

**MATHEMATICAL MODELING OF ENERGY MIX AND OPTIMIZATION
OF RENEWABLE RESOURCES**

BY

SIGEI KIPKIRUI ROBERT

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DECLARATION

Declaration by Candidate

I declare that this is my original work and has not been presented in any academic or other institution for academic award or other purposes. No part of this document should be copied or reproduced electronically or by other means without a written permission from the author, or Moi University.

Signature: Date:.....

Sigei Robert Kipkirui

MSC/AM/04/14

Declaration of the Supervisors

This thesis has been submitted with my supervision as the University Supervisor.

Signature: Date:.....

Dr. Rotich Titus

Department of Mathematics, Physics and Computing

Moi University, Eldoret

Signature: Date:.....

Dr. Koech Wesley

Department of Mathematics, Physics and Computing

Moi University, Eldoret

DEDICATION

I dedicate this Thesis to my beloved wife, **Consolata Jebet Sigei**, my children, **Marquez Kipruto Sigei** and **Barcley Kipkoech Sigei**, my family, and my friends.

May this work inspire you always.

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ABSTRACT

Energy, as both a direct and indirect fundamental life-supporting resource, has experienced a steady rise in domestic and industrial demand, driven by technological advancement, population growth, and economic expansion. Various sources of energy including fossil fuels, hydroelectric power, geothermal energy, wind, solar, and nuclear are available in different proportions, each with distinct cost structures and environmental impacts. The challenge of meeting these diverse needs while minimizing production and distribution costs, conserving the environment, and reducing wastage has evolved into a complex multi-objective problem. This research focuses on the mathematical modelling of the optimal energy mix and the optimization of renewable resources, with particular emphasis on individualized demand profiles. The objectives are threefold: first, to formulate a mathematical model for analysing the dynamics of energy demand, production, and distribution; second, to determine the parameter thresholds that guarantee stability and robustness of the optimal energy mix; and third, to develop a smart grid feedback model using adaptive neural networks capable of automatically maintaining the desired energy balance. The methodology entails formulating a system of differential equations to represent the energy system, expressing it in state-space form, and applying Laplace transforms to derive transfer functions. These will be analysed for sensitivity, stability, and robustness using Nyquist and Bode plot criteria. MATLAB–Simulink, equipped with neural network modules, will then be employed to simulate and implement an intelligent, adaptive feedback control system. Through these simulations, the study will integrate real-time learning and self-adjustment capabilities to align production with demand in the most efficient manner. The anticipated outcome is an automated, smart distribution system capable of dynamically meeting individualized energy requirements at the lowest possible cost, while enhancing the utilization of renewable sources and reducing reliance on non-renewable options. Ultimately, this approach aims to promote environmental sustainability through increased adoption of green energy technologies.

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ABBREVIATIONS

AC	-	Alternating current
ANN	-	Artificial Neural Network
BTU	-	British Thermal Units
CNN	-	Convolutional Neural Network
DC	-	Direct Current
GMSP	-	Grid Management Support Program
GWh	-	Gigawatt hour
IF	-	Infra-Red
JKIA	-	Jomo Kenyatta International Airport
KETRACO	-	Kenya Electricity Transmission Company
KIPPRA	-	Kenya Institute for Public Policy Research and Analysis
KPLC	-	Kenya Power and Lighting Company
LCOE	-	Levelized Cost of Electricity
LPG	-	Liquified Petroleum Gas
LTl	-	Linear Time Invariant
MATLAB	-	Matrix Laboratory
MSE	-	Mixed Source Equilibrium
MW	-	Megawatt
PID	-	Proportional Integral Derivative
PSH	-	Peak Sun Hours
PV	-	Photovoltaic
REA	-	Rural Electrification Authority
REREC	-	Rural Electrification and Renewable Energy Corporation
RNN	-	Recurrent Neural Network

SIMULINK	-	Graphical simulation and model-based design tool that works with MATLAB
SISO	-	Single Input Single Output
SME	-	Small and Medium Scale Enterprises
SSE	-	Self Sufficiency Equilibrium
TDE	-	Total Dependence Equilibrium
TWh	-	Terawatt-hours per year
USD	-	United States Dollar
UV	-	Ultra Violet
W	-	watts

CHAPTER ONE

INTRODUCTION

In this chapter, we capture the introduction of the study, to bring on board the readers to a common understanding of the concept under study. This is split into the following subsections; namely, background of the study, energy sources, uses and levels of demand and supply, cost and other constraints affecting energy sector, statement of the problem, objectives of the study, significance of the study, justification of the study and definition of terms used in the study.

1.1 Background of the Study

Energy is a fundamental and pervasive concept that plays a vital role in shaping the modern world and sustaining life on Earth. In its various forms, energy is essential for driving processes, fuelling activities, and powering systems across diverse domains, from the natural environment to human society. From the perspective of physics and engineering, energy is defined as the ability to do work or the capacity to cause change. This broad definition encompasses a wide array of phenomena and applications, reflecting the ubiquity and significance of energy. One of the most prevalent manifestations of energy is in the form of mechanical energy, which encompasses the kinetic energy associated with motion and the potential energy associated with position or configuration. Mechanical energy plays a central role in the operation of machinery, transportation systems, and the natural movement of objects in the environment. It is fundamental to the functioning of engines, turbines, and power plants, enabling the generation of electricity and the production of mechanical work for industrial, commercial, and residential purposes. Moreover, thermal energy, or heat energy, is a crucial form of energy that influences numerous natural and human-made processes. It is integral to activities such as heating, cooling,

and thermal power generation, as well as to phenomena like conduction, convection, and radiation. Thermal energy is a vital component of energy conversion processes, such as those occurring in thermal power plants, industrial processes, and heating systems, and is a primary driver of climate and weather patterns, influencing ecosystems and human societies. Additionally, electrical energy, derived from the movement of electric charge, is a cornerstone of modern civilization, powering infrastructure, appliances, communication systems, and a vast array of electronic devices. It facilitates the transmission, distribution, and utilization of energy across diverse applications, spanning lighting, computing, telecommunications, and industrial automation. Electrical energy is integral to the functioning of power grids, renewable energy technologies, and electric vehicles, contributing to advancements in energy efficiency, sustainability, and technological innovation. In the realm of physics, energy is further categorized into potential energy, kinetic energy, chemical energy, nuclear energy, and electromagnetic energy, reflecting the diverse mechanisms and sources through which energy can be stored, transferred, and transformed. This broad spectrum of energy forms and phenomena underpins the development of energy technologies, energy policy, and our understanding of the natural world, encompassing both macroscopic and microscopic scales of observation and analysis. Furthermore, the global conversation surrounding energy encompasses critical considerations related to energy resources, sustainability, environmental impact, and climate change. The exploration and utilization of renewable energy sources, such as solar, wind, hydroelectric, and geothermal energy, is indicative of efforts to transition towards a more sustainable and resilient energy landscape. The efficient use of energy, energy conservation, and the development of clean energy technologies play a pivotal role in addressing the challenges of energy security,

environmental stewardship, and the mitigation of global climate change. From the definitions given, it is clear that energy is a multifaceted and indispensable concept that permeates every aspect of our existence, from the fundamental workings of the natural world to the complex systems and infrastructures that support human society. Understanding, harnessing, and managing energy in all its forms are crucial imperatives for advancing sustainable development, driving innovation, and ensuring a prosperous and harmonious future for generations to come.

From other researchers, it is agreed that energy is the most essential resource to all living organisms (Atwa et al., 2010). The sun, directly or indirectly, is the source of all the energy available on earth. The primary source of energy in the environment include fuels like coal, oil, natural gas and biomass, sunlight, wind, water, geothermal and nuclear sources.

The energy sector in Kenya is diverse and encompasses a range of energy sources, including renewables and non-renewables. Kenya's energy mix includes hydropower, geothermal, wind, solar, fossil fuels, and biomass. The country has made significant strides in expanding its renewable energy capacity, particularly in geothermal and wind power generation (Kiplagat et al., 2011). Kenya's energy sector is characterized by a strong focus on renewable energy sources due to the country's abundant renewable energy potential. The development of geothermal energy has been a key success story, with Kenya being a leader in geothermal power generation in Africa. The country's vast geothermal resources have been effectively utilized to increase electricity production and reduce reliance on imported fossil fuels. In addition to geothermal, Kenya has also been investing in wind and solar energy projects. The Lake Turkana Wind Power project, one of the largest wind farms in Africa, has

significantly contributed to the country's renewable energy capacity. Solar power projects, including grid-connected solar plants and off-grid solar solutions, have also been expanding to harness Kenya's abundant sunlight. Despite the focus on renewables, Kenya still relies on non-renewable sources such as thermal power (fossil fuels) to meet its energy needs. This is due in part to the demand for reliable and consistent power supply, as well as the need to diversify the energy mix. Overall, Kenya's energy sector is dynamic, with a growing emphasis on renewable energy and sustainability. The government has been actively promoting investment in clean energy technologies and implementing policies to support the expansion of renewable energy capacity, aiming to enhance energy security, reduce greenhouse gas emissions, and improve access to electricity for its citizens.

The energy sector in Kenya is largely dominated by electricity, petroleum, and wood fuel. Wind, solar and geothermal is the other sources, which have not been exploited to their capacity. The current connected grid is about 1,400MW, as opposed to a demand of 1,600MW. The connected grid comprises of 705MW Hydropower (30.0%), 306MW fossil fuel (13.0%), 1,105MW Geothermal (47.0%) and 42MW Biogases Cogeneration (1.8%)(Power, 2015).

As the population grows, much pressure is exerted to the limited sources of energy. This depletes the non-renewable sources and exerts much pressure on the renewable resources. However, the pressure in the need for energy can be traced to two types of needs; optimal usage and inefficient use (Atwa et al., 2010).

Optimal use relates to a consumer supplied with the exact amount of energy he requires and actually using the whole of it according to their energy needs. Inefficient

use refers to energy supply being more than the demand, leading to unnecessary high cost of production.

In order to match the energy demands and supply of various individuals, there is need for a dynamic feedback mechanism, which analyses and detects the demands of an individual, and automatically, control the amount of supply as required. This can be achieved through the use of Smart Grid (Wang et al., 2009).

Smart grid is an electricity network that uses digital communications technology to detect and react to local changes in usage (Wang et al., 2009). This system uses a two-way technology between the utility and its customers. Here, utility refers to the production unit, where energy is generated. This can be hydro, geothermal, solar, wind or any other sources of energy, while the consumers refers to all points where energy is consumed, which include and not limited to the household, government institutions, industries, both small scale, medium or large enterprises, among others. Controls, digital gadgets and computers work with electrical grid to digitally change electrical demand and supply, to improve efficiency, robust supply of electricity, economical use of resources, reduced peak demand, improved security, and increased integration of large scale renewable energy systems (Carlo et al., 2014).

1.1.1 Energy Sources

As of 2021, (Ghoshray & Malki, 2021)the global energy mix comprises a diverse array of energy sources, each contributing varying proportions to the overall energy supply. The primary energy sources include fossil fuels (such as coal, oil, and natural gas), nuclear energy, and renewable energy sources (including hydroelectric, solar, wind, geothermal, and biomass energy). The proportions of these energy sources in the global energy mix reflect regional variations, technological advancements, energy

policies, and the evolving dynamics of energy demand and consumption. Fossil fuels, historically dominant in the global energy mix, continue to play a significant role in meeting energy demands. Coal, oil, and natural gas collectively account for a substantial share of the global energy supply (Ghoshray & Malki, 2021). Coal has traditionally been a leading energy source for electricity generation, especially in emerging economies, while oil is a primary fuel for transportation and industrial applications. Natural gas, prized for its versatility and lower emissions compared to coal and oil, has become increasingly prominent in power generation and various industrial processes. Nuclear energy, though representing a smaller proportion of the global energy mix, is a notable source of low-carbon electricity production in many countries. Nuclear power plants utilize controlled nuclear reactions to generate heat, which is then converted into electricity. While nuclear energy has the advantage of minimal greenhouse gas emissions, safety concerns and waste management issues have shaped its role and expansion in the energy mix. Renewable energy sources, including hydroelectric, solar, wind, geothermal, and biomass energy, have garnered increasing attention and investment due to their sustainability, environmental benefits, and declining costs. Hydroelectric power, derived from the energy of flowing water, has historically been a major contributor to global electricity generation. Solar energy, harnessed through photovoltaic systems and concentrated solar power technologies, has experienced rapid growth, driven by technological advancements and supportive policies. Wind energy, obtained from wind turbines, has also seen significant expansion, particularly in regions with favourable wind resources. Geothermal energy, derived from the natural heat of the Earth, and biomass energy, derived from organic materials, complement the renewable energy mix, contributing to electricity generation and heating applications. The proportions of these energy sources vary by

region, influenced by factors such as energy resource endowment, economic development, policy priorities, and environmental considerations. As the global energy landscape evolves, there is a growing emphasis on enhancing the share of renewable energy sources and reducing the reliance on fossil fuels to address climate change, improve energy security, and promote sustainable development. While fossil fuels continue to constitute a significant portion of the global energy mix, the increasing adoption and competitiveness of renewable energy technologies are anticipated to lead to a gradual shift in the proportions of energy sources, with renewable energy playing a larger role in meeting global energy demands in the coming years.

The ultimate source of energy is the sun. This gives life to every living thing, however other sources of energy, which can be transformed for use to solve day-to-day human needs include hydroelectric power, geothermal energy, wind energy, fuel oil, biomass and gas turbines. All these are actively explored in Kenya, but in different levels of success. According to (Karekezi & Kimani, 2002), new generation of wind energy that could come online in the main grid by 2020 is a total of approximately 5000MW up from 2300MW in 2015 through 42 installed power plants.

Considering all sources of power in Kenya, hydro takes a share of 2700MW with over 50% from geothermal and the rest from water dams (Karekezi & Kimani, 2002).

