs usually require many more points than the available minimal point designs and hence may not always be desirable. (Koske & Patel, 1985) gave the necessary and sufficient conditions for a set of points to form a rotatable design of order four. (Koske, 1989) presented features of the variance function of the difference between two estimated responses for fourth order rotatable designs. (Njui, 1985) obtained the moment and non-singularity conditions for a set of experimental points to form a fifth order rotatable design while (Njui & Patel, 1988) gave conditions for a set of points to form a rotatable design of order five in three dimensions.

Since rotatability is a highly desirable property, when considering designs for estimating slopes of a response surface it is preferable to look for designs having the property of rotatability in terms of the estimated slopes rather than the estimated response. (Hader & Park, 1978) were the first to realize this and introduced a concept of slope-rotatability, but for dealing with central composite designs in the context of second-order models only. Various possible concepts of slope-rotatability for a design of order one and above are discussed in (Huda & Chowdhury, 2004) and (Huda, 2006). Among these the ones likely to be most useful are the A- (slope)-rotatability and D- (slope)- rotatability. The concept of A- rotatability for second-order designs has also been known as slope rotatability over all directions (SROAD) (Park, 1987). While deriving minimax designs for estimating slopes (Mukerjee & Huda, 1985) observed some results concerning sufficient conditions for A-rotatability of second- and third-order designs. These sufficient conditions for second-order designs were more formally presented among the results in (Park, 1987). A- rotatability is relatively easier to deal with than D- rotatability as can be seen from the work of (Ying *et al*, 1995a, b) concerning A-rotatability of second-order designs. This study formulated and applied an A-, D-, T- optimal four factor rotatable central composite design.

2.2 Response Surface Methodology

Response surface methodology (RSM) is a collection of mathematical and statistical techniques that is useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response (Montgomery, 2005). RSM was initially developed and described by (Box & Wilson, 1951). (Hill & Hunter, 1966) conducted an extensive review of the literature for RSM emphasizing especially on the practical applications of the method. (Mead & Pike, 1975) examined the state of RSM from the biometrician's point of view and investigated the extent to which the methodology is used in applied research with particular emphasis on biometric applications. (Myers *et al*., 1989) evaluated the use of RSM between 1966 - 1988. Over the years RSM has been implemented in a wide variety of fields. Examples of the recent applications are (Madamba, 2002), (Hussain *et al*., 2011), (Pishgar-Komleh *et al*., 2012), (Anwar *et al*., 2012), (Hussain & Uddin, 2012), (Krishna *et al.*, 2013) and (Zainal *et al*., 2013). This study applied RSM in fitting second order models as well as investigating optimal conditions for the germination of *Melia volkensii*.

2.3 Germination of *Melia volkensii*

Melia volkensii (Melia) is an indigenous tree species in the plant family Meliaceae. Melia has been heavily exploited because it is highly valued as a timber tree. This trend has been worsening over the last decade owing to shortage of alternative hardwood species. As a result the tree growers are now striving to grow Melia as a plantation species.

Propagation of Melia has, however, been a major bottle neck and hindered planting of the species on large scale. The seeds of Melia fail to germinate when placed under normal conditions of air, moisture and warm temperature. The seed dormancy therefore constitutes a problem for nursery management. Some research has been done on the germination of Melia. (Milimo, 1986) studied factors which maintain seed dormancy and conditions that lead to its release. The influence of temperature on germination of *Melia volkensii* seeds was examined by (Milimo & Hellum, 1990). The effect on germination of alternating day and night temperature and constant temperature between 22° and 42°C were studied. There were significant differences in total germination and germination rates between temperatures in both regimes. Most seeds failed to germinate at temperature above 37°C. (Mwamburi *et al.*, 2005) researched on the traditional methods used by farmers to break seed dormancy of *Melia volkensii* in Eastern and Coastal provinces of Kenya. The methods found include burning of nuts, use of troughs, cracking of nuts, long-term beds, sunken beds, direct sowing of seeds and sowing of nuts. These achieved germination of between 5% - 20% in 1-10 weeks depending on the method used. (Indieka & Odee, 2005) studied vegetative propagation of *Melia volkensii*. They concluded that M*elia volkensii is* amenable to propagation by rejuvenated leafy stem cuttings and tissue culture and proposed rooting experiments to develop an *in vitro* multiplication protocol for *Melia volkensii*.

The study investigated the effect of temperature, soil pH, length of time of seed pre-treatment and chemical concentration on the germination of Melia.

CHAPTER THREE

METHODOLOGY

3.1 Research Site

The experiments were conducted at Kenya Forestry Research Institute (KEFRI) laboratories in Muguga, Kiambu County. The facilities will provided the required conditions for the accomplishment of the study objectives especially the use of temperature controlled germination chambers.

3.2 Constructing an A-, D-, T- Optimal Four Factor Rotatable Central Composite Design

3.2.1 Central Composite Design

The central composite design (CCD) is made up of three parts; the cube portion that consists of a 2^{m-h} fractional factorial design with m being the number of factors and h defines the fractional feature, the star or axial points are $2m$ points taking the values $\pm\alpha$ for a particular factor and zero for others and the centre point that has the value 0 for all the factors. The cubic, star and center point can each be replicated n_c, n_s and n_0 times respectively. Thus the sample size for the CCD is

$$n = 2^{m-h}n_c + 2mn_s + n_o. \quad (3.1)$$

The central composite design for two, three and four factors is given in table 3.1.

The table shows the coded values of the explanatory factors. Given a natural variable X, the coded values C are obtained by using the transformation

$$C = \frac{X - X_0}{d} \quad (3.2)$$

for some suitably chosen values of the center point X_0 and deviation d.

Table 3.1: Two, Three and Four Factor Central Composite Design

Two Factor CCD		**Three Factor CCD**			**Four Factor CCD**			
x_1	x_2	x_1	x_2	x_3	x_1	x_2	x_3	x_4
-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	1	-1	-1	1	-1	-1	-1
1	-1	-1	1	-1	-1	1	-1	-1
1	1	1	1	-1	1	1	-1	-1
-α	0	-1	-1	1	-1	-1	1	-1
α	0	1	-1	1	1	-1	1	-1
0	-α	-1	1	1	-1	1	1	-1
0	α	1	1	1	1	1	1	-1
0	0	-α	0	0	-1	-1	-1	1
		α	0	0	1	-1	-1	1
		0	-α	0	-1	1	-1	1
		0	α	0	1	1	-1	1
		0	0	-α	-1	-1	1	1
		0	0	α	1	-1	1	1
		0	0	0	-1	1	1	1
					1	1	1	1
					-α	0	0	0
					α	0	0	0
					0	-α	0	0
					0	α	0	0
					0	0	-α	0
					0	0	α	0
					0	0	0	-α
					0	0	0	α
					0	0	0	0

The central composite design is commonly used to fit the second order response surface model of the form:

$$y_u = \beta_0 + \sum_{i=1}^{m} \beta_i\, x_{iu} + \sum_{i=1}^{m} \beta_{ii} x_{iu}^2 + \sum_{i<j}^{m} \beta_{ij}\, x_{iu} x_{ju} + \varepsilon_u\,, \quad (u = 1,2,\dots,n) \tag{3.3}$$

where x_{iu} is the value of the variable x_i at the $u\,th$ experimental point and $\varepsilon_u{}'$s are uncorrelated random errors with mean zero and variance σ_ε^2. For a central composite design, the $x_{iu}{}'$s satisfy the following conditions

$$\sum_{u=1}^{n} \left\{ \prod_{i=1}^{m} x_{iu}^{\pi_i} \right\} = 0 \text{ if any } \pi_i \text{ is odd, for } \pi_i = 0,1,2,3 \text{ and } \sum \pi_i < 4 \tag{3.4a}$$

$$\sum_{u=1}^{N} x_{iu}^2 = n_c 2^{m-h} + 2n_s \alpha^2 \;, i = 1,2,\dots,m \tag{3.4b}$$

$$\sum_{u=1}^{N} x_{iu}^4 = n_c 2^{m-h} + 2n_s \alpha^4 \;, i = 1,2,\dots,m \tag{3.4c}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = n_c 2^{m-h} \;, i = 1,2,\dots,m \tag{3.4d}$$

(Box & Hunter, 1957) gave the condition that a CCD must satisfy to be rotatable as

$$\sum_{u=1}^{N} x_{iu}^4 = 3 \sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 \quad , i = 1,2,\dots,m \tag{3.5}$$

this gives

$$\alpha^4 = \frac{2^{m-h} n_c}{n_s}\,. \tag{3.6}$$

The various values of α, m, h, n with $n_s = n_c = 1$ that constitute a rotatable CCD are given in table 3.2.

Table 3.2: Values of α for Rotatable CCDs

m	h	2^{m-h}	$2m$	n_0	n	α
2	0	4	4	1	9	1.4142
2	1	2	4	1	7	1.1892
3	0	8	6	1	15	1.6818
3	1	4	6	1	11	1.4142
3	2	2	6	1	9	1.1892
4	0	16	8	1	25	2.0000
4	1	8	8	1	17	1.6818
4	2	4	8	1	13	1.4142
4	3	2	8	1	11	1.1892
5	0	32	10	1	43	2.3784
5	1	16	10	1	27	2.0000
5	2	8	10	1	19	1.6818
5	3	4	10	1	15	1.4142
5	4	2	10	1	13	1.1892

3.2.2 Optimal Weights on Given Support Points

For n regression vectors $x_1, x_2, \dots, x_n$ the detailed contents of the second order design matrix is given by the matrix X below made up with $x_1, x_2, \dots, x_n$ as the rows.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & \dots & x_{m1} & x_{11}^2 & x_{11}x_{21} & \cdots & x_{m-1,1}x_{m1} \\ 1 & x_{12} & \dots & x_{m2} & x_{12}^2 & x_{12}x_{22} & \cdots & x_{m-1,2}x_{m2} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1n} & \dots & x_{mn} & x_{1n}^2 & x_{1n}x_{2n} & \cdots & x_{m-1,n}x_{mn} \end{bmatrix} \tag{3.7}$$

Any design including the central composite design can be made better by varying the proportion that a particular regression vector is run. In the simplest case all the regression vectors have uniform weights implying they are run an equal number of times.

Let C be an $s \times s$ matrix and K the coefficient matrix of the parameter subsystem $K'\theta$. Then C is referred to as the information matrix for $K'\theta$ if $C = (K'M^{-}K)^{-1}$. M is the moment matrix. The matrix means are defined by:

$$\phi_p(C) = \begin{cases} \left(\frac{1}{s} trace\ C^p\right)^{1/p} & \text{for } p = \pm 1 \\ (\det C)^{1/s} & \text{for p} = 0 \\ \lambda_{min}(C) & \text{for p} = -\infty \end{cases} \tag{3.8}$$

The matrix means constitute the D-, A-, E- and T- criteria for design optimality with the value of p being respectively 0, -1, -∞ and 1. $\lambda_{min}(C)$ is the minimum eigenvalue of C.

The A-, D- and T- optimal values are given by:

$$v(\phi_p) = \begin{cases} \left(\frac{1}{s} trace\ C^p\right)^{1/p} & \text{for } p = \pm 1 \\ (\det C)^{1/s} & \text{for p} = 0 \end{cases} \tag{3.9}$$

(Pukelsheim, 2006) gave a method for finding optimal weights as follows;

Assume that the design $\xi \epsilon \Xi$ is ϕ-optimal for $K'\theta$ in Ξ. Let $E \epsilon \mathbb{R}^{s\times s}$ be a square root of the information matrix, $C_K\big(M(\xi)\big) = C = EE'$. Let $D \epsilon NND(s)$ and $G \epsilon M(\xi)^{-}$ satisfy the polarity equation $\phi\big(C_K(M)\big)\phi^{\infty}(D) = trace\ C_K(M)D = 1$ and the normality inequality $x'GKCDCK'G'x \leq 1$ for all $x \epsilon \chi$. Let the support points $x_1, x_2, \ldots, x_n$ of ξ form the rows of the matrix $X \epsilon \mathbb{R}^{n\times k}$ and let the corresponding weights $w_i = \xi(x_i)$ form the vector $w \epsilon \mathbb{R}^n$. Then, with $A = XGKE(E'DE)^{\frac{1}{2}}E'K'G'X' \epsilon NND(n)$, the weight vector w solves

$$(A \star A)w = 1_n. \tag{3.10}$$

Here $P \star Q = ((p_{ij}q_{ij}))$ denotes the hadamard matrix product of P and Q

For the matrix means ϕ_p with parameter $p \epsilon (-\infty; 1]$ the solution of the polarity equation is $D = \frac{C^{p-1}}{trace\ C^p}$. For full parameter $\theta,$ the information matrix C equals the moment matrix M. $G = M^{-1}$and $A \propto XM^{\frac{p}{2}-1}X'$. Therefore to get the optimal weights, we first compute

$$A_t = XM^{\frac{p}{2}-1}X' \tag{3.11}$$

then obtain the intermediate weights from

$$u = (A_t \star A_t)^{-1}1_n. \tag{3.12}$$

The final weights for the given support point are

$$w_i = \frac{u_i}{\sum_{i=1}^{n} u_i} \text{ for all } i = 1, \dots, n. \tag{3.13}$$

An A-, D-, T- optimal four factor rotatable central composite design was constructed using (3.13) with the corresponding optimal values given by (3.9). The specific design was achieved by setting $n_c = 2, n_s = 2$ and $n_0 = 12$. These would enable the achievement of the A- , D- and T- criteria as demonstrated in (Okango *et al*, 2014). The four factors are temperature, soil pH, chemical concentration and length of time of seed pre-treatment.

All the computations were accomplished using R 3.1.1 (R Core Team, 2014) with the raw data managed using Microsoft Excel.

3.3 Modelling the Germination of *Melia volkensii*

Germination of *Melia volkensii* was modelled using second degree model of the form:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + B_{11}x_1^2 + B_{12}x_1x_2 + B_{13}x_1x_3 + B_{14}x_1x_4 + B_{22}x_2^2 + B_{23}x_2x_3 + B_{24}x_2x_4 + B_{33}x_3^2 + B_{34}x_3x_4 + B_{44}x_4^2. \tag{3.14}$$

Four models corresponding to each one of the chemicals used for seed pre-treatment were formulated.

For all the models, x_1 denoted temperature, x_2 soil pH, x_3 chemical concentration and x_4 length of time of seed pre-treatment. The estimated response $\hat{y}$, was the number of seeds that germinate in a given petri dish that contains 20 seeds.

For each fitted model we illustrated the estimated coefficients, their standard errors, t values and their corresponding p values. A parameter was deemed to be significant if the associated p value was less than 0.05. Analyses of variance were performed to check the adequacy of the models. The output included the multiple R^2, adjusted R^2, the F-statistic and the p value of this statistic. A model was classified as being significant if the p value of its F-statistic was less than 0.05. The complete data used for modelling is displayed in appendix 1. Modelling was done in R 3.1.1 (R Core Team, 2014) using the rsm package (Lenth, 2009).

3.4 Determining Optimal Conditions for the Germination of *Melia volkensii*

The fitted models were used to find the set of operating conditions for temperature, soil pH, pre-treatment time and chemical concentrations that optimize germination of *Melia volkensii*. The fitted second order model (3.14) can be presented in matrix form as:

$$\hat{y} = b_0 + x'\hat{\boldsymbol{b}} + x'\hat{\boldsymbol{B}}x \tag{3.15}$$

where b_0, $\hat{\boldsymbol{b}}$ and $\hat{\boldsymbol{B}}$ are estimates of the intercept, linear and second order coefficients respectively. $x' = (x_1, x_2, \dots, x_p)$, $\hat{\boldsymbol{b}}' = (b_1, b_2, \dots, b_p)$ and $\hat{\boldsymbol{B}}$ is the $p * p$ symmetric matrix

$$\widehat{\boldsymbol{B}} = \begin{bmatrix} B_{11} & \frac{B_{12}}{2} & \cdots & \frac{B_{1p}}{2} \\ \frac{B_{12}}{2} & B_{22} & \cdots & \frac{B_{2p}}{2} \\ & & \ddots & \\ \frac{B_{1p}}{2} & & & B_{pp} \end{bmatrix}. \quad (3.16)$$

The stationary point is the one in which the response has an optimum value (maximum or minimum). From differential calculus, this point is obtained by differentiating the dependent variable and equating to zero to obtain the corresponding values of the independent variable.

Differentiating (3.15) with respect to x:

$$\frac{\partial \hat{y}}{\partial x} = \widehat{\boldsymbol{b}} + 2\widehat{\boldsymbol{B}}x. \quad (3.17)$$

Setting the derivative equal to 0 and solving for the stationary point of the system:

$$x_s = -\frac{1}{2}\widehat{\boldsymbol{B}}^{-\mathbf{1}}\widehat{\boldsymbol{b}}. \quad (3.18)$$

The predicted response at the stationary point is:

$$\hat{y}_s = b_0 + \frac{1}{2}x_s'\widehat{\boldsymbol{b}}. \quad (3.19)$$

There are several ways to examine the fitted second order response surface. Initially it is desirable to plot response contours. This is done by setting $\hat{y}$ to some specified value y_0 and tracing out contours relating $x_1, x_2, \ldots, x_p$. An alternative procedure is to reduce the equation to its canonical form. This is done by forming the equation:

$$\hat{y} = \hat{y}_s + \sum_{i=1}^{p} \lambda_i w_i^2 \quad (3.20)$$

where $\hat{y}_s$,the estimated stationary point is the centre of the contours and $w_i s$ are a new set of axes called the principal axes. The coefficients $\lambda_i s$ are the eigenvalues of $\widehat{\boldsymbol{B}}$ and give the shape of the surface such that if $\lambda_1, \lambda_2, \ldots, \lambda_p$ are all negative, the stationary point is a point of maximum response, if $\lambda_1, \lambda_2, \ldots, \lambda_p$ are all positive, the stationary point is a point of minimum response and if $\lambda_1, \lambda_2, \ldots, \lambda_p$ are mixed in sign, the stationary point is a saddle point.

The relative sizes of the eigenvalues also tell a great deal. For example, if most of the eigenvalues are large positive numbers but a few are near zero, then there is a ridge in the graph of the response function. Moving along that ridge will make little difference in the value of the response but might make a big difference in some other aspect of the system, like cost, for example.

The $w_i s$ are obtained as follows: For a matrix M with columns equal to the normalized eigenvectors of $\widehat{\boldsymbol{B}}$, then $M'\widehat{\boldsymbol{B}}M = \Lambda$ where Λ is a diagonal matrix with diagonal elements equal to the eigenvalues of $\widehat{\boldsymbol{B}}$.

Let $z = x - x_s$,$w = M'z$. Now, $\hat{y} = b_0 + x'\widehat{\boldsymbol{b}} + x'\widehat{\boldsymbol{B}}x$ becomes

$$\hat{y} = b_0 + (z + x_s)'\widehat{\boldsymbol{b}} + (z + x_s)'\widehat{\boldsymbol{B}}(z + x_s) \tag{3.21}$$

$$\hat{y} = b_0 + z'\widehat{\boldsymbol{b}} + {x_s}'\widehat{\boldsymbol{b}} + (z'\widehat{\boldsymbol{B}} + {x_s}'\widehat{\boldsymbol{B}})(z + x_s) \tag{3.22}$$

$$\hat{y} = b_0 + z'\widehat{\boldsymbol{b}} + {x_s}'\widehat{\boldsymbol{b}} + z'\widehat{\boldsymbol{B}}z + {x_s}'\widehat{\boldsymbol{B}}z + z'\widehat{\boldsymbol{B}}x_s + {x_s}'\widehat{\boldsymbol{B}}x_s \tag{3.23}$$

Since ${x_s}'\widehat{\boldsymbol{B}}z = z'\widehat{\boldsymbol{B}}x_s$,

$$\hat{y} = \left[b_0 + x_s'\widehat{\boldsymbol{b}} + x_s'\widehat{\boldsymbol{B}}x_s\right] + z'\widehat{\boldsymbol{b}} + z'\widehat{\boldsymbol{B}}z + 2x_s'\widehat{\boldsymbol{B}}z \tag{3.24}$$

But $x_s = -\frac{1}{2}\widehat{\boldsymbol{B}}^{-1}\widehat{\boldsymbol{b}}$. This implies $2x_s'\widehat{\boldsymbol{B}}z = -z'\widehat{\boldsymbol{B}}\widehat{\boldsymbol{B}}^{-1}\widehat{\boldsymbol{b}} = -z'\widehat{\boldsymbol{b}}$. Therefore:

$$\hat{y} = \hat{y}_s + z'\widehat{\boldsymbol{B}}z. \tag{3.25}$$

Changing the coordinate system:

$$\hat{y} = \hat{y}_s + w'M'\widehat{\boldsymbol{B}}Mw \tag{3.26}$$

$$\hat{y} = \hat{y}_s + w'\Lambda w. \tag{3.27}$$

For p = 4,

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + B_{11}x_1^2 + B_{12}x_1x_2 + B_{13}x_1x_3 + B_{14}x_1x_4 + B_{22}x_2^2 + B_{23}x_2x_3 + B_{24}x_2x_4 + B_{33}x_3^2 + B_{34}x_3x_4 + B_{44}x_4^2. \tag{3.28}$$

Its canonical form equivalent is:

$$\hat{y} = \hat{y}_s + \lambda_1 w_1^2 + \lambda_2 w_2^2 + \lambda_3 w_3^2 + \lambda_4 w_4^2 \tag{3.29}$$

where λ_1, λ_2 , λ_3 and λ_4 are the eigenvalues of $\hat{\boldsymbol{B}} = \begin{bmatrix} B_{11} & \frac{B_{12}}{2} & \frac{B_{13}}{2} & \frac{B_{14}}{2} \\ \frac{B_{12}}{2} & B_{22} & \frac{B_{23}}{2} & \frac{B_{24}}{2} \\ \frac{B_{13}}{2} & \frac{B_{32}}{2} & B_{33} & \frac{B_{34}}{2} \\ \frac{B_{14}}{2} & \frac{B_{24}}{2} & \frac{B_{34}}{2} & B_{44} \end{bmatrix}$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{12} & m_{22} & m_{32} & m_{42} \\ m_{13} & m_{23} & m_{33} & m_{43} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \begin{bmatrix} x_1 - x_{1s} \\ x_2 - x_{2s} \\ x_3 - x_{3s} \\ x_4 - x_{4s} \end{bmatrix} \tag{3.30}$$

$\begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{12} & m_{22} & m_{32} & m_{42} \\ m_{13} & m_{23} & m_{33} & m_{43} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}'$ is a matrix with columns equal to the normalized eigenvectors of $\hat{B}$.

From (3.30)

$$\begin{bmatrix} x_1 - x_{1s} \\ x_2 - x_{2s} \\ x_3 - x_{3s} \\ x_4 - x_{4s} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{12} & m_{22} & m_{32} & m_{42} \\ m_{13} & m_{23} & m_{33} & m_{43} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}^{-1} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \tag{3.31}$$

But

$$\begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{12} & m_{22} & m_{32} & m_{42} \\ m_{13} & m_{23} & m_{33} & m_{43} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}^{-1} = \begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{12} & m_{22} & m_{32} & m_{42} \\ m_{13} & m_{23} & m_{33} & m_{43} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \tag{3.32}$$

hence

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_{1s} \\ x_{2s} \\ x_{3s} \\ x_{4s} \end{bmatrix} + \begin{bmatrix} m_{11} & m_{21} & m_{31} & m_{41} \\ m_{12} & m_{22} & m_{32} & m_{42} \\ m_{13} & m_{23} & m_{33} & m_{43} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}. \qquad (3.33)$$

(3.33) gives the relationship between the original variables x_1, x_2, x_3, x_4 and the new set of variables w_1, w_2, w_3, w_4 used in the canonical model.

Optimal conditions for germination of *Melia volkensii* were obtained using the rsm package (Lenth, 2009) in R 3.1.1 (R Core Team, 2014).

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 A-, D-, T- Optimal Four Factor Central Composite Design

4.1.1 Four Factor Rotatable Central Composite Design

Consider a central composite design with m = 4, h = 0, $n_c = n_s = n_0 = 1$. From (3.1), the design consists of 25 regression vectors $x_1, x_2, \dots, x_{25}$.

For the CCD to be rotatable the value of α is computed using (3.6). This gives $\alpha = 2$.

The four factor rotatable CCD design matrix X for the second order model is shown in (4.1).

$$X = \begin{array}{c}
\begin{matrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_1^2 & x_1x_2 & x_1x_3 & x_1x_4 & x_2^2 & x_2x_3 & x_2x_4 & x_3^2 & x_3x_4 & x_4^2 \end{matrix} \\
\begin{bmatrix}
1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\
1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\
1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\
1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array} \tag{4.1}$$

The moment matrix for the general four factor central composite design defined by $M_G = \frac{X'X}{N}$ is given in (4.2).

$$M_G = \frac{1}{25}\begin{bmatrix} 25 & 0 & 0 & 0 & 0 & 24 & 0 & 0 & 0 & 24 & 0 & 0 & 24 & 0 & 24 \\ 0 & 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 24 & 0 & 0 & 0 & 0 & 48 & 0 & 0 & 0 & 16 & 0 & 0 & 16 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 24 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 48 & 0 & 0 & 16 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 \\ 24 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 16 & 0 & 0 & 48 & 0 & 16 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 24 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 16 & 0 & 0 & 16 & 0 & 48 \end{bmatrix} \tag{4.2}$$

The optimal values with respect to the A-, D- and T- criteria are computed using (3.9) with $C = M_G$ and are given in table 4.1.