Solar power is potentially estimated to supply between 20 – 30% of the total population by 2020. This will bring to a total of between 70 – 80% of the total population of people with access to electricity by 2020, up from 46% in 2015.

In order to achieve this, the government will cough a total of 14 – 18 billion USD from their annual budget in order to deliver power sector projects. The production and

distribution of power is managed by various organizations, namely; Commercial transmission lines is done by Kenya Electricity Transmission Company (KETRACO) together with Rural Electrification authority (REA) who connects rural and public facilities(Owiro et al., 2015). KETRACO is a private company, but majority of the shares is owned by the government, REA or REREC is a purely government entity, meant to distribute power to marginalized areas, where KPLC cannot reach due to less rate of return. Kenya power and lighting company KPLC is also a purely private company with 51% of the shares owned by the government.

1.1.2 Energy Demand

Global energy demand refers to the total amount of energy required worldwide to support various human activities, including industrial processes, transportation, heating, cooling, and electricity generation. It is typically measured in units of power, such as watts (W) or more commonly in units of energy over time, such as terawatt-hours (TWh) per year. The demand for energy is driven by population growth, economic development, technological advancements, and changes in energy consumption patterns. As global population increases and developing countries industrialize, the demand for energy rises. Meeting global energy demand requires a diverse energy mix, including fossil fuels (such as coal, oil, and natural gas), renewable energy sources (such as solar, wind, hydro, and biomass), and nuclear power. Increasingly, there is a shift towards cleaner and more sustainable sources of energy to mitigate environmental impacts and address climate change. Understanding and projecting global energy demand is essential for policymakers, energy companies, and researchers to make informed decisions about energy infrastructure investments, energy policies, and technological innovations to ensure a secure, reliable, and sustainable energy future.

The relationship between energy demands, industrialization, and population growth is complex and interconnected. As a population grows, the demand for energy increases to support the expanding needs of households, transportation, manufacturing, and other sectors (Owiro et al., 2015). Likewise, industrialization, which involves the growth of manufacturing and infrastructure, leads to higher energy demands due to the deployment of machinery, heating, and other energy-intensive processes. Population growth directly impacts the overall energy demand as more people require energy for their daily lives. As communities develop and urbanize, the need for energy further escalates. Similarly, industrialization fuels energy demands as factories, production facilities, and industrial processes necessitate substantial energy inputs. Consequently, the interplay between population growth, industrialization, and energy demands underscores the significance of sustainable and diverse energy strategies. As more countries undergo industrialization and experience population expansion, there is a growing need to balance energy requirements with environmental sustainability, leading to the adoption of cleaner and renewable energy sources and the advancement of energy-efficient technologies. Understanding these relationships is crucial for policymakers, energy planners, and researchers to develop strategic energy policies and infrastructure that meet burgeoning demands while addressing environmental and social concerns.

Based on 2015 survey by (Owiro et al., 2015), the estimated demand of energy is expected to double up from 2600 MW in 2015 to 5600MW by 2022. This is based on baseline survey of estimated population growth rate and increased economic activities.



Figure 1 Energy demand due to population increase, urbanization and economic growth. Source: (Stiftung, 2012)

As much as high cost is one of the major bottlenecks to economic activity in the country, it is estimated by Kenya Institute for Public Policy Research and Analysis (KIPPRA) that commercial consumption of electricity is at 89.4 per capita. (Mwakubo et al., 2007). This consumption is dominated by biomass at 68%, followed by petroleum at 21%, then electricity 9%, and the remaining at 1%, which include solar, wind among others (Mwakubo et al., 2007).

Unlike other sources of energy, electricity is a moving energy, which is generated and used without storage or delay, with an exception of solar energy. This means that the demand or energy due to connection of every new user or electric gadget, is proportional to the amount of energy drawn from the source, and thus adjustment is made at the production point to increase the power generated to match the demands. However, the rate of production may not be proportional to the rate of consumption. The difference is either a deficit (negative) or wastage (positive). Types of energy losses include heat transfer, mechanical friction, electrical resistance, and electromagnetic radiation. The wastage of energy is attributed to heating in distribution transformers, sparks due to loose connection, poor or obstruction of cables by trees and other barriers, and wastage due to no loading. Other wastage here referred to as commercial waste is associated with unmetered losses created by electricity theft, and illegal connection.

The concern of this research is more on how to reduce this wastage, by finding a feedback mechanism, which synchronizes production and consumption, so that only what is required is produced. Using a smart grid can help reduce energy losses through more efficient monitoring and management of electricity distribution. Smart grids can enable better control over energy flow, reduce transmission and distribution losses, and optimize energy consumption. By implementing advanced metering infrastructure, real-time monitoring, and automation, smart grids can help identify and address energy losses more effectively, leading to overall energy conservation and cost savings. Apart from that, this research aims at analysing various sources of energy in an attempt to find a suitable energy mix to optimize the cost of consumption and the cost of production.

Energy production capacity exceeds the demands the in Kenya. As of 2021, Kenya's energy generation capacity stands at around 2,900 megawatts, with the bulk of the electricity being generated from renewable sources such as hydroelectric, geothermal, and wind power (Takase et al., 2021). However, the country's energy consumption continues to grow due to population expansion, industrial development, and increasing urbanization. To meet the rising demand, Kenya has been investing in expanding its energy infrastructure and diversifying its energy mix to ensure a reliable and sustainable energy supply for its growing economy

The demand as at 2022 was estimated to be 10,008 GWh against the production capacity of 12,652 GWh (Takase et al., 2021). It is noted that the demand increased by around 4.5% of the production capacity, and thus the need to invest more on production and distribution. It is estimated that the demand will rise by 5.7% by 2024, meaning there will be a shortage of supply, thus leading to increase in price and

agitation for production capacity. This will be solved by increasing the import quota, from other countries. Currently AC is imported from Uganda and DC is imported from Ethiopia. The DC from Ethiopia is inverted to AC at Suswa before distribution in Kenya.

1.1.3 Energy generation, supply and cost implications

Kenya's energy generation and supply sector has experienced significant developments in recent years. The country relies on a mix of energy sources to meet its growing demand for electricity and fuel for transportation and industrial processes.

Key aspects of energy generation and supply in Kenya include:

1. **Diversified Energy Sources:** Kenya has made efforts to diversify its energy mix, embracing renewable energy sources such as geothermal, hydroelectric, wind, and solar power. This diversification aims to reduce reliance on imported fossil fuels and enhance energy security and environmental sustainability (Kiplagat et al., 2011).
2. **Geothermal Power:** Kenya ranks among the top countries globally in geothermal energy production. The country has tapped into its geothermal potential to generate a substantial portion of its electricity, with geothermal power playing a significant role in reducing reliance on fossil fuels (Kiplagat et al., 2011).
3. **Hydroelectric Power:** Kenya has a significant capacity for hydroelectric power generation, with several major dams and power plants contributing to the country's energy supply. However, hydroelectric power generation is subject to fluctuations in rainfall and water availability (Kiplagat et al., 2011).

4. **Wind and Solar Power:** The development of wind and solar power projects has gained momentum in Kenya, contributing to the country's renewable energy portfolio. These projects aim to harness Kenya's abundant wind and solar resources to add more clean energy to the national grid (Muchiri et al., 2023).
5. **Grid Expansion and Rural Electrification:** Efforts to extend electricity access to rural and underserved areas have been a priority in Kenya (Hartvigsson et al., 2018). The government has invested in expanding the national grid and promoting off-grid solutions to bring electricity to more remote communities.
6. **Energy Policy and Regulation:** Kenya has implemented policies and regulations to incentivize renewable energy investments, promote energy efficiency, and improve the overall energy sector governance. Overall, Kenya's energy generation and supply sector continue to evolve, with a focus on expanding renewable energy capacity, enhancing energy access, and ensuring a reliable and sustainable energy supply for the country's development (Hartvigsson et al., 2018).

As Kenya tries to achieve its vision 2030 of improving the economy to industrialized, middle-income country, the cost implications of energy production, transmission and management need to be met. Apart from internal sources, Kenyan government gets loans and grants from other financial institutions. Power Africa provides 1 billion USD through Grid Management Support Program (GMSP) to finance 45% production and distribution of hydro-electricity, 24% thermal energy and 31% other renewable sources (Power Africa, 2013). Based on Government of Kenya budget plans, Kenyan government has secured between 3 – 5.5 billion USD leaving a gap of between 14 – 18 billion USD. This is intended for power generation (10 – 14B USD), power transmission (1 – 2B USD), distribution (5.7 – 6.7 billion USD) and off-grid (0.9 –

1.5 billion USD) (Karekezi & Kimani, 2002). In general, benefit maximization in any normal planning problem means minimizing cost while maintaining the performance of the system within acceptable limits (Y. Atwa et al., 2009).

The cost of production of solar energy is more on the initial investment on the solar panels, mounting structural framework, security infrastructure and the distribution cables. All these can be apportioned to the lifespan of the project. If the solar plant has a 30 years guarantee, then the initial cost is distributed to 30 years, for the purpose of comparing with other forms of energy.

Kenya is located along the equator, where insolation is the highest, and thus gets the most benefit of solar power. The cost of installing a solar plant for a home or an office costs between Ksh. 30,000 and Ksh. 50,000 depending on the capacity required.

Studies done by (Rehman et al., 2007) indicated that installation of 5MW capacity of solar energy using photovoltaic cells connected to the grid in an area with average solar radiation of $2.06MWh/m^2yr^{-1}$. Analysis indicated that 70% of the cost goes to power generation equipment, including solar panels, batteries, DC-AC inverters and cables, while 28% goes into control and installation equipment. (Rehman et al., 2007) indicated that an average yield of $261kWh/m^2$ was sold for an average of 30 cents per kWh and an internal rate of return of 13.53% was realized. In addition, environmental impact assessment indicated that 8182 tons of greenhouse gases per year were avoided for the use of solar energy as compared with alternative sources of energy.

In Kenya, particularly Dagoreti and Thika, the average solar radiation emitted by the sun ranges between $3.82MWh/m^2yr^{-1}$ and $7.09MWh/m^2yr^{-1}$. This radiation is

known to reach the outer atmosphere at the rate of about 1.367 kW/m^2 , the value is called the *solar constant* (Carbonell et al., 2013). Total emission of the sun is about $3.7 \times 10^{26} \text{ W/s}$. This radiation may be divided according to its spectral distribution into UV, visible and near IR, the latter two accounts for about 90% of the total emission (Eltbaakh et al., 2011). The atmosphere distorts the solar radiation and alters the wavelength distribution; also, the solar energy actually reaching the ground varies with latitude, season, time of day and other factors such as topography, meteorological elements, atmospheric dust and contamination. The radiation available on the ground is composed of beam or direct radiation and diffuse radiation, producing half of the available energy. There is also solar reflected by the earth's surface, and long-wave re-radiation such as nocturnal radiation which is particularly significant for some cooling purposes.

Heat source include solar irradiation energy collected using photovoltaic cells or solar water heating tubes. The sun rises from the East on average from sunrise time of around 6.30am and sets in the West at sunset time of around 6.30pm, approximately 12 hours sunshine in a day. Because of geographical location of Kenya, most of the time of the year, the sun is not far from directly above the surface, and thus placing the solar collector with a tilt to face the East or the West picking maximum sunrays.

The average radiation from the sun reaches the atmosphere at a constant solar rate of 1.367 kW/m^2 . This radiation energy is subdivided into three: Ultra Violet (UV) light, Visible light and near Infra-Red (IF) light (N. W. Wasike, 2015). The solar radiation from the sun is distorted by atmospheric conditions, and depends on other factors like latitude, topography, climatic conditions, atmospheric contamination, among others. The average radiation reaching the earth surface in a sunny day is

approximately $3.14 \times 10^7 \text{ W/m}^2/\text{day}$. Diffuse solar radiation can be estimated by the empirical formula given by (Fu & Rich, 2000). It is defined as;

$$H_d = H(1.00 - 1.13K_T)$$

where H_d is the mean daily solar diffuse to the earth, H is the total global solar radiation and K_T is the sky clarity index or transmissibility.

The following data related to average received and extractable energy at 90% efficiency of the thermal system in $\text{KWhr/m}^2/\text{day}$ as recorded by (N. W. Wasike, 2015) for the period January – December, 2000 at Dagoreti, Thika and at Jomo Kenyatta International Airport (JKIA) – Nairobi, Kenya recorded at an interval of 10 minutes in a span of 12 hours in a day, from 6.am to 6.pm and tabulated in the Figure below. The solar irradiation data collected indicates an increase in received and extractable energy as the day grows, and drops again in the evening, as plotted in Figure 2 below.