Table 4.1: A-, D- and T- Optimal Values for the General Four Factor CCD

A	**D**	**T**
0.3164835	0.7672656	1.090667

4.1.2 A-, D-, T- Optimal Four Factor Rotatable Central Composite Design

We considered a design with $n_c = 2, n_s = 2$ and $n_0 = 12$. The design consists of 60 runs. The associated moment matrix defined by $M_{Exp} = \frac{1}{60} X'_{Exp} X_{Exp}$ where X_{Exp} is the design matrix for the experiment is:

$$M_{Exp} = \frac{1}{60}\begin{bmatrix} 60 & 0 & 0 & 0 & 0 & 48 & 0 & 0 & 0 & 48 & 0 & 0 & 48 & 0 & 48 \\ 0 & 48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 48 & 0 & 0 & 0 & 0 & 96 & 0 & 0 & 0 & 32 & 0 & 0 & 32 & 0 & 32 \\ 0 & 0 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 0 & 0 & 0 \\ 48 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 96 & 0 & 0 & 32 & 0 & 32 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 \\ 48 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 32 & 0 & 0 & 96 & 0 & 32 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 0 \\ 48 & 0 & 0 & 0 & 0 & 32 & 0 & 0 & 0 & 32 & 0 & 0 & 32 & 0 & 96 \end{bmatrix} \tag{4.3}$$

The optimal values with respect to the A-, D- and T- criteria are computed using (3.9) with $C = M_{Exp}$ and are given in table 4.2.

Table 4.2: A-, D- and T- Optimal Values for the Experimental Design

A	**D**	**T**
0.585366	0.720512	0.920000

4.1.3 A- Optimal Four Factor Rotatable Central Composite Design

The A- optimal four factor rotatable CCD was obtained by assigning A- optimal weights to the support points of the original experimental design. These weights were obtained using (3.11), (3.12) and (3.13) with p = -1.

The A- optimal weights obtained in this way were found to be 0.0132519, 0.0265038 and 0.01265653 for the cube, star and centre point parts respectively of the set experimental central composite design. The associated A- optimal moment matrix defined by $M_A = X'_{Exp}W_AX_{Exp}$ where W_A is a diagonal matrix with elements equal to A- optimal weights is shown in (4.4).

$$M_A = 0.424\begin{bmatrix} 2.36 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 5 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 5 \end{bmatrix} \tag{4.4}$$

The A- optimal value for M_A is 0.5144232. The A- optimal value for the general experimental design was found to be 0.585366. This represents an efficiency of 87.88% of the general design compared to the A- optimal one.

4.1.4 D- Optimal Four Factor Rotatable Central Composite Design

The D- optimal four factor rotatable CCD was obtained by assigning D- optimal weights to the support points of the original experimental design. The weights were obtained using (3.11), (3.12), (3.13) and letting $p = 0$.

The D- optimal weights obtained in this way were found to be 0.01111111, 0.01111111 and 0.03888889 for the cube, star and centre point parts respectively of the set experimental central composite design. The associated D- optimal moment matrix defined by $M_D = X'_{Exp}W_DX_{Exp}$ where W_D is a diagonal matrix with elements equal to D-optimal weights is shown in (4.5).

$$M_D = 0.1778\begin{bmatrix} 5.62 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 3 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 6 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 6 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 6 \end{bmatrix} \tag{4.5}$$

The D-optimal value for M_D is 0.5221811. The D-optimal value for the general design was found to be 0.720512. This represents an efficiency of 72.47% of the general design compared to the D- optimal one.

4.1.5 T- Optimal Four Factor Rotatable Central Composite Design

The T- optimal four factor rotatable CCD was obtained by assigning T- optimal weights to the support points of the original experimental design. These weights are obtained using (3.11), (3.12), (3.13) and letting p = 1.

The optimal weights obtained in this way were found to be 0.004279601, 0.002139800 and 0.069067998 for the cube, star and centre point parts respectively of the set experimental central composite design. The associated T- optimal moment matrix defined by $M_T = X'_{Exp}W_TX_{Exp}$ where W_T is a diagonal matrix with elements equal to T- optimal weights is shown in (4.6).

$$M_T = 0.0342\begin{bmatrix} 29.24 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 5 & 0 & 0 & 5 & 0 & 5 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 8 & 0 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 8 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 5 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 4 & 0 & 8 \end{bmatrix} \tag{4.6}$$

The T-optimal value for M_T is 0.2401331. The T-optimal value for the general design was found to be 0.920000. This represents an efficiency of 26.10 % of the general design compared to the T-optimal one.

4.1.6 Application of a Four Factor Rotatable Central Composite Design for the Germination of *Melia volkensii* Experiment

A four factor rotatable central composite design for the germination of *Melia volkensii* experiment was formulated by setting $n_c = 2$, $n_s = 2$ and $n_0 = 12$. The experiment therefore was of 60 runs. The factors under investigation were temperature, soil pH, chemical concentration and length of time of seed pre-treatment. Four different chemicals were used for seed treatment. These are Sulphuric Acid (H_2SO_4), Gibberellic Acid (GA_3), Hydrogen Peroxide (H_2O_2) and Potassium Nitrate (KNO_3). The chemical concentrations was unique to each chemical but was set as per the requirements of the design. However temperature, soil pH and length of seed pre-treatment were uniform among all the chemicals. The coded values in conformity to the design and the corresponding raw actual value setting are summarized in table 4.3.

Table 4.3: Coded Variable Setting and Corresponding Actual Values for the Germination of *Melia volkensii* Experimental Design

Coded	Temperature	Soil pH	Pre-treatment Time (Hours)	Chemical Concentration (%)			
				H_2SO_4	GA_3	H_2O_2	KNO_3
-2	15.0	3.0	4.0	20.0	0.01	1.0	0.1
-1	20.0	5.0	6.0	35.0	0.02	2.0	0.2
0	25.0	7.0	8.0	50.0	0.03	3.0	0.3
1	30.0	9.0	10.0	65.0	0.04	4.0	0.4
2	35.0	11.0	12.0	80.0	0.05	5.0	0.5

The experiment was performed by soaking 20 seeds of *Melia volkensii* in a chemical solution for a specified period of time. The seeds were then placed in a petri-dish containing soil of a particular pH. They were then placed in germination chambers of a defined temperature. The outcome was the number of seeds that germinate in a particular petri-dish. The objective was to find the temperature, soil pH, chemical concentration and pre-treatment time that maximize the germination of *Melia volkensii* seeds. The entire experiment consisted of 240 runs.

4.2 Modelling the Germination of *Melia volkensii*

4.2.1 Model for the Germination of *Melia volkensii* using Potassium Nitrate (KNO_3) Treatment

The coefficients, standard errors, t values and p values of the fitted models using KNO_3treatment are shown in table 4.4.

Table 4.4: The Fitted Germination of *Melia volkensii* Model using KNO_3 Treatment

	Coefficient	Standard Error	t value	p value
Intercept	10.500	0.8114	12.9410	0.0000
x_1	1.875	0.4057	4.6218	0.0000
x_2	-1.875	0.4057	-4.6218	0.0000
x_3	0.292	0.4057	0.7189	0.4759
x_4	0.208	0.4057	0.5135	0.6101
x_1^2	-2.208	0.3795	-5.8193	0.0000
x_1x_2	-1.938	0.4969	-3.8994	0.0003
x_1x_3	0.813	0.4969	1.6353	0.1090
x_1x_4	0.250	0.4969	0.5032	0.6173
x_2^2	-1.396	0.3795	-3.6782	0.0006
x_2x_3	0.000	0.4969	0.0000	1.0000
x_2x_4	-0.063	0.4969	-0.1258	0.9005
x_3^2	-0.896	0.3795	-2.3606	0.0226
x_3x_4	-0.563	0.4969	-1.1321	0.2636
x_4^2	-0.708	0.3795	-1.8666	0.0685

The fitted model is therefore:

$$\hat{y} = 10.500 + 1.875x_1 - 1.875x_2 + 0.292x_3 + 0.208x_4 - 2.208x_1^2 - 1.938x_1x_2 + 0.813x_1x_3 + 0.250x_1x_4 - 1.396x_2^2 - 0.063x_2x_4 - 0.896x_3^2 - 0.563x_3x_4 - 0.708x_4^2. \tag{4.7}$$

The significant factors were $x_1, x_2, x_1^2, x_1x_2, x_2^2$ and x_3^2 as shown in table 4.4.

Consequently the final model consisting of only the significant factors is:

$$\hat{y} = 9.691 + 1.875x_1 - 1.875x_2 - 2.107x_1^2 - 1.938x_1x_2 - 1.295x_2^2 - 0.795x_3^2. \tag{4.8}$$

The analysis of variance for the model is shown in table 4.5.

Table 4.5: Analysis of Variance Table for the Germination of *Melia volkensii* Model using KNO_3 Treatment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F value	P value
Model	843.84	14	60.274	7.630	0.0000
First Order	343.67	4	85.917	10.876	0.0000
Two way interaction	153.50	6	25.583	3.238	0.0099
Pure Quadratic	346.67	4	86.667	10.971	0.0000
Residuals	355.50	45	7.900		
Lack of fit	130.50	10	13.050	2.030	0.0598
Pure error	225.00	35	6.429		
Total	1199.34	59			
Multiple R^2	0.7036				
Adjusted R^2	0.6114				

The F-statistic value was found to be 7.630 with a p value of 0.0000. This indicates that the model is significant. The adjusted R^2 shows that 61.14% of the variation in the response is explained by the model. The test of lack of fit has an F value of 2.030 with a p value of 0.0598. This shows that the model fits the data well. The results indicate that the second order model adequately represents the germination of *Melia volkensii*. However the reliability of the fitted model is 61.14%.

4.2.2 Model for the Germination of *Melia volkensii* using Hydrogen Peroxide ($\boldsymbol{H_2O_2}$) Treatment

The coefficients, standard errors, t values and p values of the fitted models using H_2O_2 treatment are shown in table 4.6.

The fitted model is therefore:

$$\hat{y} = 11.667 + 1.229x_1 - 1.271x_2 + 0.063x_3 + 0.146x_4 - 1.766x_1^2 - 2.094x_1x_2 - 0.156x_1x_3 + 0.844x_1x_4 - 1.516x_2^2 + 0.219x_2x_3 + 0.719x_2x_4 - 1.203x_3^2 + 0.406x_3x_4 - 0.328x_4^2. \tag{4.9}$$

The only significant factors were $x_1, x_2, x_1^2, x_1x_2, x_2^2$ and x_3^2 as displayed in table 4.6.

Consequently the final model consisting of only the significant factors is:

$$\hat{y} = 11.292 + 1.229x_1 - 1.271x_2 - 1.719x_1^2 - 2.094x_1x_2 - 1.469x_2^2 - 1.156x_3^2. \tag{4.10}$$

Table 4.6: The Fitted Germination of *Melia volkensii* Model using H_2O_2 Treatment

	Coefficient	Standard Error	t value	p value
Intercept	11.667	0.8330	14.0054	0.0000
x_1	1.229	0.4165	2.9511	0.0050
x_2	-1.271	0.4165	-3.0512	0.0038
x_3	0.063	0.4165	0.1501	0.8814
x_4	0.146	0.4165	0.3501	0.7279
x_1^2	-1.766	0.3896	-4.5318	0.0000
x_1x_2	-2.094	0.5101	-4.1045	0.0002
x_1x_3	-0.156	0.5101	-0.3063	0.7608
x_1x_4	0.844	0.5101	1.6540	0.1051
x_2^2	-1.516	0.3896	-3.8902	0.0003
x_2x_3	0.219	0.5101	0.4288	0.6701
x_2x_4	0.719	0.5101	1.4090	0.1657
x_3^2	-1.203	0.3896	-3.0881	0.0034
x_3x_4	0.406	0.5101	0.7964	0.4300
x_4^2	-0.328	0.3896	-0.8422	0.4041

The analysis of variance for the model is shown in table 4.7.

The F-statistic value was found to be 5.458 with a p value of 0.0000. This indicates that the model is significant. The adjusted R^2 shows that 51.41% of the variation in the response is explained by the model. The test of lack of fit has an F value of 0.728 with a p value of 0.6929. This shows that the model fits the data well. The results indicate that the second order model adequately represents the germination of *Melia volkensii*. However the reliability of the fitted model is 51.41%.

Table 4.7: Analysis of Variance Table for the Germination of *Melia volkensii* Model using H_2O_2 Treatment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F value	P value
Model	636.28	14	45.449	5.458	0.0000
First Order	151.25	4	37.812	4.541	0.0036
Two way interaction	187.19	6	31.198	3.747	0.0042
Pure Quadratic	297.84	4	74.459	8.942	0.0000
Residuals	374.71	45	8.327		
Lack of fit	64.54	10	6.454	0.728	0.6929
Pure error	310.17	35	8.862		
Total	1010.99	59			
Multiple R^2	0.6294				
Adjusted R^2	0.5141				

4.2.3 Model for the Germination of *Melia volkensii* using Gibberellic Acid (GA_3) Treatment

The coefficients, standard errors, t values and p values of the fitted models using GA_3 treatment are shown in table 4.8.