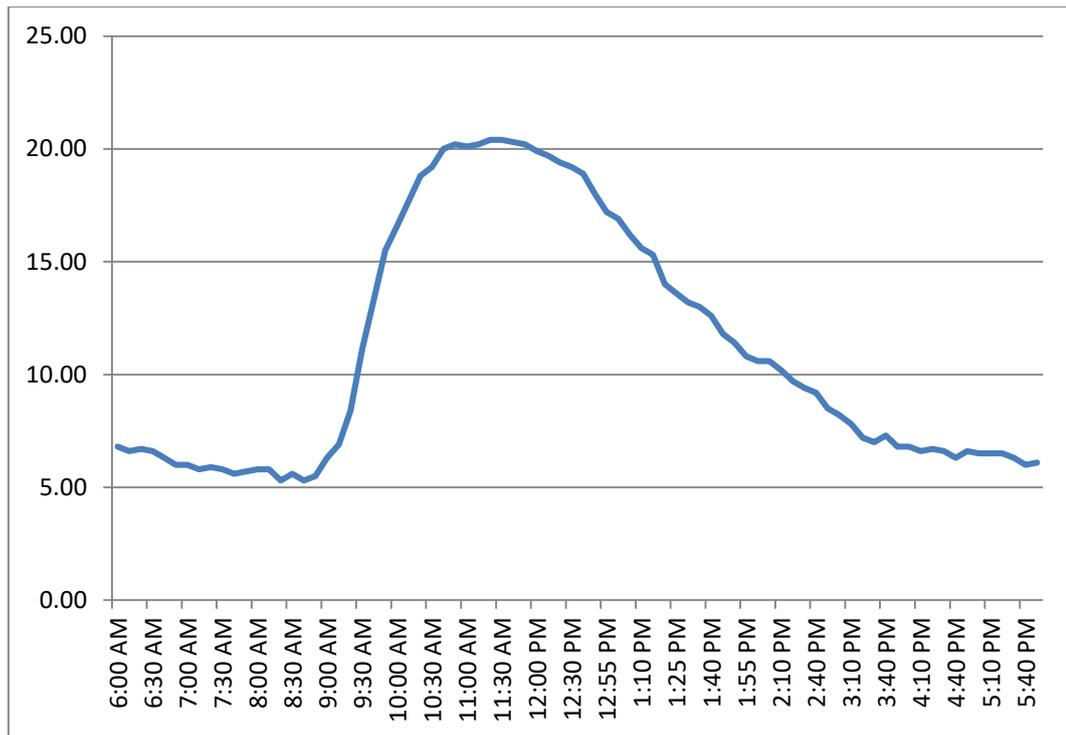


Figure 2 Solar Irradiation Energy received and extractable in a 12 hours sunny day in Kenya (W. Wasike, 2015)

1.2 Statement of the Problem

Environmental conservation and satisfaction of energy demands are contradicting needs, difficult to balance, because of their inverse relationship. Increase in production of energy, for example through the use fuel leads to destruction of the environment. In Kenya for example, wood fuel and other biomass account for 68% of the total energy consumption. The massive deficit in fuel wood supply has led to high rates of deforestation in both exotic and indigenous vegetation resulting to adverse environmental effects. Increased industrialization activities and urbanization has led to increased demand for energy and correspondingly increased environmental degradation. Cities for example, which cover 3% of the earth's surface, consume 75% of the available energy and produce between 60% - 80% of the greenhouse gases (Owiro et al., 2015). Viable response methods of addressing the energy consumption demands are limited by finances and relevant policy framework and mandate. The

plan for green energy and integration into the national power grid has been minimally implemented through projects and policies like zero rated import and production of solar power panels, Ngong plant of 5.1MW wind energy, Turkana solar plant, among others. Despite the costly sacrifice to invest in green sources of power there is still a lot of wastage of existing power, due to leakages, theft, and excessive or insufficient supply not commensurate to desired consumption rates(Y. Atwa et al., 2009). It is for this reason that this research study is proposed to formulate a mathematical model to describe the dynamics of energy supply and demands and determine parameter thresholds for the control and optimization of energy with an attempt to enhance green energy and improve efficient power use. This will provide guidelines and strategies, to control and optimize energy mix to enhance cost effective production of environmentally friendly energy sources, and utilize the available energy without wastage.

1.3 Objectives of the Study

1.3.1 The general Objective of the study

The general objective of this study is to design an energy mix optimization model, which uses neural networks to simulate the use of smart grid in regulating individual's energy supply from demands data, and control the production and distribution of energy, in order to achieve an environmentally friendly energy mix.

1.3.2 Specific Objectives

The specific objectives of this study were:

- i) To formulate an optimization mathematical model for analysing energy demand patterns, distribution cost and production dynamics of energy mix.
- ii) To determine the desired parameter thresholds that guarantees stability and robustness of the optimal energy mix model parameters.

- iii) To build a smart grid mathematical model using neural networks and train so as to control and maintain optimal energy mix.

1.4 Significance of the study

The process of conducting practical experiments in a laboratory and waiting for results to materialize is often costly, time-consuming, and limited to small-scale items that can fit within a laboratory building or controlled environment. A more efficient and faster alternative is to simulate the practical process through a mathematical model using computer programs. In this study, MATLAB–SIMULINK is employed to create accurate and adaptable computer-based models, enabling detailed analysis of stability conditions, persistence, and allowable parameter thresholds essential for the success of the project. By leveraging such simulation technology, a project that might otherwise require years of physical trials can be completed within hours in a virtual environment, significantly reducing cost, time, and risk while ensuring reliable, data-driven outcomes.

1.5 Justification of the study

Following the high demand of energy, the limited production and cost associated with transmission, and distribution, optimization of production of renewable resource and injecting the power to the main grid is necessary but not sufficient. There is need to minimizing energy losses and wastes, at the same time satisfy the consumers. This can only be done by use of smart grid technologies, which provides feedback and makes desired proportional adjustment.

The success of this study, will lead to development of guiding principles for energy production and supply, and provide guiding strategies for the future energy systems. Integration of renewable energy into the smart grid will help improve environmental quality, better management of ecosystem and sustainability. This will be achieved

through formulation of mathematical model using Neural Networks and Smart Grid, which optimizes energy production and supply (Dragana, 1997) and (Torriti, 2012; Warren, 2011).

The results of this study will be beneficial to the society at large. This include the business community and the general public, who will switch to the use of solar powered gadgets, which are environmentally friendly and satisfactory power production, especially in the equatorial region.

CHAPTER TWO

LITERATURE REVIEW

This chapter covers the detailed review of other researchers' findings related to energy and mathematical techniques of modelling energy dynamics. Literature on energy types, production, distribution, demand and supply is discussed. Methodologies of monitoring and analyzing energy including transfer function analysis, neural networks are discussed. The chapter also includes discussion on the mathematical methods and SIMULINK procedures used in Modelling and analysis of models. This chapter is divided into three sections; namely, literature related to renewable solar energy, mathematical Modelling and simulation of energy, smart grid technologies and literature related to the application of neural networks in Modelling smart grid controls.

2.1 Literature related to Solar Energy

Solar energy is a renewable and sustainable source of power that has garnered increasing attention and utilization in recent year. This clean energy source is harnessed from the radiation emitted by the sun and is considered one of the most abundant sources of renewable energy available to us (Hosseini & Wahid, 2020). As the world seeks to reduce dependence on fossil fuels and mitigate the impacts of climate change, solar energy has emerged as a viable and crucial component of the transition to a more sustainable energy future (Bakhsh et al., 2024).

The utilization of solar energy is achieved through the deployment of photovoltaic (PV) panels, which convert sunlight directly into electricity, and solar thermal systems, which capture the sun's heat to produce steam, driving turbines that generate electricity (Maka & Alabid, 2022). The widespread adoption of solar energy has been

facilitated by technological advancements, leading to increased efficiency and reduced costs, making it more accessible to a broader range of consumers and businesses (Saraswat et al., 2024).

One of the key advantages of solar energy is its environmental impact. Unlike traditional fossil fuels, solar energy production does not release harmful greenhouse gases or pollutants into the atmosphere (Foster et al., 2009). This makes it an essential contributor to efforts to combat climate change and improve air quality (Change, 2022). Additionally, the minimal water usage in solar energy production sets it apart from other forms of power generation, addressing concerns related to water scarcity in many regions (Zakariazadeh et al., 2024).

Beyond its environmental benefits, solar energy offers significant economic advantages. The declining costs of solar panels and associated technology have made solar power increasingly competitive with conventional energy sources (Timilsina et al., 2012). As a result, rooftop solar installations, utility-scale solar farms, and community solar projects have proliferated, providing consumers and businesses with opportunities to reduce energy costs and achieve energy independence (Gerlach et al., 2014).

Furthermore, the deployment of solar energy systems stimulates economic growth and job creation. The solar industry supports a wide range of employment opportunities, including manufacturing, installation, maintenance, and research and development. This not only contributes to local and global economies but also drives innovation and technological advancements in the renewable energy sector (Renner et al., 2022).

In addition to its environmental and economic benefits, solar energy plays a crucial role in enhancing energy security. By diversifying the energy mix and reducing reliance on imported fossil fuels, solar power contributes to a more resilient energy infrastructure. This is particularly relevant in regions where energy supply disruptions or geopolitical instabilities pose significant challenges to energy security (Y. M. Atwa et al., 2009). The integration of solar energy into the power grid has also led to advancements in energy storage technologies, enabling surplus solar power generated during peak sunlight hours to be stored for use during periods of low solar availability. This has addressed one of the primary challenges of solar energy – its intermittent nature – and has enhanced its reliability as a consistent power source (Hassan et al., 2023).

As governments and organizations worldwide prioritize sustainable development and the transition to clean energy, policies, incentives, and supportive measures have been implemented to accelerate the uptake of solar energy. This includes feed-in tariffs, tax credits, rebates, and net metering programs designed to facilitate investment in solar installations and make renewable energy more financially appealing to consumers and businesses (Lu et al., 2020).

In conclusion, solar energy stands as a cornerstone of the global shift towards a more sustainable and environmentally conscious energy source (Lu et al., 2020). Its capacity to generate clean, renewable electricity, coupled with its economic advantages and contribution to energy security, positions solar energy as a pivotal component of our energy future (Trieb, 2009). With ongoing technological advancements and supportive policies, the widespread integration of solar energy

holds the promise of a brighter, cleaner, and more sustainable world for generations to come (Razykov et al., 2011).

World energy consumption provided by wind, solar, geothermal and fuel all together is less than 2% (De Castro et al., 2013). The average power that the sun shines on the earth surface is 174,000TW, of which 86,000TW is on the land and 21,840TW on the non-ice land. Of this energy, only less than 10% of this is captured and turned to electricity (De Castro et al., 2013)

Solar irradiation is suns radiant energy incident on surface of unit area. Solar irradiation (energy) is equal to average irradiance (power) multiplied by time Peak sun hours (PSH) is average daily amount of solar energy received on surface. Solar irradiance is the power per unit area, received from the Sun in the form of electromagnetic radiation as reported in the wavelength range of the measuring instrument. Solar irradiance is often integrated over a given time period in order to report the radiant energy emitted into the surrounding environment, during that time period. This integrated solar irradiance is called solar irradiation, solar exposure, solar insolation, or insolation. Irradiance on the Earth's surface additionally depends on the tilt of the measuring surface, the height of the sun above the horizon, and atmospheric conditions (Lee & Levermore, 2020; MUTIE, 2020).

Electrical signals produced by a solar panel per square meter is measured in watts with units of kWh Measurement of solar energy are used when sizing a Photovoltaic systems and heaters. There two methods which symbolize solar radiation of a given place; namely, solar radiance and Insolation. Solar radiance consists of global or direct radiation measurements, while solar insolation is the total amount of solar energy received at a particular location during a specified time period. These two

measurements are affected by the environmental factors like clouds, smoke, obstacles like buildings, trees and other factors,(Schafer et al., 2002; Yeom et al., 2012).

Apart from photovoltaic extraction of solar energy in form of electrical energy, solar panels can also be used to extract thermal energy for the purpose of heating and cooking. Solar thermal collectors are currently on a high demand as a way of supplementing electric heating systems(Xiaowu & Ben, 2005).

2.2 Research Design

Due to the complexity of energy fundamental price drivers, and high frequency of events coupled with market seasonality and significant impact of storage and delivery, energy mix models are formulated to solve problems involving finding an optimal link between the energy users and energy producers, which takes into account the production, transmission and distribution constraints, as well as demand and efficiency constraints(Augustis et al., 2015).

Taking market variables as model parameters, the satisfaction of consumers is achieved if the cost and utility functions are considered. Workable mathematical models have been formulated using differential equations and neural networks (Awodele & Jegedo, 2009). These model equations were then transformed to obtain transfer functions, which can be coupled to build optimization controls (Atwa et al., 2010; Augustis et al., 2015).

A survey on mathematical models applied to energy management using smart grids was studied by(Deng et al., 2015). Most of the methods outlined in this paper are inclined to operations research methodologies, which include game theory, Markov decision processes, dynamic programming, particle swarm optimization, among others.

2.3 Mathematical Modelling and Simulation of Solar Energy

Mathematical modelling of solar energy plays a pivotal role in the design, analysis, and optimization of solar energy systems, contributing to the efficient harnessing of solar power for various applications. This interdisciplinary field encompasses the use of mathematical and computational techniques to simulate, predict, and evaluate the performance of solar energy technologies, taking into account factors such as solar radiation, system components, energy conversion processes, and environmental conditions (Ali, 2011). By employing mathematical models, researchers and engineers can gain valuable insights into the behaviour and characteristics of solar energy systems, leading to the development of more effective and reliable energy solutions (Y. M. Atwa et al., 2009).

One of the fundamental aspects of mathematical modelling in the context of solar energy is the representation of solar radiation and its variability. Solar radiation data, including direct normal irradiance, diffuse horizontal irradiance, and global horizontal irradiance, are essential inputs for solar energy modelling. Mathematical models are used to analyse and predict solar radiation patterns, considering factors such as geographical location, time of day, time of year, atmospheric conditions, and the impact of shading or obstructions (Abad-Alcaraz et al., 2025). These models enable the estimation of solar resource availability and the optimization of solar energy system placement and orientation to maximize energy capture.

In addition, mathematical modelling is instrumental in simulating the behaviour of solar photovoltaic (PV) systems, which convert sunlight into electricity. PV modelling encompasses the characterization of PV module performance, the interaction of PV arrays with inverters and the electrical grid, and the impact of factors such as

temperature, soiling, and degradation over time. Mathematical models allow for the prediction of PV system output under different operating conditions, aiding in the design, sizing, and performance assessment of solar PV installations. Furthermore, these models support the development of advanced control strategies for maximizing energy yield and enhancing system reliability (Coakley et al., 2014).

Another area of focus within mathematical modelling of solar energy lies in solar thermal systems, which utilize the sun's heat to generate electricity or provide thermal energy for heating and cooling applications. Mathematical models are employed to simulate the thermal behaviour of solar collectors, heat transfer fluids, energy storage systems, and power conversion processes. These models enable engineers to analyse system performance, evaluate the impact of design parameters, and optimize the operation of solar thermal installations for enhanced efficiency and cost-effectiveness (Soni et al., 2021).

Moreover, mathematical modelling plays a critical role in the integration of solar energy systems with energy storage technologies, enabling the prediction of energy storage requirements, the optimization of charging and discharging strategies, and the assessment of overall system performance. This is particularly pertinent in addressing the intermittent nature of solar energy and enhancing the reliability and dispatchability of solar power (Bamisile et al., 2024).

Furthermore, mathematical modelling supports the techno-economic analysis of solar energy projects, aiding in the assessment of investment returns, levelized cost of electricity (LCOE), and overall project feasibility. By incorporating parameters such as capital costs, operational and maintenance expenses, energy yield projections, and

financial incentives, these models provide valuable decision-support tools for project developers, investors, and policymakers (Yang et al., 2009).

Notably, advancements in computational techniques, data analytics, and machine learning have expanded the capabilities of mathematical modelling in the realm of solar energy. The integration of big data, real-time monitoring, and predictive analytics contributes to the development of more accurate and adaptive solar energy models, enabling real-time performance optimization, fault detection, and predictive maintenance for solar energy systems (Mohammad & Mahjabeen, 2023; Wahid & Hermawan, 2020).

In conclusion, the mathematical modelling of solar energy represents a dynamic and evolving field that underpins the advancement of solar energy technologies and applications. Through the use of mathematical models, researchers, engineers, and stakeholders can gain a deeper understanding of solar energy dynamics, optimize system performance, and drive innovation in the pursuit of a more sustainable and renewable energy future. As the demand for clean energy solutions continues to grow, mathematical modelling will continue to play a vital role in shaping the development and deployment of solar energy systems across diverse sectors and applications (Wahid & Hermawan, 2020).

A mathematical model is a representation of a real situation using mathematical equations. These are either algebraic, differential, or trigonometric equations, or a combination of all. A physical phenomenon is transformed with stated assumptions and conditions into a mathematical system of equations, then solved, and the results interpreted back to the physical situations. Various mathematical models have been used to find a balance of energy mix so as to minimize costs and improve customer

satisfaction. Among them is the use of multi-objective optimization algorithms to simulate energy supply and demand. Other methods of optimization were used to analyse consumption of energy and production of carbon (Mickens, 2022).

The work done by Wahid and Hermawan (2020) aimed at determining electricity demand projections and fulfilment of the obtained needs using renewable energy. It was found that the optimum energy mix had the proportion of renewable energy at 52%. The mathematical model used was of the type of a regression model, with data obtained from the government. In an attempt to find an optimal solution to the energy mix, it is always faced with multiple options, constraints, and objectives, which leads to a multi-objective problem. The most common and latest technology form of analysing such complex systems is by use of a smart grid and neural networks. Here, the smart grid is not viewed as a global device for controlling and balancing optimal energy mix, but seen as a mathematical technique used to simulate and determine the optimal energy mix (Wahid & Hermawan, 2020).

2.4 Smart Grid Technologies

Smart grid technology and energy optimization have emerged as key components in the modernization of energy infrastructure and the pursuit of more efficient, sustainable, and reliable power systems. The smart grid integrates advanced digital communication and control technologies into traditional electricity grids, enabling real-time monitoring, automation, and optimization of energy generation, distribution, and consumption. One of the primary objectives of smart grid technology is to enhance energy optimization by dynamically managing the supply and demand of electricity. Through the deployment of smart meters, sensors, and control systems, the smart grid enables utilities to gather real-time data on energy usage patterns, grid

performance, and environmental conditions. This wealth of information empowers utilities and energy operators to optimize energy distribution, reduce transmission losses, and implement demand response strategies to balance supply and demand more effectively (Khalid, 2024). Energy optimization within the smart grid also encompasses the integration of renewable energy resources and energy storage systems. By leveraging predictive analytics, forecasting algorithms, and advanced energy management tools, the smart grid facilitates the seamless integration of solar, wind, and other renewable energy sources into the grid. Furthermore, energy optimization algorithms enable the efficient utilization of energy storage systems, such as batteries and grid-scale storage, to store excess renewable energy and discharge it during peak demand periods, thus enhancing grid stability and resilience (Pramila et al., 2024). Moreover, the smart grid facilitates the implementation of advanced grid management techniques, such as voltage and frequency regulation, load balancing, and predictive maintenance. By leveraging real-time data analytics and predictive Modelling, energy operators can optimize grid operations, reduce system inefficiencies, and proactively address potential issues, leading to improved reliability and reduced energy waste. In essence, the convergence of smart grid technology and energy optimization represents a paradigm shift in the way electricity grids are operated and managed. By harnessing digital intelligence, automation, and data-driven decision-making, the smart grid enables more efficient energy use, reduces environmental impact, and lays the groundwork for a more resilient and adaptive energy infrastructure capable of meeting the evolving needs of a modern, dynamic energy landscape. Smart grid is a technology that has let us dreamt about solution of maximum power problems including generation to distribution ends. In this time of technological advancement where all the governments are facing challenges of

meeting the power demand of the consumers, smart grid can bring harmony in demand supply ratio (Paul et al., 2014).

Smart Grid is an electricity grid network enabling a two-way flow of electricity and data with digital communications technology enabling to detect, react and pro-act to changes in usage and multiple issues. Traditionally Electrical grid has been used with its challenges ranging from failure to detect the electrical energy required by every consumer. This failure has caused a lot of energy wastage and the cost of production of electrical energy is high (Momoh, 2012).

Smart Grid Technology, being an advanced computational intelligence analytic tool, needed to solve optimization problem of bulk energy supply and demand needs, taking into account various constraints and objectives(Wang et al., 2009). Smart grid therefore, is the best solution in addressing the energy flow from the production to the consumer. The energy flowing to the consumer is per consumer ratings/needs. The components of smart grid technologies include; Smart appliances, Smart substations, Smart meters and integrated communications. Here, smart is used to refer to cutting edge programmable intelligent technologies which can automatically sense, regulate control and communicate reports to respective recipients.

The traditional grid is well known to consist of power generation, power transmission, power distribution and power consumption centres. In the modern smart grid, additional elements are linked to the traditional grid for efficiency, and monitoring. The following is a conceptual framework of the smart grid technologies.

From Figure 3 below, it is clear that apart from the traditional grid, we have additional three domains, namely; the market domain, the service provider domain and the operations domain. These three domains are what makes a shift from the traditional

grid to the modern smart grid. These three domains are all interconnected and works in synchrony with all the other sectors of the traditional grid. Mathematical model of such grid will involve representation of each domain and their inter connectedness with other domains.

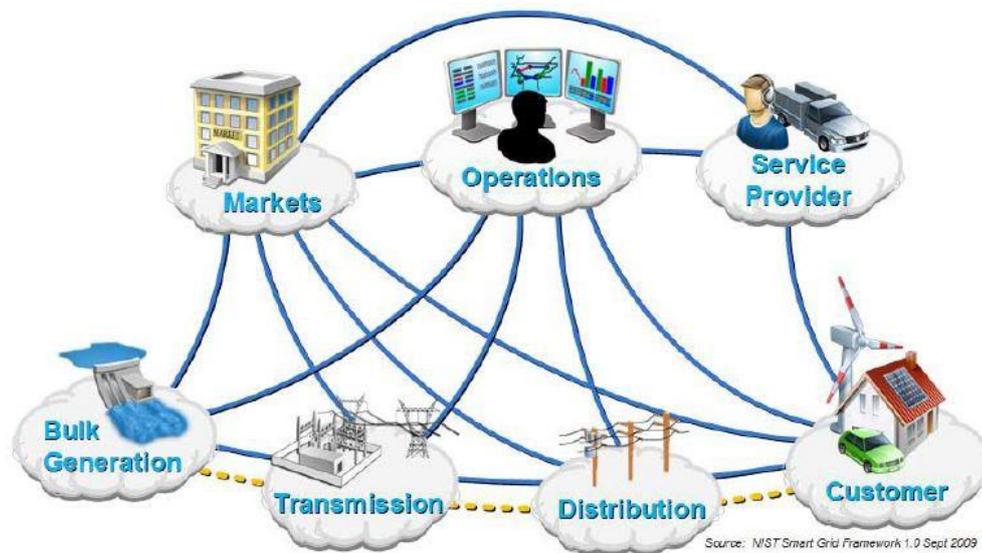


Figure 3: Conceptual framework of a modern Smart Grid technology. Source: (Koepke, 2011)

The mathematical construction of automated smart grid systems intended for the control of electric power systems requires high-speed mathematical tools. The proposed Algorithms for the construction of automated systems to control active components of the electrical network is the Neural Networks. This will allow the injection of artificial intelligence in the system so as to automatically determine and control the energy production and demands. The method applied is a universal approach to the mathematical Modelling of nonlinear dynamic systems which allow an active experiment using test sets of input signals (Suslov et al., 2015). This makes it possible for the input and output characteristics of the object to obtain an adequate and fast mathematical description. The results enable us to assess the applicability of

this mathematical apparatus to the control of active components of electric power system in which power quality parameters are taken into account as an objective vector.

2.5 Neural Networks and Optimization

Neural networks, a fundamental concept in the field of artificial intelligence and machine learning, are structured to mimic the workings of the human brain and process complex patterns and data in a manner akin to human cognition. Comprising interconnected nodes or "neurons," these networks are designed to recognize underlying relationships and patterns within data, making them adept at diverse tasks such as pattern recognition, classification, regression, and decision-making. The basic building block of a neural network is the artificial neuron, also known as a node or perceptron. These neurons receive input data, perform computations using weighted connections, and produce an output signal. Through the collective behaviour of interconnected neurons arranged in layers, neural networks can effectively process and learn from data. Neural networks typically consist of multiple layers, including an input layer to receive data, one or more hidden layers to process information, and an output layer to produce the network's prediction or decision. The strength of neural networks lies in their ability to learn and adapt from data, a process referred to as training. During training, the network adjusts the weights of its connections in response to input data and expected output, optimizing its ability to make accurate predictions or classifications. One of the most prevalent types of neural networks is the feedforward neural network, wherein data moves in a unidirectional flow from the input layer to the output layer through the hidden layers. These networks are commonly used for tasks such as image recognition, speech recognition, and natural language processing. Another significant class of neural networks is recurrent neural

networks (RNNs), which are designed to process sequences of data and have feedback loops that allow them to retain information from previous inputs. RNNs are well suited for applications involving time-series data, language modelling, and speech recognition, due to their ability to capture temporal dependencies and contextual information. Furthermore, convolutional neural networks (CNNs) are specialized for tasks involving grid-like data, such as images. By leveraging convolutional layers to extract features and hierarchical representations, CNNs excel in image recognition, object detection, and image segmentation tasks, making them pivotal in computer vision applications. In recent years, advancements in neural network architectures, training algorithms, and parallel computing capabilities have led to the development of deep neural networks, characterized by their numerous hidden layers. This has significantly enhanced the capacity of neural networks to learn complex representations and abstract features from large-scale datasets, ushering in breakthroughs in fields such as natural language processing, medical diagnosis, autonomous vehicles, and more. In conclusion, neural networks represent a powerful and versatile framework for machine learning, driving innovation and transformation across a wide array of industries and applications. By leveraging their capacity to learn from data, recognize patterns, and make informed decisions, neural networks continue to push the boundaries of what is possible in artificial intelligence and contribute to advancements that are reshaping our interactions with technology and the world around us.

In this section, the neural network discussed refers to artificial neural networks and not the human body neural networks. ANN for (Artificial Neural Networks) denotes this. This is discussed in terms of its application in energy control systems. Artificial neural networks are Computer networks or programming structures made in likeness

of the brain neural architecture. They typically consist of many hundreds of simple decision-making units, which are connected, in a circuit into a complex network. Each unit of ANN represents a real neuron which is responsible of making a decision or classification between two options; A or B, Yes or No, Right or Wrong, Up or Down, left or right, true or false among others. Two signals are input, and basing on the bias, the output will either be any one of the two options. This is in form of a simplified neuron, which fires if it receives depending on the nature of the input stimuli. A significantly strong input signal from the other nodes to which it is connected, will yield an equivalent output signal to the next connected node.

An Artificial Neural Network is an information-processing paradigm that is motivated by biological nervous systems, which is responsible in processing information and giving back the desired response to the input stimulus. The key element of this methodology is the unique structure of the input stimuli processing system. A neural is composed of a large number of complex but definite interconnected neurons working in unison to give solution to specific problems. Artificial Neural Networks, like people, learn and improve the response the next time the same stimuli is presented. This learning is done by comparing the input, output and the expected output, so that the error is evaluated and minimized in the subsequent response. The neural network is structured to perform nonlinear Bayesian classification, and many other mathematical problems.

The fundamental building block for neural networks is the single-input neuron, such as this example in figure 4 below. There are three distinct functional operations that take place in this example neuron. First, the scalar input p is multiplied by the scalar weight w to form the product wp , again a scalar. Second, the weighted input wp is

added to the scalar bias b to form the net input n . In this case, you can view the bias as shifting the function $f(\cdot)$ to the left by an amount b . The bias is much like a weight, except that it has a constant contribution to the function to obtain $f(wp + b)$. Finally, the net input is passed through the transfer function $f(\cdot)$, which produces the scalar output $a(\cdot)$. The names given to these three processes are: the weight function, the net input function and the transfer function (Marvin, 2016).

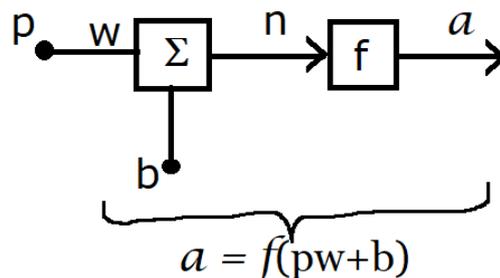


Figure 4: A Perceptron, as a unit of a Neural Network. Source: (Marvin, 2016)

A neuron could be a single layer, or multiple layer with a single input or a vector of inputs. It can be instantaneous or with delay. Some of the transfer functions include linear, threshold function or a sigmoid function defined as;

$$f(x) = \frac{1}{1 + e^{-mx}}$$

Where m is the slope parameter, and x is the variable representing the input.