Table 4.8: The Fitted Germination of *Melia volkensii* Model using GA_3 Treatment

	Coefficient	Standard Error	t value	p value
Intercept	14.250	0.7082	20.1215	0.0000
x_1	1.583	0.3541	4.4714	0.0001
x_2	-2.167	0.3541	-6.1188	0.0000
x_3	0.125	0.3541	0.3530	0.7257
x_4	0.375	0.3541	1.0590	0.2952
x_1^2	-2.969	0.3312	-8.9628	0.0000
x_1x_2	-0.813	0.4337	-1.8735	0.0675
x_1x_3	1.313	0.4337	3.0264	0.0041
x_1x_4	0.438	0.4337	1.0088	0.3185
x_2^2	-1.781	0.3312	-5.3777	0.0000
x_2x_3	0.875	0.4337	2.0176	0.0496
x_2x_4	1.000	0.4337	2.3058	0.0258
x_3^2	-1.969	0.3312	-5.9438	0.0000
x_3x_4	-0.250	0.4337	-0.5765	0.5672
x_4^2	-0.594	0.3312	-1.7926	0.0798

The fitted model is therefore:

$$\hat{y} = 14.250 + 1.583x_1 - 2.167x_2 + 0.125x_3 + 0.375x_4 - 2.969x_1^2 - 0.813x_1x_2 + 1.313x_1x_3 + 0.438x_1x_4 - 1.781x_2^2 + 0.875x_2x_3 + x_2x_4 - 1.969x_3^2 - 0.250x_3x_4 - 0.594x_4^2. \qquad (4.11)$$

The non-significant factors were $x_3, x_4, x_1x_2, x_1x_4, x_3x_4$ and x_4^2 as indicated in table 4.8.

Hence the final model consisting of only the significant factors is:

$$\hat{y} = 13.751 + 1.583x_1 - 2.167x_2 - 2.884x_1^2 + 1.313x_1x_3 - 1.696x_2^2 + 0.875x_2x_3 + x_2x_4 - 1.884x_3^2. \qquad (4.12)$$

The analysis of variance for the model is shown in table 4.9.

The F-statistic value was found to be 14.117 with a p value of 0.0000. This indicates that the model is significant. The adjusted R^2 shows that 75.69% of the variation in the response is explained by the model. The test of lack of fit has an F value of 1.019 with a p value of 0.4476. This shows that the model fits the data well. The results indicate that the second order model adequately represents the germination of *Melia volkensii*. However the reliability of the fitted model is 75.69%.

Table 4.9: Analysis of Variance Table for the Germination of *Melia volkensii* Model using GA_3 Treatment

Source	Sum of Squares	Degrees of Freedom	Mean Square	F value	P value
Model	1189.57	14	84.969	14.117	0.0000
First Order	353.17	4	88.292	14.670	0.0000
Two way interaction	140.88	6	23.479	3.901	0.0032
Pure Quadratic	695.52	4	173.881	28.891	0.0000
Residuals	270.83	45	6.019		
Lack of fit	61.08	10	6.108	1.019	0.4476
Pure error	209.75	35	5.993		
Total	1460.40	59			
Multiple R^2	0.8145				
Adjusted R^2	0.7569				

4.2.4 Model for the Germination of *Melia volkensii* using Sulphuric Acid (H_2SO_4) Treatment

The coefficients, standard errors, t values and p values of the fitted models using H_2SO_4 treatment are shown in table 4.10.

The fitted model is therefore:

$$\hat{y} = 9.667 + x_1 - 1.208x_2 - 0.375x_3 - 1.125x_4 - 2.115x_1^2 - 1.375x_1x_2 + 0.188x_1x_3 + 0.438x_1x_4 - 1.427x_2^2 + 0.938x_2x_3 + 0.563x_2x_4 - 0.865x_3^2 - 0.615x_4^2. \tag{4.13}$$

The significant factors were $x_1, x_2, x_4, x_1^2, x_1x_2, x_2^2$ and x_3^2 as shown in table 4.10.

Thus the final model is:

$$\hat{y} = 8.964 + x_1 - 1.208x_2 - 1.125x_4 - 2.027x_1^2 - 1.375x_1x_2 - 1.339x_2^2 - 0.777x_3^2. \quad (4.14)$$

Table 4.10: The Fitted Germination of *Melia volkensii* Model using H_2SO_4 Treatment

	Coefficient	Standard Error	t value	p value
Intercept	9.667	0.8595	11.2475	0.0000
x_1	1.000	0.4297	2.3271	0.0245
x_2	-1.208	0.4297	-2.8119	0.0073
x_3	-0.375	0.4297	-0.8726	0.3875
x_4	-1.125	0.4297	-2.6179	0.0120
x_1^2	-2.115	0.4020	-5.2605	0.0000
x_1x_2	-1.375	0.5263	-2.6126	0.0122
x_1x_3	0.188	0.5263	0.3563	0.7233
x_1x_4	0.438	0.5263	0.8313	0.4102
x_2^2	-1.427	0.4020	-3.5502	0.0009
x_2x_3	0.938	0.5263	1.7813	0.0816
x_2x_4	0.563	0.5263	1.0688	0.2909
x_3^2	-0.865	0.4020	-2.1509	0.0369
x_3x_4	0.000	0.5263	0.0000	1.0000
x_4^2	-0.615	0.4020	-1.5289	0.1333

The analysis of variance for the model is shown in table 4.11.

Table 4.11: Analysis of Variance Table for the Germination of *Melia volkensii* Model using H_2SO_4 Treatment.

Source	Sum of Squares	Degrees of Freedom	Mean Square	F value	P value
Model	618.77	14	44.198	4.986	0.0000
First Order	185.58	4	46.396	5.234	0.0015
Two way interaction	106.00	6	17.667	1.993	0.0865
Pure Quadratic	327.19	4	81.798	9.228	0.0000
Residuals	398.88	45	8.864		
Lack of fit	166.71	10	16.671	2.513	0.0214
Pure error	232.17	35	6.633		
Total	1017.65	59			
Multiple R^2	0.6080				
Adjusted R^2	0.4861				

The F-statistic value was found to be 4.986 with a p value of 0.0000. This indicates that the model is significant. The adjusted R^2 shows that 48.61% of the variation in the response is explained by the model. The test of lack of fit has an F value of 2.513 with a p value of 0.0214. This shows that the model does not fit the data well. The results indicate that the second order model formulated does not adequately represent the germination of *Melia volkensii*. However, the model has a reliability of 48.61% and can provide some information regarding germination of *Melia volkensii.*

4.2.5 Comparison of the Models

The GA_3 model was found to be the best one to represent germination of *Melia volkensii* having an adjusted R^2 value of 0.7569 followed by the KNO_3 model with an adjusted R^2 value of 0.6114 then the H_2O_2 model with an adjusted R^2 value of 0.5141. The H_2SO_4 model had the lowest adjusted R^2 value of 0.4861 and was the only one where the lack of fit test was significant.

4.3 Optimal Conditions for the Germination of *Melia volkensii*

4.3.1 Optimal Conditions for the Germination of *Melia volkensii* using Potassium Nitrate (KNO_3) Treatment

Figure 4.1 shows the response surface plot for the model whereas figure 4.2 shows the contour plot of the fitted model (4.8).

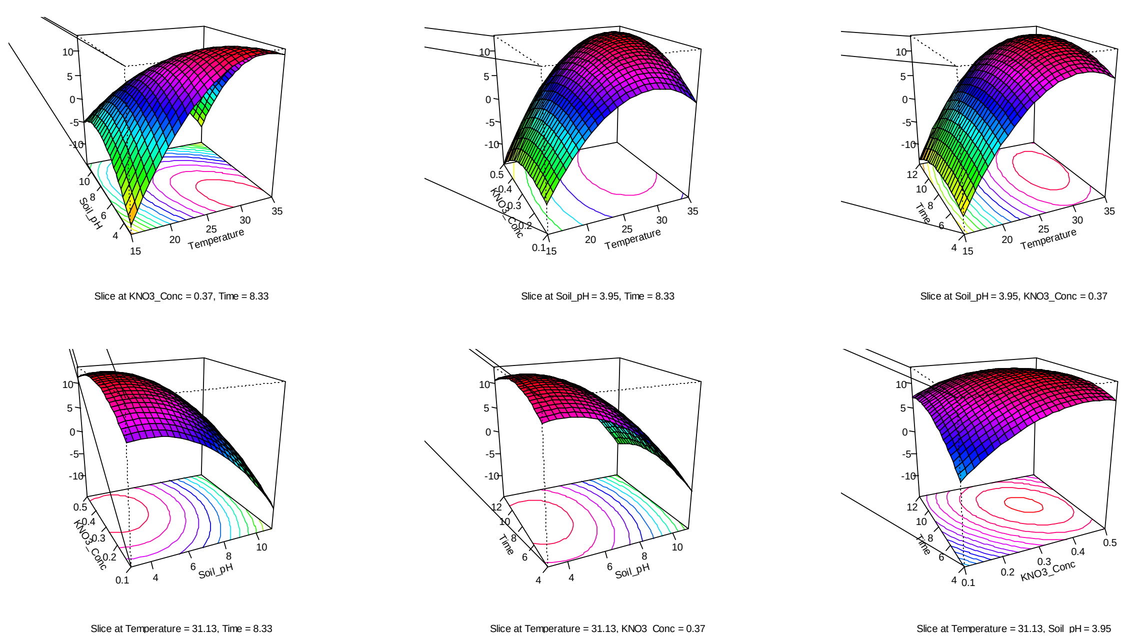

Figure 4.1: Response Surface Plot for the Germination of *Melia volkensii* Model using KNO_3 Treatment

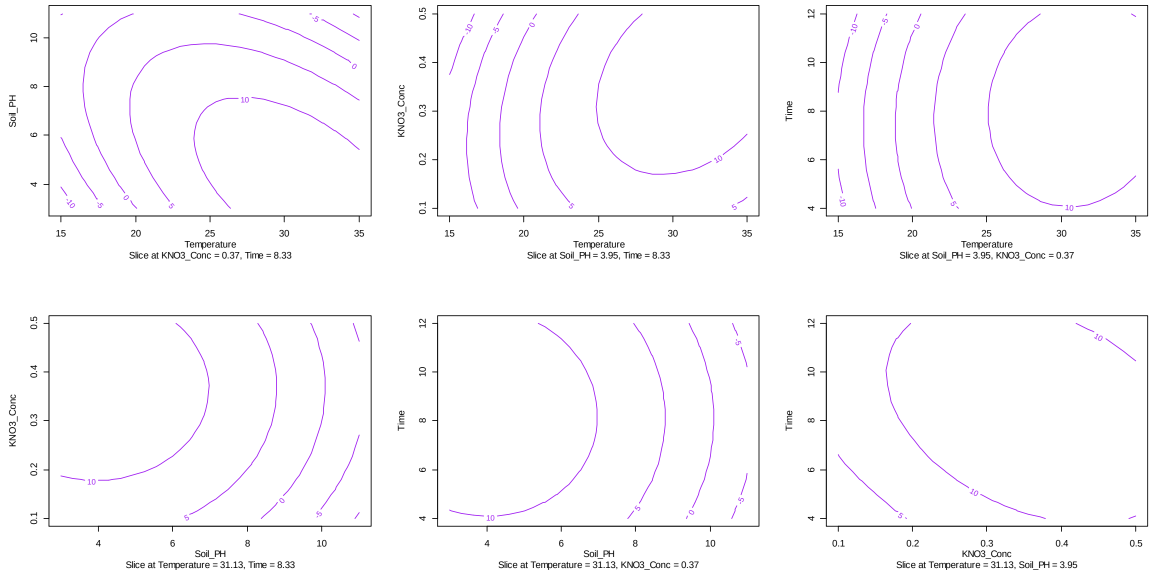

Figure 4.2: Contour Plot for the Germination of *Melia volkensii* Model using KNO_3 Treatment

The stationary point and eigenvalues of the model are shown in table 4.12.

Table 4.12: Stationary point and Eigenvalues for the Germination of *Melia volkensii* Model using KNO_3 Treatment

Stationary Points				Eigenvalues			
x_1	x_2	x_3	x_4				
1.2262	-1.5263	0.6667	0.1661	-0.4768	-0.6421	-1.1710	-2.9184

Since all the eigenvalues are negative, the response surface is a maximum one.

The canonical equivalent form of the fitted model is:

$$\hat{y} = 13.1951 - 0.4768w_1^2 - 0.6421w_2^2 - 1.1710w_3^2 - 2.9184w_4^2. \tag{4.15}$$

The stationary point for the model is (1.2262, -1.5263, 0.6667, 0.1661). In terms of the natural variable this is (31.13, 3.95, 0.37, 8.33). Thus the optimal temperature is 31.13°C, the optimal soil pH is 3.95, the optimal concentration of KNO_3 is 0.37% and the optimal pre-treatment time is 8.33 hours.

Suppose the investigator is interested in finding where to run the experiment to obtain a response that is close to 13 as possible. This can be obtained from the canonical equivalent model (4.15).

The region is presented as a contour plot of the canonical model in figure 4.3.

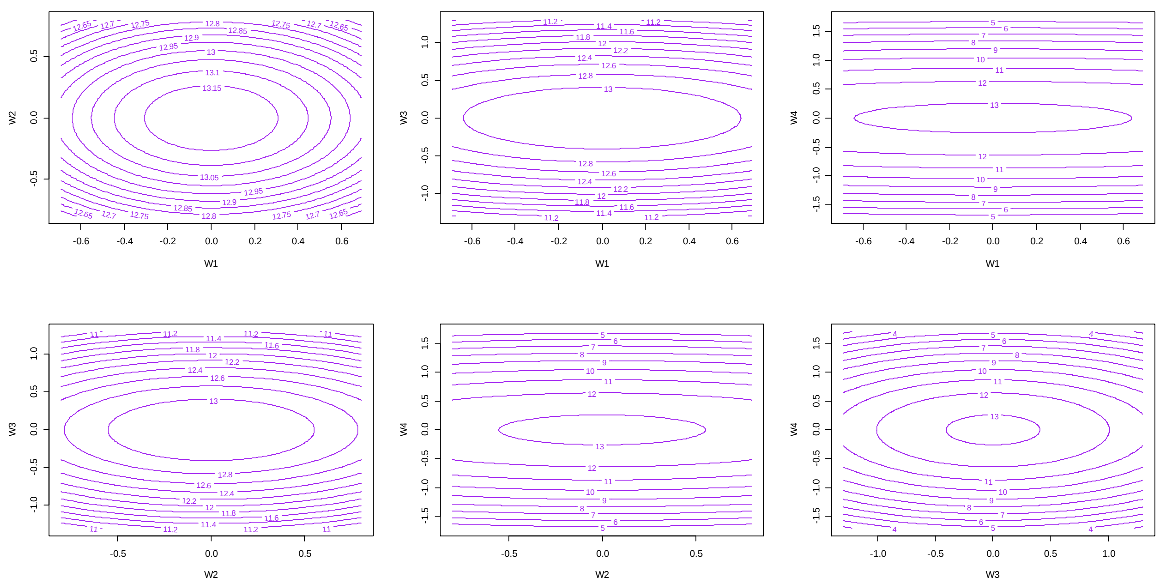

Figure 4.3: Contour Plot for Expected Response of 13 of the KNO_3 Model

From (4.15)

$$x_1 = 1.2262 - 0.2392w_1 + 0.2297w_2 - 0.6746w_3 + 0.6595w_4 \quad (4.16)$$

$$x_2 = -1.5263 + 0.4871w_1 - 0.6494w_2 + 0.1562w_3 + 0.5627w_4 \quad (4.17)$$

$$x_3 = 0.6667 + 0.1316w_1 - 0.4982w_2 - 0.6999w_3 - 0.4946w_4 \quad (4.18)$$

$$x_4 = 0.1661 + 0.8296w_1 + 0.5266w_2 - 0.1752w_3 - 0.0618w_4 \quad (4.19)$$

Table 4.13 gives values of w_1, w_2, w_3, w_4, x_1, x_2 , x_3 and x_4 for which $\hat{y} = 13.1951 - 0.4768w_1^2 - 0.6421w_2^2 - 1.1710w_3^2 - 2.9184w_4^2$. The table is obtained form (4.16), (4.17), (4.18) and (4.19). When the experiment is run at the given values of the temperature, soil pH, concentration of KNO_3 and pre-treatment time the expected response is close to 13.

Table 4.13: Operating Conditions for Expected Response of 13 of the KNO_3 Model

w_1	w_2	w_3	w_4	x_1	x_2	x_3	x_4	$\hat{y}$
0.68	0.80	0.00	0.00	1.247	-1.715	0.358	0.023	12.5
0.00	0.80	0.00	0.00	1.410	-2.046	0.268	0.587	12.4
-0.68	0.00	0.00	0.00	1.389	-1.858	0.577	0.730	12.3
0.00	0.00	0.00	0.00	1.226	-1.526	0.667	0.166	12.2
-0.68	0.80	0.00	0.00	1.573	-2.377	0.179	1.151	12.1
0.68	0.00	0.00	0.00	1.064	-1.195	0.756	-0.398	12.0
-0.68	0.00	1.08	0.00	0.660	-1.689	-0.179	0.541	11.9
-0.68	-0.80	1.08	0.00	0.477	-1.169	0.220	0.120	11.8
0.00	0.00	1.08	0.00	0.498	-1.358	-0.089	-0.023	11.8
-0.68	-0.80	0.00	0.00	1.205	-1.338	0.976	0.309	11.7
0.68	0.80	1.08	0.00	0.519	-1.546	-0.398	-0.166	11.6
0.68	0.00	1.08	0.00	0.335	-1.026	0.000	-0.587	11.6
0.00	0.80	1.08	0.00	0.681	-1.877	-0.488	0.398	11.5
0.00	-0.80	1.08	0.00	0.314	-0.838	0.309	-0.444	11.4
0.00	-0.80	0.00	0.00	1.042	-1.007	1.065	-0.255	11.3
-0.68	0.80	1.08	0.00	0.844	-2.208	-0.577	0.962	11.2

4.3.2 Optimal Conditions for the Germination of *Melia volkensii* using Hydrogen Peroxide (H_2O_2) Treatment

Figure 4.4 shows the response surface plot for the model whereas figure 4.5 shows the contour plot of the fitted model (4.10).

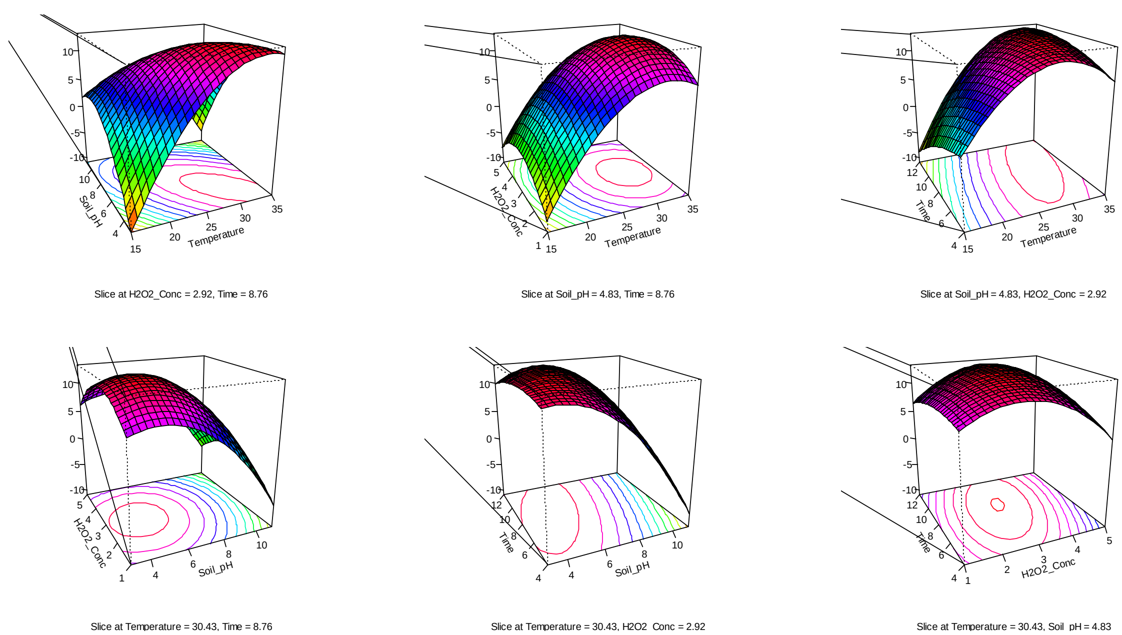

Figure 4.4: Response Surface Plot for the Germination of *Melia volkensii* Model using H_2O_2 Treatment

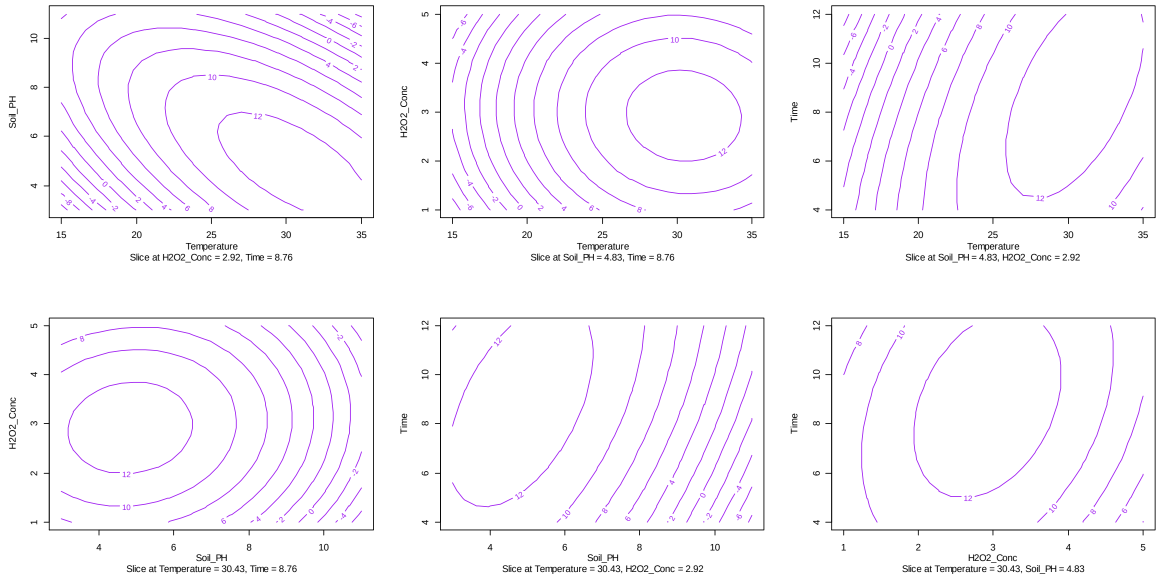

Figure 4.5: Contour Plot for the Germination of *Melia volkensii* Model using H_2O_2 Treatment

The stationary point and eigenvalues of the model are shown in table 4.14.

Table 4.14: Stationary point and Eigenvalues for the Germination of *Melia volkensii* Model using H_2O_2 Treatment

Stationary Points				**Eigenvalues**			
x_1	x_2	x_3	x_4				
1.0857	-1.0843	-0.0786	0.3819	-0.1652	-0.5608	-1.2677	-2.8188

Since all the eigenvalues are negative, the response surface is a maximum one.

The canonical equivalent form of the fitted model is:

$$\hat{y} = 13.0486 - 0.1652w_1^2 - 0.5608w_2^2 - 1.2677w_3^2 - 2.8188w_4^2. \qquad . \qquad (4.20)$$

The stationary point for the model is (1.0857, -1.0843, -0.0786, 0.3819). In terms of the natural variable this is (30.43, 4.83, 2.92, 8.76). Thus the optimal temperature is 30.43^{o}C, the optimal soil pH is 4.83, the optimal concentration of H_2O_2 is 2.92% and the optimal pre-treatment time is 8.76 hours.

Suppose the investigator is interested in finding where to run the experiment to obtain a response that is close to 13 as possible. This can be obtained from the canonical equivalent model (4.20).

The region is presented as a contour plot of the canonical model in figure 4.6.

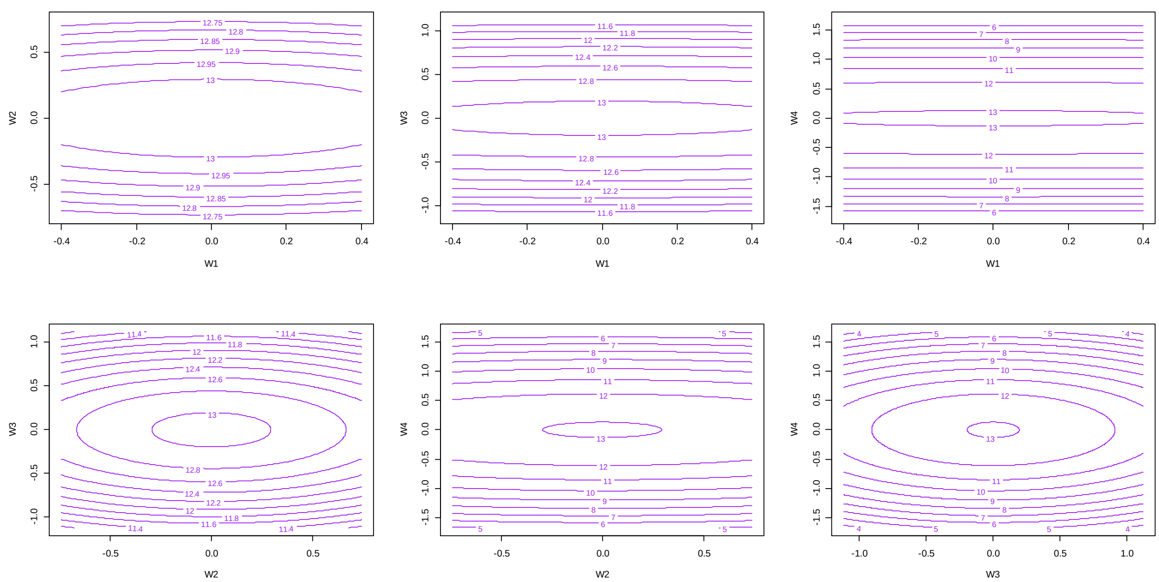

Figure 4.6: Contour Plot for Expected Response of 13 of the H_2O_2 Model

From (4.20)

$$x_1 = 1.0857 + 0.1336w_1 + 0.1669w_2 + 0.1949w_3 + 0.9572w_4 \tag{4.21}$$

$$x_2 = -1.0843 + 0.6638w_1 - 0.7221w_2 - 0.1815w_3 + 0.0702w_4 \tag{4.22}$$

$$x_3 = -0.0786 + 0.0835w_1 - 0.1826w_2 + 0.9637w_3 - 0.1760w_4 \tag{4.23}$$

$$x_4 = 0.3819 + 0.7311w_1 + 0.6460w_2 + 0.0191w_3 - 0.2186w_4 \tag{4.24}$$

Table 4.15 gives values of w_1, w_2, w_3, w_4, x_1, x_2 , x_3 and x_4 for which $\hat{y} = 13.0486 - 0.1652w_1^2 - 0.5608w_2^2 - 1.2677w_3^2 - 2.8188w_4^2$. The table is obtained form (4.21), (4.22), (4.23) and (4.24). When the experiment is run at the given values of the temperature, soil pH, concentration of H_2O_2 and pre-treatment time the expected response is close to 13.