Once the neural network architecture has been built, a set of learning algorithm is passed, through a process called training, so as to enable the network respond to inputs as desired. The training process is meant to improve the response rate in future occurrence of the same input, just the way the human brain responds to various stimulus, after learning (Jones, 2014).

Using a multi-layered neural network, training is defined as a process of selecting appropriate values of parameters (weights and biases) which best approximates a given function. Using personal energy consumption data, which is assumed to have a certain consistent pattern, neural network is trained to produce the same pattern, and therefore control the flow of energy to such an individual automatically. This is an application of neural network in control systems(Hagan et al., 2002).

2.6 Optimization of Energy Mix and Smart Grid

The integration of smart grid technologies within the energy mix has brought about transformative changes in the optimization and management of energy resources, contributing to enhanced efficiency, reliability, and sustainability. Smart grids encompass an array of advanced digital communication, automation, and control systems that enable real-time monitoring, analysis, and optimization of energy generation, distribution, and consumption. One of the key contributions of smart grid technology to the energy mix is its role in facilitating the seamless integration of renewable energy sources, such as solar and wind power. The variability and intermittency of renewable energy poses challenges to grid stability and energy balancing. Smart grid technologies address these challenges by leveraging advanced forecasting, predictive analytics, and real-time monitoring to anticipate fluctuations in renewable energy generation and proactively adjust energy distribution and grid operations. This facilitates the effective deployment of renewable energy resources within the energy mix while maintaining grid stability and reliability. Furthermore, smart grid technologies enable the dynamic management of energy demand and supply through demand response programs, real-time monitoring of energy usage, and grid balancing mechanisms. By integrating smart meters, sensors, and advanced metering infrastructure, utilities can gather real-time data on energy consumption

patterns and grid conditions, allowing for the optimization of energy distribution, load balancing, and voltage control. Additionally, demand response initiatives empower consumers to adjust their energy usage in response to pricing signals or grid conditions, contributing to more efficient utilization of energy resources within the overall energy mix. Energy optimization within the smart grid also extends to the integration of energy storage technologies, enabling the efficient capture and utilization of surplus energy and the provision of energy during peak demand periods. By incorporating grid-scale battery storage, pumped hydroelectric storage, and other energy storage solutions, smart grids can store excess energy from renewable sources, alleviate grid congestion, and provide essential grid services, such as frequency regulation and voltage support. This flexibility in energy storage and deployment optimizes the utilization of renewable energy resources and enhances grid resilience. Moreover, the deployment of smart grid technologies supports the implementation of advanced energy management and efficiency initiatives, including predictive maintenance, asset management, and grid modernization. By leveraging data-driven insights, predictive analytics, and automation, energy operators can optimize the performance of grid infrastructure, minimize energy losses, and reduce operational costs. This contributes to the overall optimization of the energy mix by maximizing the efficiency and reliability of energy generation, transmission, and distribution systems. In conclusion, the incorporation of smart grid technologies in the energy mix represents a pivotal advancement in the modernization and optimization of energy systems. By enabling the seamless integration of renewable energy sources, dynamic energy management, and enhanced grid resilience, smart grids play a central role in optimizing the utilization of diverse energy resources and driving the transition to a more sustainable, flexible, and efficient energy mix. As smart grid technologies

continue to evolve, their impact on energy optimization and the dynamic management of energy resources will be instrumental in shaping a more resilient and sustainable energy.

From the literature reviewed, it is clear that energy demands in the world are far from being satisfied. The continued use of fuel, and nuclear energy increases the production of carbon, which contributes to the global warming. The alternative green energy can be obtained by harnessing the wind and solar power, which can be supplied to the main power grid and its consumption controlled so that the environmental conservation and cost-effective technologies are used, and at the same time wastage of energy is avoided. It is for this reason that this research intends to fill the gap of optimized energy mix and exploitation of renewable energy through formulation of mathematical model of a smart grid using neural networks, and running simulations to show the effect of using smart grid in the optimal control and management of energy.

2.6.1 Optimization of energy

Mathematical optimization methodologies encompass a diverse set of techniques and algorithms designed to identify the best solution from a range of feasible options, subject to specific constraints and objectives. These methodologies are utilized across various fields, including operations research, engineering, economics, finance, and machine learning, to address complex decision-making problems and enhance efficiency and performance. One prevalent optimization methodology is linear programming, which is employed to address problems characterized by linear relationships and constraints. Linear programming aims to optimize a linear objective function subject to a set of linear equality and inequality constraints. By utilizing techniques such as the simplex method and interior-point methods, linear

programming facilitates the allocation of resources, production planning, supply chain optimization, and other decision-making processes in diverse applications. Another prominent optimization technique is nonlinear programming, which addresses problems with nonlinear objective functions and constraints. Nonlinear programming involves the use of optimization algorithms such as gradient-based methods, genetic algorithms, and simulated annealing to find optimal solutions in cases where relationships between variables are nonlinear. Nonlinear programming is instrumental in optimizing complex systems, including engineering designs, financial Modelling, and parameter estimation in machine learning. Integer programming and mixed-integer programming methodologies are employed to tackle problems where decision variables are required to take integer values, adding an additional layer of complexity compared to linear programming. These methodologies are crucial in addressing optimization challenges related to discrete decision-making, such as resource allocation, scheduling, and combinatorial optimization problems in logistics, transportation, and telecommunications. Furthermore, optimization methodologies extend to stochastic programming, which accounts for uncertainty and risk in decision-making processes. Stochastic programming models incorporate probabilistic distributions and scenarios to address optimization problems under uncertainty, making them essential in risk management, portfolio optimization, and decision-making in the presence of random variables and fluctuations. Additionally, metaheuristic optimization techniques, including genetic algorithms, particle swarm optimization, and simulated annealing, offer efficient approaches to solving complex optimization problems by leveraging population-based search strategies and iterative improvement mechanisms. These methodologies are particularly effective in addressing combinatorial optimization, global optimization, and multi-objective

optimization challenges in diverse domains. Moreover, convex optimization methodologies are pivotal in addressing optimization problems characterized by convex objective functions and constraints. Convex optimization techniques, such as interior-point methods and gradient descent algorithms, enable efficient and scalable solutions for a wide range of applications, including machine learning, signal processing, and control systems. In conclusion, mathematical optimization methodologies encompass a rich array of techniques and algorithms that play a critical role in addressing complex decision-making problems across diverse domains. By leveraging linear programming, nonlinear programming, integer programming, stochastic programming, metaheuristic optimization, and convex optimization methodologies, researchers, engineers, and decision-makers can derive optimal solutions, enhance efficiency, and make informed decisions that drive progress and innovation in a wide range of industries and applications.

In this research study, optimization is done using Pontryagin's maximum principle, which is the most convenient method in solving the optimal control problem involving the decision variables $x(t)$ and the objective variable $u(t)$. This involves the process of determining control and state trajectories for a dynamic system over a period of time in order to minimize a performance index.

Consider a system of dynamical systems which evolve according to the state equation

$$x'(t) = f(t, x(t), u(t))$$

Where $u = (u_1, u_2, u_3, \dots, u_m)$ and $x = (x_1, x_2, x_3, \dots, x_n)$ represents the control variables exercised on the system, and the state variables respectively. Using the

restrictions in section 3.3 above, the objective cost function measuring how good a given control $u(t)$ is, is of the form

$$J(x, u) = \int_{t_0}^T f(t, x(t), u(t)) dt$$

The main objective of this Optimal Control problem is finding the piecewise continuous control $u(t)$ and the associated state variable $x(t)$ to maximize the given objective functional $J(x, u)$.

The desired Optimal Control problem is in the form $\max_u J(x, u)$

Such that the following constraints hold

$$x'(t) = g(t, x(t), u(t))$$

$$x(t_0) = x_0, (x(T) = x_T)$$

where the final condition on x is written in brackets to mean that sometimes it is omitted when the final value is not restricted. Considering a weight parameter b to show different efforts on the application of a control variable, we represent the optimal control problem in the form

$$J(u) = \int_0^T x(t) dt + b \int_0^T u^2(t) dt$$

where b is the weight parameter describing the importance of implementing the control variable $u(t)$.

In this study, once the model differential equations are formulated, the system will be representing the objective function and the associated constraints, are then solved using Pontryagin's algorithm for efficient solution of differential equations

constrained optimization, identification and control problems sensitivity analysis of the optimal solar heater variables by considering an objective form which describes the distribution of irradiation intensity and output temperature. In order to solve this optimal control problem, the adjoint functional H here referred to as Hamiltonian is defined as;

$$H(t, x(t), u(t), \lambda(t)) = f(t, x(t), u(t)) + \lambda(t)g(t, x(t), u(t))$$

with $\lambda(t)$ as the multiplier of the state variable.

The Hamiltonian function then satisfies the equations

$$\frac{\partial H}{\partial u} = 0; \quad \lambda'(t) = -\frac{\partial H}{\partial x}$$

whose integral gives the desired solution $H(t, x, u)$ and the multiplier $\lambda(t)$.

2.6.2 Smart Grid Technologies

Smart grid technologies encompass a range of advanced digital and automated solutions designed to modernize and optimize electricity grids. These technologies include smart meters, sensors, and automation systems that enable real-time monitoring, control, and communication within the grid infrastructure. Additionally, smart grid technologies incorporate advanced analytics, grid management software, and integration of renewable energy sources and energy storage systems to enhance grid reliability, efficiency, and resilience. By leveraging these technologies, smart grids facilitate dynamic energy management, demand response, and seamless integration of distributed energy resources, contributing to a more flexible, sustainable, and reliable energy infrastructure.

The grid is the network of cables together with the attached gadgets that forms a transmission vector of energy from the source (production point) to subscribers (users) spread all over the country. A good grid should be robust, reliable and cost effective. The performance of the grid is affected by resistance of conductors, which causes heating and sometimes cables snap. The use of less resistant conductors come with other factors like weight, cost among others and thus the most economical way of reducing costs associated with the grid is through stepping up voltage, because high-voltage lines allow significant push of power to a good distance without losses, where the voltage will then be stepped down to desired levels.

The electrical grid is divided into three major areas, namely; production, distribution and transmission; and consumption. These components are linked and must work together. The last component describes the end user which could be institutions, households, government agencies, transport sector, industries, business enterprises among others.

The concept of smart grid includes the three major components of the grid and an additional component which can intelligently integrate the activities of all users connected and improve efficiency, robustness and secured economically sustainable energy. In order to achieve this fourth component, a set of controls and feedback network is required together with the use of smart meters, smart switches, smart sockets, and smart electrical appliances at customer service point. All the four components make up a smart grid(Melhem, 2018).

CHAPTER THREE

METHODOLOGY AND RESEARCH DESIGN

In this chapter, an overview and choice of mathematical methodology and design is provided and justified. The chapter is subdivided into three sections, namely; research design, neural networks formulation and model analysis methodologies.

3.1 Research Design

In this research thesis, a mathematical model is formulated, to account for the dynamics of energy production and use in the national electricity grid. This involves the production, transmission and consumption. A smart grid is investigated, where a feedback mechanism is fitted using smart gadgets; switches, sockets, meters etc., which have the capacity of collecting usage data and transmitting back to the source for the purpose of adjustment of supply to meet the needs. It is also fitted in the smart grid, a device that can allow backward flow of energy, for the case of a consumer who is domestically producing excess power, and therefore selling back to the main grid. Due to the complex feedback and response dynamics of the problem, neural network model is used in the study to analyse the model.

3.2 Mathematical Model Formulation

Mathematical modelling of an energy mix involves developing quantitative representations of the various energy sources and technologies that contribute to the overall energy supply. This modelling aims to analyse and optimize the combination of energy sources, such as fossil fuels, renewable energy, nuclear power, and other forms of energy, to meet demand while considering factors such as cost, environmental impact, and reliability. The process of mathematical modelling typically includes formulating equations and algorithms that describe the behaviour of

individual energy sources and their integration into the broader energy system. This may involve considering factors such as energy generation capacity, energy storage, transmission and distribution losses, and energy conversion efficiency. The models may also account for factors such as variable renewable energy output, energy demand patterns, and potential constraints on the energy system. By using mathematical models, energy planners and policymakers can evaluate the optimal combination of energy sources to meet demand while minimizing costs and environmental impacts. These models can also assess the impact of different policy interventions, technological advancements, and changes in energy consumption patterns on the overall energy mix. Overall, mathematical modelling of the energy mix provides valuable insights for long-term energy planning, infrastructure investment, and policy development to ensure a sustainable, reliable, and affordable energy supply.

The mathematical model under study is formulated by representing the physical dynamics of power supply in mathematical equations. The model domain involves the dynamics related to power supply, that is, the transmission of power by the government to the consumers. This power is hereby in this study recognized to be generated from three sources; namely, government generated electric power (from hydro, geothermal, wind and photovoltaic sources), imported power (from neighbouring countries) and domestic (private) production of power.

Once electric power is produced, the voltage is connected through cables with step up and step-down transformers, from the source point to the terminal point of consumption, as illustrated in Figure 2. The task at hand is to represent the changes in electric energy as it flows from the source to the consumption point. This

mathematical equation formulation is done by first describing the model domain, variables, parameters, assumptions and representation of the model on a compartmental flow chart.

3.2.1 Model Domain and Assumptions

In this research, a mathematical model is formulated to represent the consumption of electric energy in a home, by a learning institution and by small and medium scale commercial enterprises (SME). It is here assumed that all the homes are homogeneous, and use the same amount of power. A typical home is here assumed to be consuming electric power by use of electrical appliances for cooling, heating, lighting, entertainment and other power draining gadgets. An assumption is also made that every home is generating its own power using either photovoltaic cells or a wind cruiser. Once electric power is produced domestically, the generated power will be channelled first as a priority, and the government power supplied is utilized only if the domestic power is insufficient. This means that domestic electric power generated is only in excess if it is greater than the amount consumed. Any channelling back of domestic power to the main grid happens only after satisfying the domestic needs. In this study, it is also assumed that power substations have multidirectional transmission of power to any direction as long as the power is flowing from high to low concentration.

3.2.2 Model Flow chart

Modelling using compartmental techniques involves the classification of the energy distribution into unique categories with similar characteristics, so that a definite number of compartments is created, and linked using the flow rates from one

compartment to the next. The flow rates from one compartment to the next is represented mathematically by the change described below.

Consider a substance flowing from left to right through a cube as illustrated below. From the law of conservation of mass, flow out equals to flow in plus internal creation minus internal decayed or consumed inside the cube, and this is depicted in the figure below.

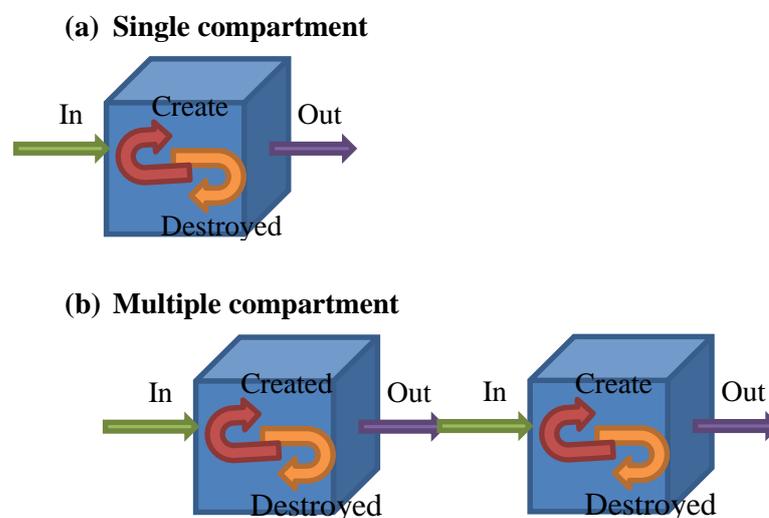


Figure 5: Compartmental Model flow chart. Source: Author

In the first scenario (a), there is one compartment, and the changes of matter inside the compartment is described mathematically as;

$$\frac{da}{dt} = in1 + created1 - destroyed1 - out1$$

If the output in compartment (a) then another set of equation is created to form a system as;

$$\frac{da}{dt} = in1 + created1 - destroyed1 - out1$$

$$\frac{db}{dt} = in2 + created2 - destroyed2 - out2$$

Using this idea, compartmental Modelling is used to formulate differential equations representing production, distribution, and consumption of energy.

In this study, transfer of electric energy forms the production point to consumption point is described in compartmental Modelling as described below. The distribution is classified into two compartments, namely; production point and consumption point. Other peripherals or sub-compartments are created basing on differentiated levels of consumption; namely home, institution, and small business enterprises. Additional feedback component is also added to make it a smart grid.

The transmission of electric energy from the point of generation to the consumer and the communication back is depicted by the following compartmental flow chart. Three categories of consumers are placed in the grid, namely homes, institutions and SME's according to the amount of supply they need.

In the flow chart, the complete lines denote the flow of energy, with green denoting small-scale energy flow from domestic generators to their respective usage points. The black arrows show the flow of energy from or to the national grid. The blue lines denote the communication paths, to and from homes and power sources through smart grid technologies. The green rectangles represent domestic small-scale production points, while the grey rectangles represent the consumption points and the national grid source point.

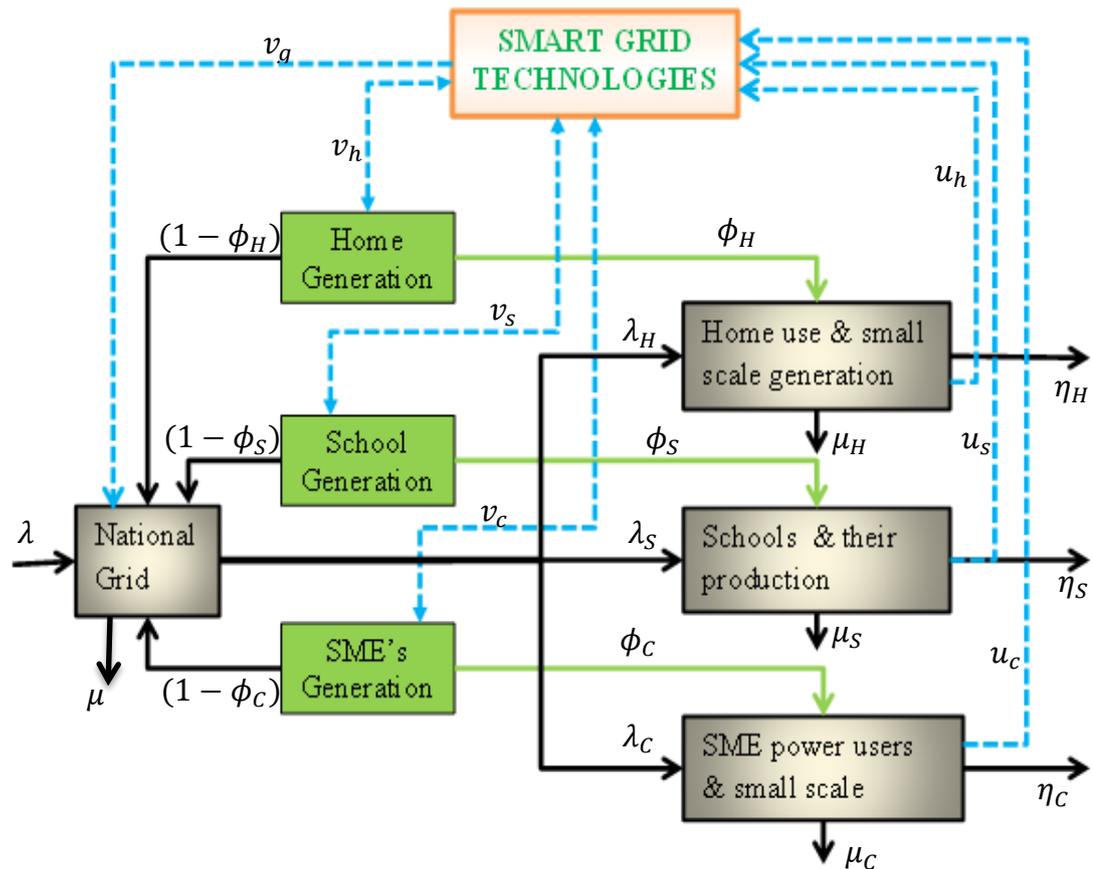


Figure 6. Flow chart showing the dynamics of electric energy and feedback communication through smart technologies. Source: Author

3.2.3 Definition of Model variables and Parameters

In order to formulate the energy, mix mathematical model equations, the description of variables and parameters is hereby presented. Four compartments of energy source and consumption points are defined as follows: The national grid representing the main source of electricity is represented by the variable $G(t)$, with the amount of electric energy received from generation points and import denoted by the parameter λ . The second set of consumption points compartments are the home denoted by $H(t)$, the learning institutions or schools denoted by $S(t)$ and the small and medium enterprises (SME) denoted by $C(t)$.

The parameters denoting the flow rate of energy from the generation source to other points are presented in the table below.

Table 1: Parameters denoting the flow rate of energy from the generation source to other points

	Parameter	Definition on the usage
1	λ	Total power received from import and that generated by the government
2	λ_i	Power transmitted to user $i = H, S, C$ from the national grid
3	ϕ_i	Amount of energy generated domestically by the i^{th} user, $i = H, S, C$
4	$(1 - \phi_i)$	The complementary energy supplied by the i^{th} domestic producer to the national grid. This is in excess of the respective consumption needs
5	μ	The proportion of leakage of power lost from the national grid, due to environmental and technical factors
6	μ_i	The proportion of lost energy at i^{th} user point due to leakage or no load, with, $i = H, S, C$.
7	η_i	The proportion of energy consumed by the i^{th} user, $i = H, S, C$
8	u_i	Data on power demands communication signal from the i^{th} , $i = h, s, c$ power user to the SMART control.
9	v_i	Control signal from the SMART control to the i^{th} , $i = h, s, c$ power source on the corresponding supply needs of individual users

3.2.4 Mathematical Model Equations

From the flow chart and definition of variables and parameters in the previous sections, the concept of an ordinary derivative to physically represent the rate of change is hereby used. From compartmental Modelling principles of conservation of energy, the following differential equations are formulated.

$$\frac{dG}{dt} = \lambda + \sum_i (1 - \phi_i)(H + S + C) - \sum_i \lambda_i G - \mu G \quad (3.1a)$$

$$\frac{dH}{dt} = \lambda_H G + \phi_H \lambda_H G - \eta_H H - \mu_H H \quad (3.1b)$$

$$\frac{dS}{dt} = \lambda_S G + \phi_S \lambda_S G - \eta_S S - \mu_S S \quad (3.1c)$$

$$\frac{dC}{dt} = \lambda_C G + \phi_C \lambda_C G - \eta_C C - \mu_C C \quad (3.1d)$$

where $\sum_i(1 - \phi_i)(H + S + C)$ represents the sum $(1 - \phi_H)H + (1 - \phi_S)S + (1 - \phi_C)C$ and $\sum_i \lambda_i G = \lambda_H G + \lambda_S G + \lambda_C G$. The first equation (3.1a) describes the dynamics of electric power in and out of the national grid. Here, λ denotes the total power input from generating points, which could be imported or locally produced from hydro, geothermal, wind, solar or other sources. Additional proportion of energy $\sum_i(1 - \phi_i)(H + S + C)$ is injected into the national grid from domestic private producers. The total energy is then distributed to various users as described in equation (3.1b-d). Natural leakage from the national grid is accounted for by the parameter μ . Equations (3.1b-d) describes the consumption, generation and wastage of the same energy by the respective consumers.

3.3 Stability Analysis of the Model

Mathematical model stability analysis involves the assessment of the stability properties of mathematical models representing dynamic systems. This analysis is crucial in various fields, including engineering, physics, and biology, where understanding the behaviour of systems over time is essential for predicting their response to different inputs and disturbances. Stability analysis often involves examining the equilibrium points, eigenvalues, and stability criteria of a system to determine its long-term behaviour and response to perturbations. By evaluating the stability of mathematical models, researchers and practitioners can gain insights into

the behaviour and performance of systems, enabling them to make informed decisions and design effective control strategies.

Apart from the model existence and well boundedness, it is necessary to study its' stability, with respect to perturbation. The model stability depends on the nature of the spectral values around the fixed point. Stability is determined by the eigenvalues of community matrix M obtained by evaluating the Jacobian of the system, evaluated at the fixed point.

$$M = \left. \frac{\partial G(X,t)}{\partial X} \right|_{X_i} \quad i = 0, e$$

Let the spectrum of the matrix M be defined as

$$|M| = \lambda_j, j = 1, 2, 3, \dots, n$$

Then the system in question is classified as stable, unstable or can't be determined at the equilibrium point as follows;

$$\left. \begin{array}{l} \text{Unstable} \\ \text{Stable} \\ \text{Undetermined} \end{array} \right\} \text{ if for all } i, \quad \begin{cases} \lambda_i > 0 \\ \lambda_i < 0 \\ \lambda_i = 0 \end{cases}$$

If the nature of eigenvalues are zero, the stability of the system can be determined using Lyapunov type numbers (eigenvalue functionals), or using the Poincare map (describing the proximity of periodic solutions to each other after one period), if the solution is periodic.

Stability of the system is an important concept to consider, because disturbances, which mathematically are referred to as perturbation in inevitable in life, and many dynamical systems are prone to perturbation, making stability analysis necessary. A

set of differential equations is stable if the eigenvalues of the associated linearization matrix has negative eigenvalues. In stability analysis, two fixed points are considered; that is, self-generation/independence equilibrium, and the dependence (on government connectivity) equilibrium.

When evaluating information matrix, it's crucial to consider the stability of the resulting system locally, or around the fixed points. Without loss of generality, stability around a local fixed point gives information on global stability of the system outside the fixed point. Thus, the approximating of the system and assessing the stability about a fixed point is essential for understanding its global dynamic behaviour. This is facilitated by the evaluation of stability at a local point, usually a fixed point of the linearized system.

Stability matrix of the system (3.1 a-d) is obtained from the Jacobian matrix defined as;

$$M = \frac{\partial(f_G, f_H, f_S, f_C)}{\partial(G, H, S, C)}$$

This can be evaluated directly to obtain the same coefficient matrix, because the system is linear. The coefficient matrix is given by;

$$M = \begin{pmatrix} \frac{\partial f_G}{\partial G} & \frac{\partial f_G}{\partial H} & \frac{\partial f_G}{\partial S} & \frac{\partial f_G}{\partial C} \\ \frac{\partial f_H}{\partial G} & \frac{\partial f_H}{\partial H} & \frac{\partial f_H}{\partial S} & \frac{\partial f_H}{\partial C} \\ \frac{\partial f_S}{\partial G} & \frac{\partial f_S}{\partial H} & \frac{\partial f_S}{\partial S} & \frac{\partial f_S}{\partial C} \\ \frac{\partial f_C}{\partial G} & \frac{\partial f_C}{\partial H} & \frac{\partial f_C}{\partial S} & \frac{\partial f_C}{\partial C} \end{pmatrix}$$

Where the notation (f_G, f_H, f_S, f_C) represents equation (3.1a) - (3.1d) respectively.