Table 4.15: Operating Conditions for Expected Response of 13 of the H_2O_2 Model

w_1	w_2	w_3	w_4	x_1	x_2	x_3	x_4	$\hat{y}$
0.00	0.00	0.00	0.00	1.086	-1.084	-0.079	0.382	12.7
0.40	0.74	0.00	0.00	1.263	-1.353	-0.180	1.152	12.7
-0.40	0.00	0.00	0.00	1.032	-1.350	-0.112	0.089	12.7
-0.40	-0.74	0.00	0.00	0.909	-0.815	0.023	-0.389	12.6
0.00	0.74	0.00	0.00	1.209	-1.619	-0.214	0.860	12.5
0.40	0.00	0.00	0.00	1.139	-0.819	-0.045	0.674	12.5
0.00	-0.74	0.00	0.00	0.962	-0.550	0.057	-0.096	12.2
-0.40	0.74	0.00	0.00	1.156	-1.884	-0.247	0.568	12.1
0.40	0.74	1.12	0.00	1.481	-1.556	0.899	1.174	11.7
0.40	-0.74	0.00	0.00	1.016	-0.284	0.090	0.196	11.6
0.00	0.74	1.12	0.00	1.427	-1.822	0.866	0.881	11.6
-0.40	0.00	1.12	0.00	1.251	-1.553	0.967	0.111	11.6
0.00	0.00	1.12	0.00	1.304	-1.288	1.001	0.403	11.5
-0.40	-0.74	1.12	0.00	1.127	-1.019	1.102	-0.367	11.3
0.40	0.00	1.12	0.00	1.357	-1.022	1.034	0.696	11.2
-0.40	0.74	1.12	0.00	1.374	-2.087	0.832	0.589	11.2

4.3.3 Optimal Conditions for the Germination of *Melia volkensii* using Gibberellic Acid (GA_3) Treatment

Figure 4.7 shows the response surface plot for the model whereas figure 4.8 shows the contour plot of the fitted model (4.12).

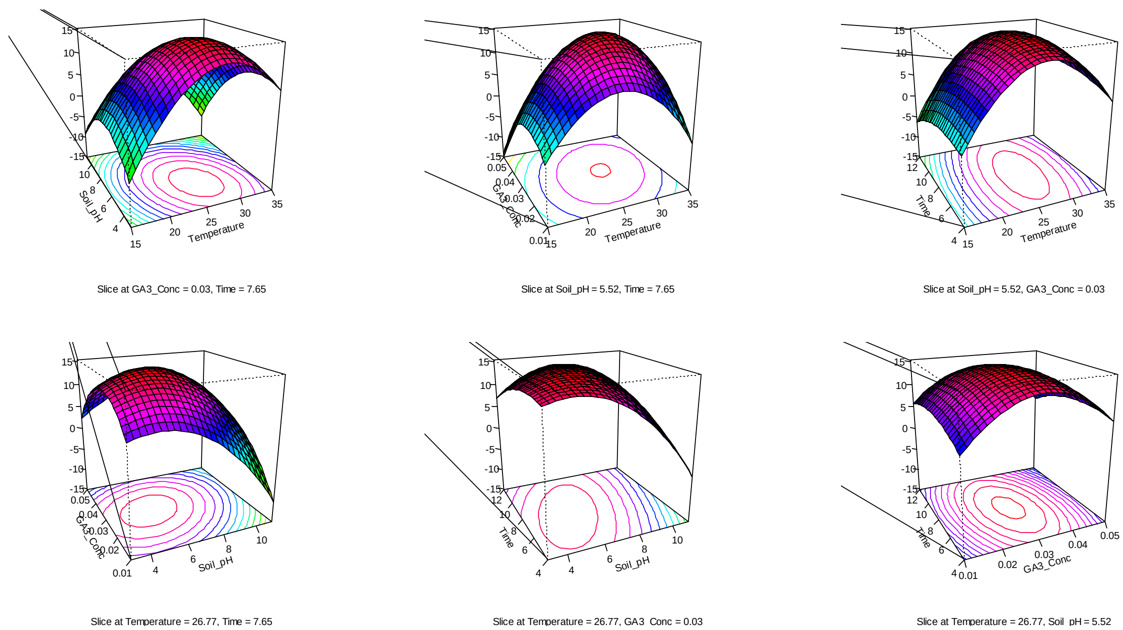

Figure 4.7: Response Surface Plot for the Germination of *Melia volkensii* Model using GA_3 Treatment

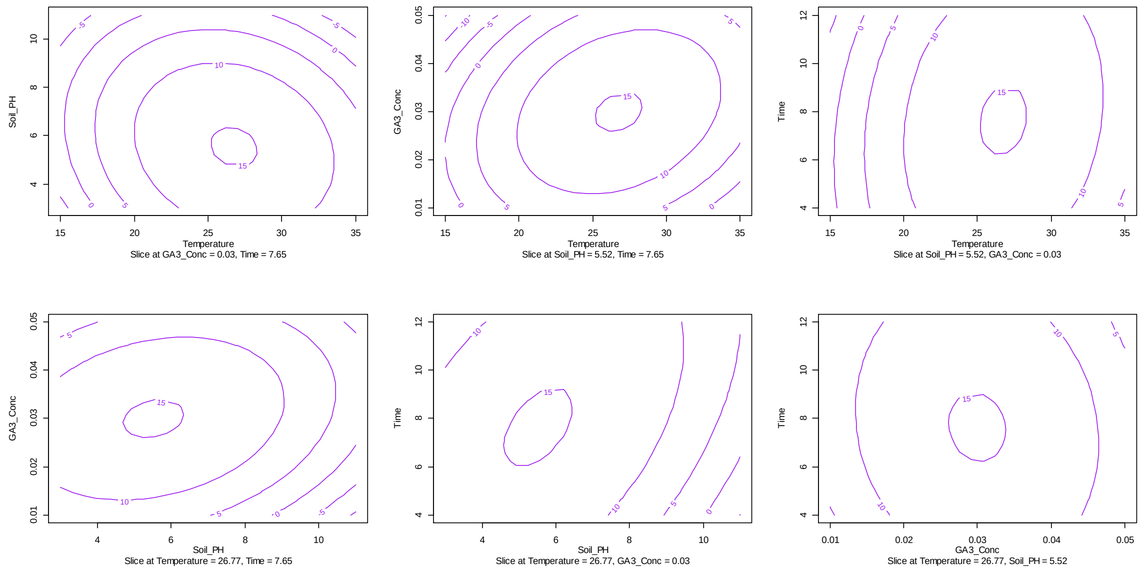

Figure 4.8: Contour Plot for the Germination of *Melia volkensii* Model using GA_3 Treatment

The stationary point and eigenvalues of the model are shown in table 4.16.

Table 4.16: Stationary point and Eigenvalues for the Germination of *Melia volkensii* Model using GA_3 Treatment

Stationary Points				Eigenvalues			
x_1	x_2	x_3	x_4				
0.3541	-0.7390	-0.0033	-0.1753	-0.4078	-1.4899	-1.8798	-3.5350

Since all the eigenvalues are negative, the response surface is a maximum one.

The canonical equivalent form of the fitted model is:

$$\hat{y} = 15.2979 - 0.4078w_1^2 - 1.4899w_2^2 - 1.8798w_3^2 - 3.5350w_4^2. \quad (4.25)$$

The stationary point for the model is (0.3541, -0.7390,- 0.0033, -0.1753). In terms of the natural variable this is (26.77, 5.52, 0.03, 7.65). Thus the optimal temperature is 26.77^{o}C, the optimal soil pH is 5.52, the optimal concentration of GA_3 is 0.03% and the optimal pre-treatment time is 7.65 hours.

Suppose the investigator is interested in finding where to run the experiment to obtain a response that is close to 15 as possible. This can be obtained from the canonical equivalent model (4.25).

The region is presented as a contour plot of the canonical model in figure 4.9.

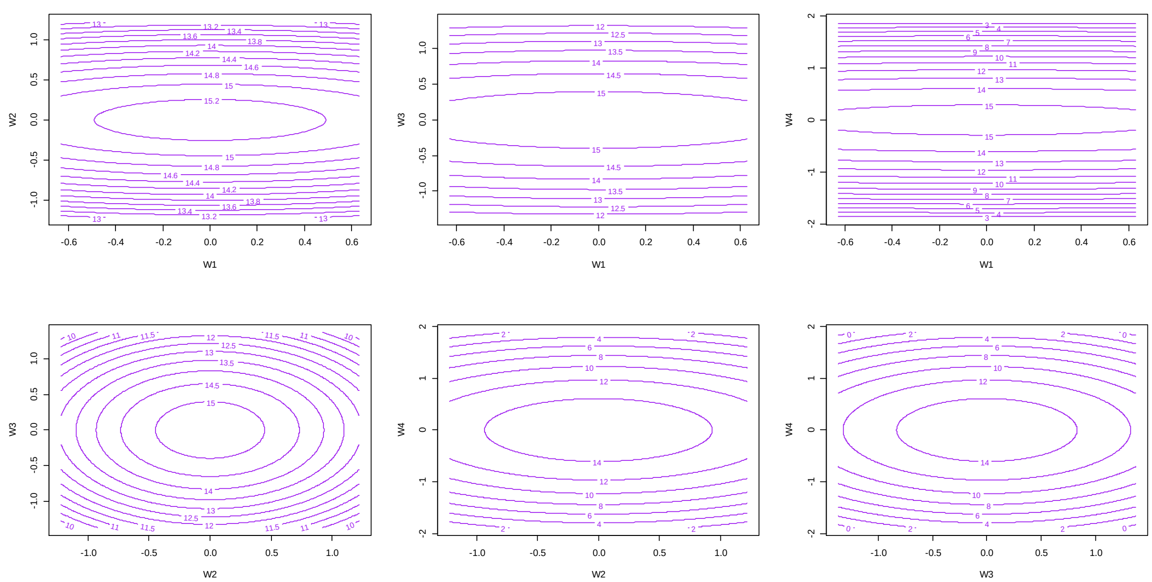

Figure 4.9: Contour Plot for Expected Response of 15 of the GA_3 Model

From (4.25)

$$x_1 = 0.3541 - 0.0349w_1 - 0.3425w_2 - 0.0356w_3 - 0.9382w_4 \quad (4.26)$$

$$x_2 = -0.7390 - 0.1635w_1 - 0.5477w_2 - 0.7860w_3 + 0.2358w_4 \quad (4.27)$$

$$x_3 = -0.0033 + 0.5539w_1 - 0.6836w_2 + 0.4250w_3 + 0.2129w_4 \quad (4.28)$$

$$x_4 = -0.1753 + 0.8156w_1 + 0.3398w_2 - 0.4476w_3 - 0.1374w_4 \quad (4.29)$$

Table 4.17 gives values of $w_1,\ w_2,\ w_3,\ w_4, x_1, x_2\ ,\ x_3$ and x_4 for which $\hat{y} = 15.2979 - 0.4078w_1^2 - 1.4899w_2^2 - 1.8798w_3^2 - 3.5350w_4^2$. The table is obtained form (4.26), (4.27), (4.28) and (4.29). When the experiment is run at the given values of the temperature, soil pH, concentration of GA_3 and pre-treatment time the expected response is close to 15.

Table 4.17: Operating Conditions for Expected Response of 15 of the GA_3 Model

w_1	w_2	w_3	w_4	x_1	x_2	x_3	x_4	$\hat{y}$
-0.63	0.00	0.00	0.00	0.376	-0.636	-0.352	-0.689	14.9
0.00	0.00	0.00	0.00	0.354	-0.739	-0.003	-0.175	14.8
-0.63	0.00	1.37	0.00	0.327	-1.713	0.230	-1.302	14.6
0.63	0.00	0.00	0.00	0.332	-0.842	0.346	0.339	14.0
-0.63	-1.22	1.37	0.00	0.745	-1.045	1.064	-1.717	13.5
0.63	0.00	-1.37	0.00	0.381	0.235	-0.237	0.952	13.3
-0.63	-1.22	0.00	0.00	0.794	0.032	0.482	-1.104	13.2
0.00	-1.22	0.00	0.00	0.772	-0.071	0.831	-0.590	12.9
0.00	1.22	0.00	0.00	-0.064	-1.407	-0.837	0.239	12.8
-0.63	1.22	0.00	0.00	-0.042	-1.304	-1.186	-0.275	12.7
0.00	0.00	1.37	0.00	0.305	-1.816	0.579	-0.789	12.4
0.63	1.22	0.00	0.00	-0.086	-1.510	-0.488	0.753	12.1
0.63	1.22	-1.37	0.00	-0.037	-0.433	-1.071	1.366	12.0
0.63	-1.22	0.00	0.00	0.750	-0.174	1.180	-0.076	12.0
0.00	0.00	-1.37	0.00	0.403	0.338	-0.586	0.438	12.0
-0.63	1.22	1.37	0.00	-0.091	-2.381	-0.604	-0.888	11.9

4.3.4 Optimal Conditions for the Germination of *Melia volkensii* using Sulphuric Acid (H_2SO_4) Treatment

Figure 4.10 shows the response surface plot for the model whereas figure 4.11 shows the contour plot of the fitted model (4.12).