Evaluation of matrix M yields

$$M = \begin{pmatrix} -\sum_i \lambda_i - \mu & (1 - \phi_H) & (1 - \phi_S) & (1 - \phi_C) \\ \lambda_H \phi_H & \phi_H - \eta_H - \mu_H & 0 & 0 \\ \lambda_S \phi_S & 0 & \phi_S - \eta_S - \mu_S & 0 \\ \lambda_C \phi_C & 0 & 0 & \phi_C - \eta_C - \mu_C \end{pmatrix} \quad (3.2)$$

Matrix M in equation (3.2) is coefficient matrix used for approximating the behaviour of the system around a particular fixed point. This linear approximation is often used to analyse the behaviour of the original linear system around a specific operating point. In the context of this study. At the fixed points, the dynamical system becomes redundant, that is; the system stops changing, and thus the derivative with respect to spatial or temporal parameter is equal to zero.

3.3.1 Self Sufficiency Equilibrium (SSE)

We define self-sufficiency as a situation where a consumer produces enough power as much as they are using, or in excess and sells the rest to the national grid. This equilibrium is referred to self-sufficient because, even with the connection to national grid, the consumer does not draw power from the grid, but instead produces power enough for their needs and the excess channelled back to the national grid for use by other subscribers.

This is a point where each energy user attempts to produce their own power supply, and consume a proportion of ϕ , and the complementary excess proportion $(1 - \phi)$ is sold to the government, through the national grid.

The new system of equations describing this SSE status is given by substituting the value of $\lambda_i, i = H, S, C$;

$$\frac{dG}{dt} = \lambda + \sum_i(1 - \phi_i)(H + S + C) - \mu G \quad (3.3a)$$

$$\frac{dH}{dt} = \phi_H \lambda_H - \eta_H H - \mu_H H \quad (3.3b)$$

$$\frac{dS}{dt} = \phi_S \lambda_S - \eta_S S - \mu_S S \quad (3.3c)$$

$$\frac{dC}{dt} = \phi_C \lambda_C - \eta_C C - \mu_C C \quad (3.3d)$$

Stability of this state is determined by the nature of the eigenvalues of the information matrix evaluated at the fixed points.

The fixed points of system (3.3) is obtained by evaluating the equation $\frac{d}{dt}(:) = 0$ to get;

$$\lambda + \sum_i(1 - \phi_i)(H + S + C) - \mu G = 0$$

$$\phi_H \lambda_H G - \eta_H H - \mu_H H = 0$$

$$\phi_S \lambda_S G - \eta_S S - \mu_S S = 0$$

$$\phi_C \lambda_C G - \eta_C C - \mu_C C = 0$$

Which yields the equilibrium points $E^0 = (G^0, H^0, S^0, C^0)$ defined as;

$$(G^0, H^0, S^0, C^0) = \left(\frac{\lambda + \sum_i(1 - \phi_i)(H^0 + S^0 + C^0)}{\mu}, \frac{\phi_H \lambda_H}{\eta_H + \mu_H}, \frac{\phi_S \lambda_S}{\eta_S + \mu_S}, \frac{\phi_C \lambda_C}{\eta_C - \mu_C} \right) \quad (3.4)$$

The information matrix which helps determine the stability of SSE is extracted from equation (3.3) as;

$$M_{E^0} = \begin{pmatrix} -\mu & (1 - \phi_H) & (1 - \phi_S) & (1 - \phi_C) \\ 0 & \phi_H \lambda_H - \eta_H - \mu_H & 0 & 0 \\ 0 & 0 & \phi_S \lambda_S - \eta_S - \mu_S & 0 \\ 0 & 0 & 0 & \phi_C \lambda_C - \eta_C - \mu_C \end{pmatrix}$$

The stability of this matrix depends on the characteristic equation

$$|M_{E^0} - I\lambda_i| = 0$$

Which yields;

$$\lambda_1 = -\mu, \lambda_2 = \phi_H \lambda_H - \eta_H - \mu_H, \lambda_3 = \phi_S \lambda_S - \eta_S - \mu_S, \lambda_4 = \phi_C \lambda_C - \eta_C - \mu_C \quad (3.5)$$

Clearly, the SSE state is stable if and only if;

$$\phi_i > \frac{(\eta_i + \mu_i)}{\lambda_i}; \quad i = H, S, C \quad (3.6)$$

The condition given in equation (3.6) is interpreted as follows; that, the SSE state is stable if and only if the self-production is greater than the total of consumption and leakage and/or wastage. This sounds logical and satisfactory.

3.3.2 Total Dependence Equilibrium (TDE)

This is the equilibrium, where the energy users solely depend on the national grid, without having their own power source or production. In this case, the parameter ϕ is set equal to zero and their corresponding complementary proportions $(1 - \phi_i) = 0$ and all the power is assumed to be accounted for by the parameter λ .

The new set of equations for this equilibrium point is given by system (3.3a-d)

$$\frac{dG}{dt} = \lambda - (\lambda_H + \lambda_S + \lambda_C)G - \mu G \quad (3.7a)$$

$$\frac{dH}{dt} = \lambda_H G - \eta_H H - \mu_H H \quad (3.7b)$$

$$\frac{dS}{dt} = \lambda_S G - \eta_S S - \mu_S S \quad (3.7c)$$

$$\frac{dC}{dt} = \lambda_C G - \eta_C C - \mu_C C \quad (3.7d)$$

The fixed points of system (3.7a-d) is defined as $E^1 = (G^1, H^1, S^1, C^1)$ and are obtained by equating the derivative to zero and solving for the values of the respective variables as follows;

From equation (3.7a), the solution for

$$\lambda - (\lambda_H + \lambda_S + \lambda_C)G - \mu G = 0$$

yields

$$G^1 = \frac{\lambda}{(\lambda_H + \lambda_S + \lambda_C + \mu)}$$

While from equation (3.7b), the solution of the fixed point H^0 is obtained from solving the equation

$$\lambda_H G - \eta_H H - \mu_H H = 0$$

Which on evaluation of the value of H yields;

$$H^1 = \frac{\lambda_H(\lambda_H + \lambda_S + \lambda_C + \mu)}{\lambda(\eta_H + \mu_H)}$$

Similarly, the equilibrium points for the other two variables S^0, C^0 are obtained from equation (3.7 c, d) as;

$$\lambda_S G - \eta_S S - \mu_S S = 0$$

$$\lambda_C G - \eta_C C - \mu_C C = 0$$

Which yields the equilibrium points

$$S^1 = \frac{\lambda_S(\lambda_H + \lambda_S + \lambda_C + \mu)}{\lambda(\eta_S + \mu_S)}$$

and

$$C^1 = \frac{\lambda_C(\lambda_H + \lambda_S + \lambda_C + \mu)}{\lambda(\eta_C + \mu_C)}$$

The dependence equilibrium point is therefore given by the equation

$$E^1 = \left(\frac{\lambda}{(\lambda_H + \lambda_S + \lambda_C + \mu)}, \frac{\lambda\lambda_H}{(\eta_H + \mu_H)(\lambda_H + \lambda_S + \lambda_C + \mu)}, \frac{\lambda\lambda_S}{(\eta_S + \mu_S)(\lambda_H + \lambda_S + \lambda_C + \mu)}, \frac{\lambda\lambda_C}{(\eta_C + \mu_C)(\lambda_H + \lambda_S + \lambda_C + \mu)} \right) \quad (3.8)$$

Stability of SSFE is determined by substituting equation (3.8) into matrix M in equation (3.7) and then the sign of the resulting eigenvalues are determined. The system is stable, if all the eigenvalues negative.

From equation (3.8), the resulting linearization matrix about the SSFE point is obtained as;

$$M^0 = \begin{pmatrix} -(\lambda_H + \lambda_S + \lambda_C) - \mu & 0 & 0 & 0 \\ \lambda_H & -\eta_H - \mu_H & 0 & 0 \\ \lambda_S & 0 & -\eta_S - \mu_S & 0 \\ \lambda_C & 0 & 0 & -\eta_C - \mu_C \end{pmatrix} \quad (3.9)$$

whose eigenvalues are;

$$\Lambda_1 = -(\lambda_H + \lambda_S + \lambda_C) - \mu, \quad \Lambda_2 = -\eta_H - \mu_H, \quad \Lambda_3 = -\eta_S - \mu_S, \quad \Lambda_4 = -\eta_C - \mu_C \quad (3.10)$$

Clearly, all the eigenvalues are negative, and thus the dependency equilibrium (DE) is stable.

3.3.3 Mixed Source Equilibrium (MSE)

This equilibrium describes a condition where each household, school or industry is at liberty to generate their own power. In most cases, not all can produce adequate power to meet their consumption needs and therefore connect to the national grid to supplement. At the same time, a few users are able to generate more than enough power and thus sale excess to the national grid. This dynamic of this state is described by the system of equation in (3.1) with fixed points evaluated as;

$$\lambda + \sum_i (1 - \phi_i)(H + S + C) - \sum_i \lambda_i G - \mu G = 0 \quad (3.11a)$$

$$\lambda_H G + \phi_H \lambda_H G - \eta_H H - \mu_H H = 0 \quad (3.11b)$$

$$\lambda_S G + \phi_S \lambda_S G - \eta_S S - \mu_S S = 0 \quad (3.11c)$$

$$\lambda_C G + \phi_C \lambda_C G - \eta_C C - \mu_C C = 0 \quad (3.11d)$$

The equilibrium point (G^e, H^e, S^e, C^e) defined as follows. From equation (3.11a), the solution of the equation

$$\lambda + \sum_i (1 - \phi_i)(H + S + C) - \sum_i \lambda_i G - \mu G = 0$$

or equally expressed as

$$\lambda + (1 - \phi_H)H + (1 - \phi_S)S + (1 - \phi_C)C - (\lambda_H + \lambda_S + \lambda_C + \mu)G = 0$$

$$\frac{\lambda}{(\lambda_H + \lambda_S + \lambda_C + \mu)} + \frac{(1 - \phi_H)H}{(\lambda_H + \lambda_S + \lambda_C + \mu)} + \frac{(1 - \phi_S)S}{(\lambda_H + \lambda_S + \lambda_C + \mu)} + \frac{(1 - \phi_C)C}{(\lambda_H + \lambda_S + \lambda_C + \mu)} = G$$

$$G^0 + \frac{(1 - \phi_H)HG^0}{\lambda} + \frac{(1 - \phi_S)SG^0}{\lambda} + \frac{(1 - \phi_C)CG^0}{\lambda} = G^e$$

$$G^e = \frac{1}{\lambda} G^0 [\lambda + (1 - \phi_H)H^e + (1 - \phi_S)S^e + (1 - \phi_C)C^e]$$

From equation (3.11b), the fixed point H^e is obtained as;

$$\lambda_H G + \phi_H H - \eta_H H - \mu_H H = 0$$

Or equally expressed as;

$$H^e = \frac{\lambda_H G^e}{-(\phi_H - \eta_H - \mu_H)}$$

Similarly, from equation (3.11c), the fixed point S^e is given by

$$\lambda_S G + \phi_S S - \eta_S S - \mu_S S = 0$$

or

$$S^e = \frac{\lambda_S G^e}{-(\phi_S - \eta_S - \mu_S)}$$

And lastly the fixed point from equation (3.11d), is given by

$$C^e = \frac{\lambda_C G^e}{-(\phi_C - \eta_C - \mu_C)}$$

The set of points described as DE is therefore defined as;

$$(G^e, H^e, S^e, C^e) = \left(G^e, \frac{\lambda_H G^e}{-(\phi_H - \eta_H - \mu_H)}, \frac{\lambda_S G^e}{-(\phi_S - \eta_S - \mu_S)}, \frac{\lambda_C G^e}{-(\phi_C - \eta_C - \mu_C)} \right) \quad (3.12)$$

where $G^e = \frac{1}{\lambda} G^0 [\lambda + (1 - \phi_H)H^e + (1 - \phi_S)S^e + (1 - \phi_C)C^e]$. Equation (3.12) is to be substituted into the matrix (3.2) to determine the stability of the SSE. It is noted that matrix (3.2) is independent of the value of the variables at various fixed point. Therefore, the system is unconditionally stable independent of either fixed point.

3.4 Neural Network Model

Neural networks are designed using mathematical equations to mimic the biological neurons in the body system. Each neuron is designed to pick a signal, transmit to interpretation centre, and send response signal, either to stimulate or suppress the original source of stimulus. In the context of our study, a set of several artificial neural network is created to monitor and give feedback on the household demand and usage of energy, so that the data generated will be used to control the amount of energy to be supplied (Dillon & Niebur, 1996).

The model formulated, receives feedback signals from energy utility terminals, which will be optimized, and used to adjust the desired power input from Integrated Power plants, for various energy terminals (Ben, 2010). The link between energy production and energy consumption points and all the processes involved is here referred to as Smart Grid. (See Figure 7 below).