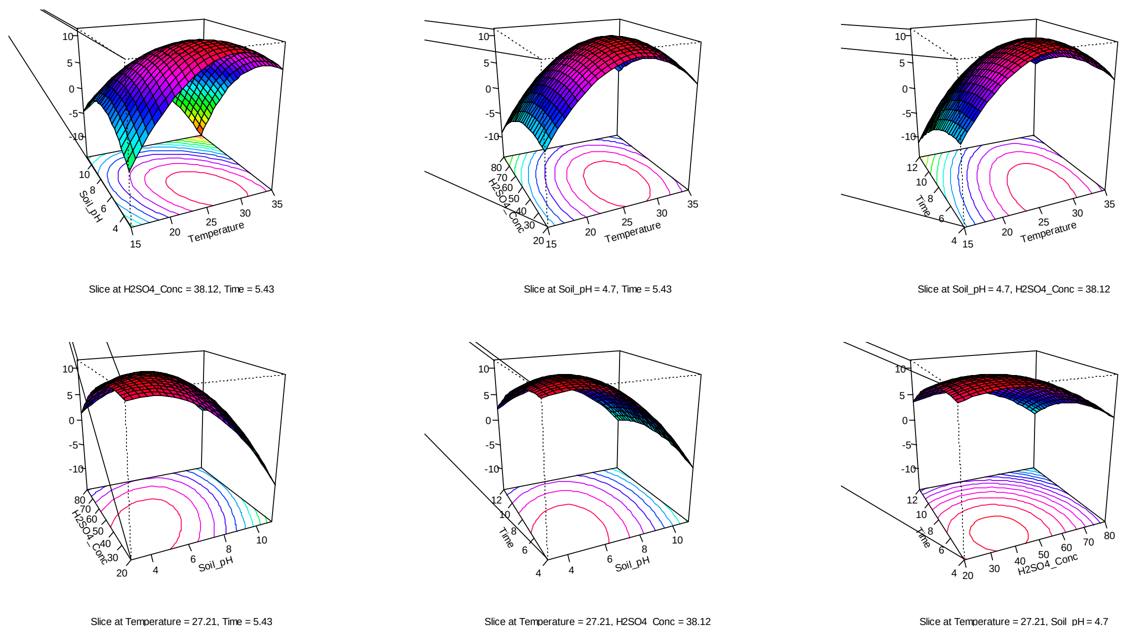

Figure 4.10: Response Surface Plot for the Germination of *Melia volkensii* Model using H_2SO_4 Treatment

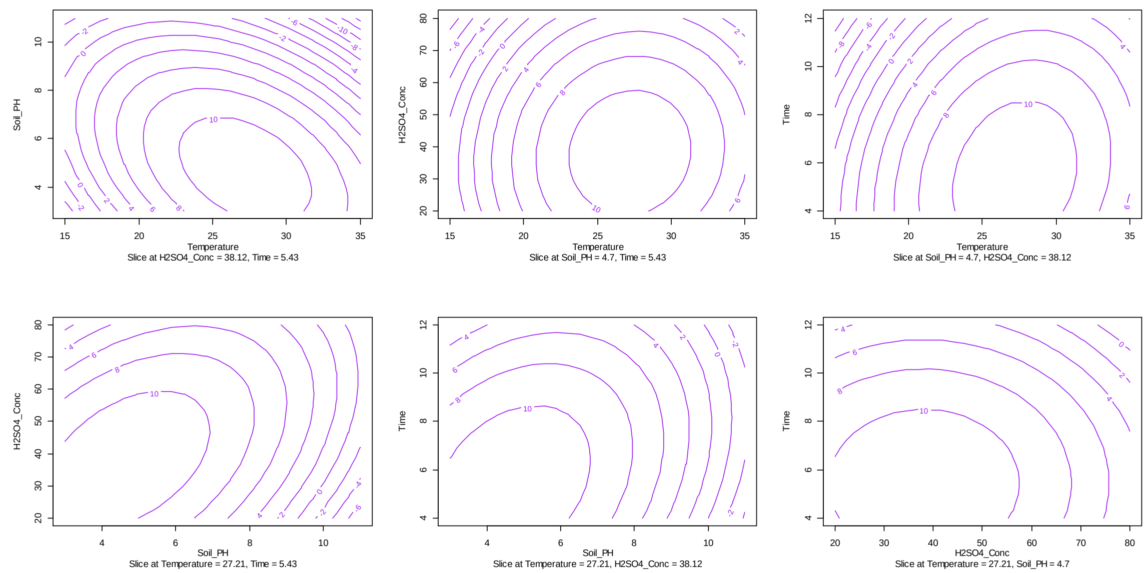

Figure 4.11: Contour Plot for the Germination of *Melia volkensii* Model using H_2SO_4 Treatment

The stationary point and eigenvalues of the model are shown in table 5.7.

Table 4.18: Stationary point and Eigenvalues for the Germination of *Melia volkensii* Model using H_2SO_4 Treatment

Stationary Points				**Eigenvalues**			
x_1	x_2	x_3	x_4				
0.4423	-1.1497	-0.7922	-1.2840	-0.4345	-0.6668	-1.2618	-2.6578

Since all the eigenvalues are negative, the response surface is a maximum one.

The canonical equivalent form of the fitted model is:

$$\hat{y} = 11.4534 - 0.4345w_1^2 - 0.6668w_2^2 - 1.2618w_3^2 - 2.6578w_4^2. \quad (4.30)$$

The stationary point for the model is (0.4423, -1.1497, -0.7922, -1.2840). In terms of the natural variable this is (27.21, 4.70, 38.12, 5.43). Thus the optimal temperature is 27.21°C, the optimal soil pH is 4.70, the optimal concentration of H_2SO_4 is 38.12% and the optimal pre-treatment time is 5.43 hours.

Suppose the investigator is interested in finding where to run the experiment to obtain a response that is close to 11 as possible. This can be obtained from the canonical equivalent model (4.30).

The region is presented as a contour plot of the canonical model in figure 5.12.

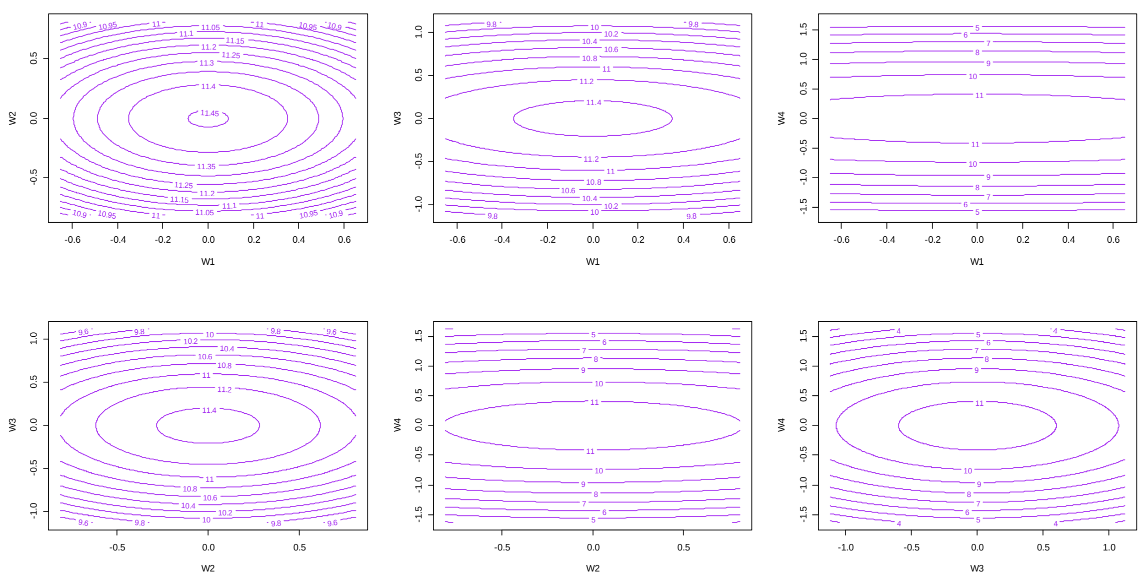

Figure 4.12: Contour Plot for Expected Response of 11 of the H_2SO_4 Model

From (4.30)

$$x_1 = 0.4423 - 0.0879w_1 + 0.5027w_2 + 0.5287w_3 + 0.6782w_4 \quad (4.31)$$

$$x_2 = -1.1497 + 0.2091w_1 - 0.2950w_2 - 0.6001w_3 + 0.7136w_4 \quad (4.32)$$

$$x_3 = -0.7922 - 0.5608w_1 + 0.5958w_2 - 0.5708w_3 - 0.0694w_4 \quad (4.33)$$

$$x_4 = -1.2840 + 0.7963w_1 + 0.5525w_2 - 0.1861w_3 - 0.1613w_4 \quad (4.34)$$

Table 4.19 gives values of $w_1,\ w_2,\ w_3,\ w_4, x_1, x_2\ ,\ x_3$ and x_4 for which $\hat{y} = 11.4534 - 0.4345w_1^2 - 0.6668w_2^2 - 1.2618w_3^2 - 2.6578w_4^2$. The table is obtained form (4.31), (4.32), (4.33) and (4.34). When the experiment is run at the given values of the temperature, soil pH, concentration of H_2SO_4 and pre-treatment time the expected response is close to 11.

Table 4.19: Operating Conditions for Expected Response of 11 of the H_2SO_4 Model

w_1	w_2	w_3	w_4	x_1	x_2	x_3	x_4	$\hat{y}$
-0.65	0.00	0.00	0.00	0.499	-1.286	-0.428	-1.802	11.1
-0.65	-0.81	0.00	0.00	0.092	-1.047	-0.910	-2.249	10.9
-0.65	0.00	-1.12	0.00	-0.093	-0.614	0.212	-1.593	10.8
-0.65	0.81	-1.12	0.00	0.314	-0.852	0.694	-1.146	10.4
-0.65	0.81	0.00	0.00	0.907	-1.525	0.055	-1.354	10.4
0.00	0.00	0.00	0.00	0.442	-1.150	-0.792	-1.284	10.3
-0.65	-0.81	-1.12	0.00	-0.500	-0.375	-0.271	-2.041	10.2
0.00	0.81	-1.12	0.00	0.257	-0.717	0.330	-0.628	10.1
0.00	0.00	-1.12	0.00	-0.150	-0.478	-0.153	-1.076	10.1
0.00	0.81	0.00	0.00	0.849	-1.389	-0.310	-0.836	9.9
0.00	-0.81	0.00	0.00	0.035	-0.911	-1.275	-1.732	9.7
0.65	0.81	-1.12	0.00	0.200	-0.581	-0.035	-0.110	9.6
-0.65	-0.81	1.12	0.00	0.684	-1.719	-1.550	-2.458	9.3
0.65	0.81	0.00	0.00	0.792	-1.253	-0.674	-0.319	9.3
0.65	0.00	0.00	0.00	0.385	-1.014	-1.157	-0.766	9.3
0.65	0.00	-1.12	0.00	-0.207	-0.342	-0.517	-0.558	9.2

4.3.5 Comparison of the Optimal Conditions for the Germination of *Melia volkensii.*

We established that in general, germination rates of Melia seeds are low. For the four chemicals used in the experiment the germination rates were found to be 31.67% for KNO_3, 39.08% for H_2O_2, 42.00% for GA_3 and 28.25% for H_2SO_4. The overall germination rate was found to be 35%.

The germination rate was optimized at 76.49% for GA_3 whereby the soil pH was 5.52, the temperature was 26.77 ^oC, the GA_3 concentration was 0.03% and the seed treatment time was 7.65 hours. The germination rate was optimized at 65.98% for KNO_3 whereby the soil pH was 3.95, the temperature was 31.13 ^oC, the KNO_3 concentration was 0.37% and the seed treatment time was 8.33 hours. For H_2O_2, germination rate was optimized at 65.24 % whereby the soil pH was 4.83, the temperature was 30.43 ^oC, the H_2O_2 concentration was 2.92% and the seed treatment time was 8.76 hours. For H_2SO_4, germination rate was optimized at 57.26 % whereby the soil pH was 4.70, the temperature was 27.21 ^oC, the H_2SO_4 concentration was 38.13% and the seed treatment time was 5.43 hours.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The study formulated an A-, D-, T- optimal four factor rotatable central composite design and proceeded to implement this design in a germination of *Melia volkensii* experiment. This design can be directly used by researchers investigating the influence of four factors on some outcome. With some adjustments, the design can be extended to any number of factors. Further it was found the A- optimal, D- optimal and T- optimal designs were more efficient than the basic design on the A-, D- and T- criteria respectively.

Second order models were found to be sufficient in representing germination of *Melia volkensii*. For all the models developed, their corresponding p values were found to be less than 0.05. However, the adjusted R^2 values were found to be 0.6114 for the KNO_3, 0.5141 for the H_2O_2 model, 0.7569 for the GA_3 and 0.4861 for the H_2SO_4 model. This showed that the percentage of variations which could be explained by the models ranged from 75.69% for the GA_3 model to 48.61% for the H_2SO_4 model. Except for the H_2SO_4 model, adequacy of fit tests showed that the models fitted the data well.

We established that in general, germination rates of Melia seeds are low. For the four chemicals used in the experiment, the germination rates were found to be 31.67% for KNO_3, 39.08% for H_2O_2, 42.00% for GA_3 and 28.25% for H_2SO_4. The overall germination rate was found to be 35%. However when the conditions are favorable and set correctly germination rates can be optimized at 57% for H_2SO_4, 65% for H_2O_2, 66% for KNO_3 and 76% for GA_3.

5.2 Recommendations

The study recommends the use of A-, D- or T- optimal four factor rotatable central composite design for researchers investigating the effect of four numerical explanatory variables on some dependent variable. With some minor modifications the design used in the study can be extended to any number of numerical independent variables.

Germination of *Melia volkensii* can be adequately modeled using second order models with the independent variables being temperature, soil pH, chemical concentration and seed pre-treatment time. It is recommended to use GA_3 for seed pre-treatment. To increase the reliability of the second order model, further research should be done such as adding more independent variables or varying the conditions at which the experiment is run.

The study recommends the use of response surface methodology to find optimal settings of explanatory factors in second order models. To maximize germination of *Melia volkensii*, it is recommended to use soil of pH was 5.5, temperature of 26.8 ^{o}C, GA_3 concentration of 0.03% and the seed pre-treatment time of 8 hours.

5.3 Suggestions for Further Research

The study considered A-, D- and T- optimal central composite design for modeling second order models. Further research can be done on other optimality criteria such as C-, E-, G-, I- and V-.

More research can be done to increase the reliability of the models for the germination of *Melia volkensii*. This may include investigating models of higher order and studying other explanatory that may influence germination of *Melia volkensii*.

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APPENDIX 1: Germination of *Melia volkensii* Experiment Data

Coded Values				Raw Values			Response (Outcome)			
x_1	x_2	x_3	x_4	Temperature	Soil pH	Time	KNO_3	H_2O_2	GA_3	H_2SO_4
-1	-1	-1	-1	20	5	6	3	6	12	3
-1	-1	-1	-1	20	5	6	1	3	6	9
1	-1	-1	-1	30	5	6	6	11	9	11
1	-1	-1	-1	30	5	6	9	14	11	10
-1	1	-1	-1	20	9	6	3	7	2	0
-1	1	-1	-1	20	9	6	0	8	1	1
1	1	-1	-1	30	9	6	4	3	2	0
1	1	-1	-1	30	9	6	1	2	5	1
-1	-1	1	-1	20	5	6	3	7	6	3
-1	-1	1	-1	20	5	6	4	7	5	2
1	-1	1	-1	30	5	6	12	10	11	6
1	-1	1	-1	30	5	6	16	7	16	11
-1	1	1	-1	20	9	6	3	2	2	3
-1	1	1	-1	20	9	6	2	7	5	9
1	1	1	-1	30	9	6	3	4	2	5
1	1	1	-1	30	9	6	3	2	8	0
-1	-1	-1	1	20	5	10	0	3	9	6
-1	-1	-1	1	20	5	10	8	4	7	0
1	-1	-1	1	30	5	10	13	14	12	8
1	-1	-1	1	30	5	10	10	12	9	5
-1	1	-1	1	20	9	10	0	7	5	1
-1	1	-1	1	20	9	10	4	7	4	2
1	1	-1	1	30	9	10	2	8	2	4
1	1	-1	1	30	9	10	0	3	7	1
-1	-1	1	1	20	5	10	0	3	2	1
-1	-1	1	1	20	5	10	1	5	0	1
1	-1	1	1	30	5	10	14	7	12	10
1	-1	1	1	30	5	10	10	16	12	7
-1	1	1	1	20	9	10	1	9	6	3
-1	1	1	1	20	9	10	2	6	2	1
1	1	1	1	30	9	10	4	13	10	1
1	1	1	1	30	9	10	6	4	10	7
-2	0	0	0	15	7	8	2	1	1	1
-2	0	0	0	15	7	8	1	3	2	2
2	0	0	0	35	7	8	5	8	7	0
2	0	0	0	35	7	8	4	6	2	6
0	-2	0	0	25	3	8	13	7	11	3
0	-2	0	0	25	3	8	4	10	14	8
0	2	0	0	25	11	8	6	3	2	4

Coded Values				Raw Values			Response (Outcome)			
x_1	x_2	x_3	x_4	Temperature	Soil pH	Time	KNO_3	H_2O_2	GA_3	H_2SO_4
0	2	0	0	25	11	8	2	2	4	5
0	0	-2	0	25	7	8	7	5	4	12
0	0	-2	0	25	7	8	11	7	10	9
0	0	2	0	25	7	8	7	6	4	5
0	0	2	0	25	7	8	8	9	10	3
0	0	0	-2	25	7	4	10	12	9	16
0	0	0	-2	25	7	4	6	12	13	10
0	0	0	2	25	7	12	9	11	14	4
0	0	0	2	25	7	12	11	6	14	3
0	0	0	0	25	7	8	10	8	15	8
0	0	0	0	25	7	8	10	11	15	7
0	0	0	0	25	7	8	14	5	10	10
0	0	0	0	25	7	8	11	17	15	6
0	0	0	0	25	7	8	12	9	13	11
0	0	0	0	25	7	8	9	11	12	13
0	0	0	0	25	7	8	11	15	15	7
0	0	0	0	25	7	8	7	14	16	11
0	0	0	0	25	7	8	10	16	15	8
0	0	0	0	25	7	8	7	15	17	12
0	0	0	0	25	7	8	16	9	11	12
0	0	0	0	25	7	8	9	10	17	11

APPENDIX 2: R Code

```
#Importing the data
data<-
read.csv("C:\\Psn\\Education\\MU\\Thesis\\2015\\Data\\data.csv")
names(data)
attach(data)

#Call required packages
library(MASS)
library(expm)
library(rsm)

#Format the data as a matrix
X<-
matrix(cbind(x0,x1,x2,x3,x4,x11,x12,x13,x14,x22,x23,x24,x33,x34,x
44),60,15)

#Computing the experimental moment matrix
M<-t(X)%*%X/60

#Computing the experimental moment matrix A-, D- and T optimal
values
a_opt_exp<-(sum(diag(solve(M)))/15)^-1
d_opt_exp
d_opt_exp<-det(M)^(1/15)
d_opt_exp
t_opt_exp<-sum(diag(M))/15
t_opt_exp

#Creating a 60 by 1 matrix 0f 1s
One_60<-matrix(cbind(x0))
One_60

#Creating a 60 by 60 matrix 0f 0s
Xero_60<-One_60-1
Xero_60_mt<-Xero_60%*%t(Xero_60)

#Computing A- optimal weights
M_3<-solve(M)%*%solve(M)%*%solve(M)
A_A<-X%*%sqrtm(M_3)%*%t(X)
w_a_1<-ginv(A_A*A_A)%*%One_60
w_a_1
w_a<-w_a_1/sum(w_a_1)
w_a

#Computing A- optimal moment matrix
W_A<-Xero_60_mt
for(i in 1:60)W_A[i,i]<-w_a[i,1]
M_A<-t(X)%*%W_A%*%X
round(M_A/0.424,3)
```

```
#Computing A- optimal value for A -optimal design
a_opt_a<-(sum(diag(solve(M_A)))/15)^-1
a_opt_a

#Computing D- optimal weights
A_D<-X%*%solve(M)%*%t(X)
w_d_1<-ginv(A_D*A_D)%*%One_60
w_d<-w_d_1/sum(w_d_1)
w_d

#Computing D- optimal moment matrix
W_D<-Xero_60_mt
for(i in 1:60)W_D[i,i]<-w_d[i,1]
M_D<-t(X)%*%W_D%*%X
round(M_D/0.1778,2)

#Computing D- optimal value for D -optimal design
d_opt_d<-det(M_D)^(1/15)
d_opt_d

#Computing T- optimal weights
M_1<-solve(M)
A_T<-X%*%sqrtm(M_1)%*%t(X)
w_t_1<-ginv(A_T*A_T)%*%One_60
w_t_1
w_t<-w_t_1/sum(w_t_1)
w_t

#Computing T- optimal moment matrix
W_T<-Xero_60_mt
for(i in 1:60)W_T[i,i]<-w_t[i,1]
M_T<-t(X)%*%W_T%*%X
round(M_T,6)
round(M_T/0.0342,2)

#Computing T- optimal value for T -optimal design
t_opt_t<-sum(diag(M_T))/15
t_opt_t


#Modeling the second order KNO3 model with coded values
model_KNO3<- rsm(KNO3 ~ SO(x1,x2,x3,x4), data=data)

#Displaying the summary for the KNO3 model
summary(model_KNO3)

#Modeling the second order KNO3 model with only the significant
factors
final_KNO3<-lm(KNO3 ~ 1+x1+x2+x11+x12+x22+x33)
summary(final_KNO3)
```

```
#Modeling the second order KNO3 model with raw values
model_KNO3_1<- rsm(KNO3 ~ SO(Temperature,Soil_pH,KNO3_Conc,Time),
data=data)
summary(model_KNO3_1)

#Displaying the response surface plot for the KNO3 model
par(mfrow = c(2,3))
persp(model_KNO3_1, ~Temperature+Soil_pH+KNO3_Conc+Time,
col=rainbow(100) , at=canonical(model_KNO3_1)$xs,
contour=("colors"))

#Displaying the contour plot for the KNO3 model
par(mfrow = c(2,3))
contour(model_KNO3_1, ~Temperature+Soil_pH+KNO3_Conc+Time,
col="purple", at=canonical(model_KNO3_1)$xs)

#Modeling the second order H2O2 model with coded values
model_H2O2<- rsm(H2O2 ~ SO(x1,x2,x3,x4), data=data)

#Displaying the summary for the H2O2 model
summary(model_H2O2)

#Modeling the second order H2O2 model with only the significant
factors
final_H2O2<-lm(H2O2 ~ 1+x1+x2+x11+x12+x22+x33)
summary(final_H2O2)

#Modeling the second order H2O2 model with raw values
model_H2O2_1<- rsm(H2O2 ~ SO(Temperature,Soil_pH,H2O2_Conc,Time),
data=data)
summary(model_H2O2_1)

#Displaying the response surface plot for the H2O2 model
par(mfrow = c(2,3))
persp(model_H2O2_1, ~Temperature+Soil_pH+H2O2_Conc+Time,
col=rainbow(100), at=canonical(model_H2O2_1)$xs,
contour=("colors"))

#Displaying the contour plot for the H2O2 model
par(mfrow = c(2,3))
contour(model_H2O2_1, ~Temperature+Soil_pH+H2O2_Conc+Time,
col="purple",at=canonical(model_H2O2_1)$xs)

#Modeling the second order GA3 model with coded values
model_GA3<- rsm(GA3 ~ SO(x1,x2,x3,x4), data=data)

#Displaying the summary for the GA3 model
summary(model_GA3)

#Modeling the second order GA3 model with only the significant
factors
final_GA3<-lm(GA3 ~ 1+x1+x2+x11+x13+x22+x23+x24+x33)
```

```
summary(final_GA3)

#Modeling the second order GA model with raw values
model_GA3_1<- rsm(GA3 ~ SO(Temperature,Soil_pH,GA3_Conc,Time),
data=data)
summary(model_GA3_1)

#Displaying the response surface plot for the GA3 model
par(mfrow = c(2,3))
persp(model_GA3_1, ~Temperature+Soil_pH+GA3_Conc+Time,
col=rainbow(100), at=canonical(model_GA3_1)$xs,
contour=("colors"))

#Displaying the contour plot for the GA3 model
par(mfrow = c(2,3))
contour(model_GA3_1, ~Temperature+Soil_pH+GA3_Conc+Time,
col="purple",at=canonical(model_GA3_1)$xs)

#Modeling the second order H2SO4 model with coded values
model_H2SO4<- rsm(H2SO4 ~ SO(x1,x2,x3,x4), data=data)

#Displaying the summary for the H2SO4 model
summary(model_H2SO4)

#Modeling the second order H2SO4 model with only the significant
factors
final_H2SO4<-lm(H2SO4 ~ 1+x1+x2+x4+x11+x12+x22+x33)
summary(final_H2SO4)

#Modeling the second order H2SO4 model with raw values
model_H2SO4_1<- rsm(H2SO4 ~
SO(Temperature,Soil_pH,H2SO4_Conc,Time), data=data)
summary(model_H2SO4_1)

#Displaying the response surface plot for the H2SO4 model
par(mfrow = c(2,3))
persp(model_H2SO4_1, ~Temperature+Soil_pH+H2SO4_Conc+Time,
col=rainbow(100), at=canonical(model_H2SO4_1)$xs,
contour=("colors"))

#Displaying the contour surface plot for the H2SO4 model
par(mfrow = c(2,3))
contour(model_H2SO4_1, ~Temperature+Soil_pH+H2SO4_Conc+Time,
col="purple",at=canonical(model_H2SO4_1)$xs)
```