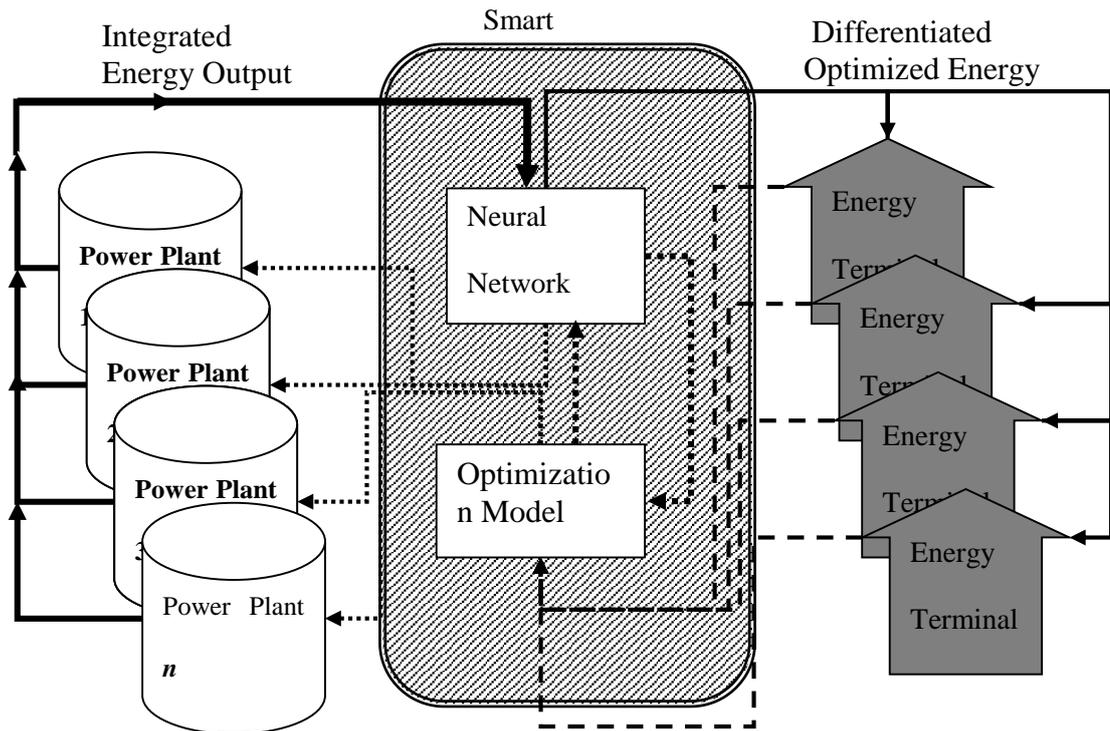


Figure 7: Smart Grid: An Optimal Energy Mix Model. Source: Author

The Neural network model formulated is then designed using MATLAB - SIMULINK, and used to simulate various model parameters, in order to determine their sensitivity, stability and optimal ranges(Ascione et al., 2016; Awodele & Jegedo, 2009).

The optimization model and the Neural Network Model, forms a system of equations, which analyses and sends feedback to energy production plant, on the intensity of use and volume of energy required by each consumer. The Smart grid will then adjust the source and transmission, so that each user obtains power, equivalent to the needs(Carlo et al., 2014).

3.4.1 Neural Network model Formulation and Analysis

The formulation of neural network model equations involves establishing the mathematical relationships that describe the behaviour of the neural network. These

equations typically include the activation functions, weights, biases, and connections between the neurons in the network. The specific form of the equations depends on the architecture of the neural network, such as whether it's a feedforward neural network, a convolutional neural network, a recurrent neural network, or a more complex architecture. The equations are used to represent the process of data propagation, learning, and decision-making within the neural network.

A neural network model was formulated using differential equations. A system of differential equations will be of the form;

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t)\end{aligned}\tag{3.13}$$

where the variables A, B, C, D are constant matrices representing evolution. Control, observation and direct transmission matrix respectively and $x(t)$ denotes the state variable, while $u(t)$ and $y(t)$ represents the input variables and response variable respectively (Dreyfus, 2005). The formulated system is schematically represented by Figure 8 below. The power supplied to the household from the distribution line is compared with predicted power needs by Neural Network Algorithms. The error is used for Neural training, and once the difference is zero, stabilizing controller relays a signal through a feedback control to the power control plant, which will adjust the power supply according to the household needs. This model can be minimized and applied to each household electrical appliance, so as to control the power needs of each gadget. The same system can be installed to control the power needs of each household or community.

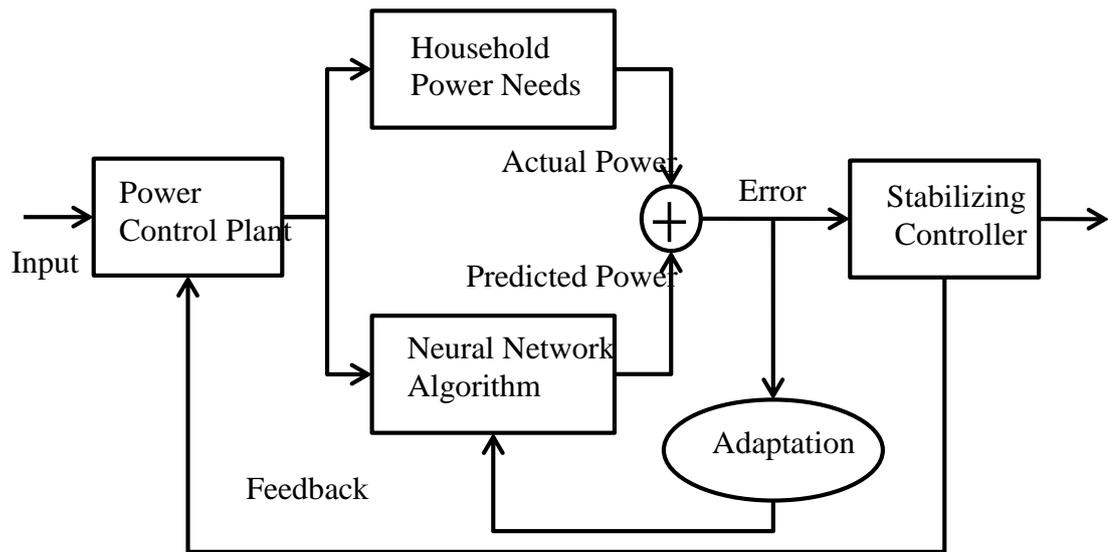


Figure 8: A Power Stabilizing Neural Network Controller. Source: Author

3.4.2 Neural Network of the Energy Model Equations

From the discussion of Neural Network presented, the energy model equation (3.1) can be expressed in form of equation (3.13) as,

$$\begin{pmatrix} H'(t) \\ S'(t) \\ C'(t) \end{pmatrix} = \begin{pmatrix} -\eta_H - \mu_H & 0 & 0 \\ 0 & -\eta_S - \mu_S & 0 \\ 0 & 0 & -\eta_C - \mu_C \end{pmatrix} \begin{pmatrix} H(t) \\ S(t) \\ C(t) \end{pmatrix} + \begin{pmatrix} \lambda_H \\ \lambda_S \\ \lambda_C \end{pmatrix} u(t) \quad (3.14)$$

$$y(t) = (\phi_H \quad \phi_S \quad \phi_C) \begin{pmatrix} H(t) \\ S(t) \\ C(t) \end{pmatrix} + (0)u(t)$$

The analytic solution of equation (3.14) can be simply obtained because the coefficient matrix is diagonal, and thus the equivalent solution to equation (3.14) with the coefficient matrices A and B given as;

$$\mathbf{x}(t) = e^{A(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^t e^{A(t-\tau)}B\mathbf{x}(\tau) d\tau$$

where

$$A = \begin{pmatrix} -(\eta_H + \mu_H) & 0 & 0 \\ 0 & -(\eta_S + \mu_S) & 0 \\ 0 & 0 & -(\eta_C + \mu_C) \end{pmatrix}, \quad B(\tau) = \begin{pmatrix} \lambda_H \\ \lambda_S \\ \lambda_C \end{pmatrix}$$

Which yields;

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = \begin{pmatrix} e^{-(\eta_H + \mu_H)t} & 0 & 0 \\ 0 & e^{-(\eta_S + \mu_S)t} & 0 \\ 0 & 0 & e^{-(\eta_C + \mu_C)t} \end{pmatrix} \begin{pmatrix} x_1(t_0) \\ x_2(t_0) \\ x_3(t_0) \end{pmatrix} + \int_a^b \begin{pmatrix} e^{-(\eta_H + \mu_H)(t-\tau)} & 0 & 0 \\ 0 & e^{-(\eta_S + \mu_S)(t-\tau)} & 0 \\ 0 & 0 & e^{-(\eta_C + \mu_C)(t-\tau)} \end{pmatrix} \begin{pmatrix} \lambda_H \\ \lambda_S \\ \lambda_C \end{pmatrix} \begin{pmatrix} x_1(\tau) \\ x_2(\tau) \\ x_3(\tau) \end{pmatrix} d\tau$$

3.4.3 Laplace Transform and Transfer functions

The Laplace transform is a powerful mathematical tool used in engineering, physics, and mathematics to simplify the analysis of linear time-invariant systems and functions. It transforms a function of time, often denoted as $f(t)$, into a function of a complex variable s , denoted as $F(s)$. The transform is defined by the integral:

$$F(s) = \int_0^{\infty} e^{-st} * f(t) dt$$

where e^{-st} is the exponential kernel and s is a complex frequency parameter. By applying the Laplace transform, differential equations can be converted into algebraic equations, making it easier to analyse and solve problems related to control systems, circuits, and signal processing. The Laplace transform provides a method to analyse

the behaviour and response of linear systems to different inputs and disturbances. The inverse Laplace transform, denoted as

$$f(t) = \mathcal{L}^{-1}[F(s)],$$

allows one to convert the transformed function back into the time domain, enabling the interpretation of the system's response or behaviour in terms of time. The Laplace transform also offers the benefits of linearity and time-shifting, making it a valuable tool for solving differential equations, convolution integrals, and initial value problems. Additionally, it is widely used in the analysis of stability, transient response, and frequency response characteristics of systems. In summary, the Laplace transform serves as a fundamental tool for simplifying the analysis of linear time-invariant systems, enabling the representation and solution of complex problems in a more manageable mathematical framework.

As a powerful mathematical technique used to solve complex differential equations, Laplace transform involves converting differential equations to algebraic equations, which can then be solved algebraically and inversely transformed back to original variables so as to obtain the solution of the differential equation.

Taking the Laplace transform of the state model in equation (3.13) above, and assuming that $x(0) = x_0$ we obtain

$$\hat{y}(s) = C(sI - A)^{-1}x_0 + [C(sI - A)^{-1}B + D]\hat{u}(s) \quad (3.15)$$

The first term on the right-hand side is the natural response, also called zero input response depending on the initial state x_0 , while the second term is the zero-state response which depends on the input $\hat{u}(s)$. In the case of zero initial state, the transformed system becomes

$$\hat{y}(s) = [C(sI - A)^{-1}B + D]\hat{u}(s) := \hat{H}(s)\hat{u}(s)$$

or presented in a transfer function form as

$$\hat{H}(s) = [C(sI - A)^{-1}B + D] \quad (3.16)$$

whose solution after taking the inverse Laplace transform is the impulse response matrix

$$H(t) = Ce^{At}B + D\delta(t)$$

The state variable representation of dynamical system specifies both the internal as well as external behaviour of the system. The transfer function on the other hand specifies the external behaviour only thus information on the initial state is assumed to be relaxed at $t = 0$.

The transfer function matrix in equation (3.16) can be expressed as;

$$\hat{H}(s) = \frac{C(sI-A)^{-1}B}{|sI-A|} + D = \frac{C(sI-A)^{-1}B + D|sI-A|}{|sI-A|} := \frac{Y(s)}{U(s)} \quad (3.17)$$

Where $Y(s) = (a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_2s^2 + a_1s + a_0)E(s)$ and $U(s) = (s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \dots + b_2s^2 + b_1s + b_0)E(s)$ are polynomials of order $n - 1$ and n respectively, while

$$\begin{aligned} E(s) &= e(t) = x_1(t) \\ sE(s) &= \dot{e}(t) = x_2(t) \\ s^2E(s) &= \ddot{e}(t) = x_3(t) \\ &\vdots \\ s^nE(s) &= e^{(n)}(t) = x_n(t) \end{aligned} \quad (3.18)$$

Note that from the system (3.13), we obtain the following system of differential equations

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= x_4(t) \\
&\vdots \\
\dot{x}_n(t) &= -b_0x_1(t) - b_1x_2(t) - b_2x_3(t) - \dots - b_{n-1}x_n(t) + u(t)
\end{aligned}$$

which in matrix form, can be expressed as;

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \vdots \\ \dot{x}_n(t) \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ -b_0 & -b_1 & -b_2 & -b_3 & -b_4 & \dots & -b_n \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ \vdots \\ x_n(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u(t) \quad (3.19)$$

Which in vector form yields the system in equation (3.8), here reproduced as

$$\dot{X}(t) = AX(t) + BU(t) \quad (3.20)$$

with the solution to state equation (3.13) obtained using variation of parameters method of calculus as;

$$\mathbf{x}(t) = e^{A(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^t e^{A(t-\tau)}B\mathbf{x}(\tau) d\tau$$

Equation (3.15) gives a set of solutions of the vector $\mathbf{x} = x_1, x_2, x_3, \dots, x_n$ which describes the output solution of the control system, describing the set of values of energy inputs required by the individual consumer, as controlled by the smart grid technology, using neural networks.

Depending on the initial inputs (conditions) from the data collected from consumers, the set of solution values obtained by equation (3.15) will control the input or power supply from production, and thus optimize the cost of production and associated distribution costs.

3.5 Stability of the System

Stability of the system is a very important characteristic of the system that determines whether results obtained will remain robust in case of any perturbation. In everyday life, every system, especially that of power is prone to multiple perturbation and any variation of fluctuation beyond acceptable limits will result in devastating effect which include power surge among others.

In this regard, stability is also viewed in terms of output response for every input. A system is stable if and only if its output in response to every bounded input remains bounded. If a linear system is stable, it is stable for any input, regardless of size. System (3.14) is said to be stable if the eigenvalues of the system has negative real part; that is the condition that the eigenvalues λ_i of matrix A is such that $\Re(\lambda_i) \leq 0$.

In this case, the $\sigma(A)$ are given by;

$$\lambda_1 = -(\eta_H + \mu_H); \lambda_2 = -(\eta_S + \mu_S), \text{ and } \lambda_3 = -(\eta_C + \mu_C)$$

Clearly, for $\eta_i > 0$, $\mu_i > 0$; $i = H, S, C$ and the system (3.14) is stable.

3.6 Settling time of the system

Settling time in a response function refers to the time it takes for the system's response to reach and remain within a specified range around its final steady-state value after a disturbance. It is an essential parameter for assessing the performance and stability of dynamic systems. A shorter settling time indicates a faster response to disturbances, while a longer settling time may result in slower dynamics and potentially lead to instability or oscillations in the system. Therefore, settling time is a critical measure in evaluating the transient behaviour of response functions in various engineering and control applications.

Apart from stability of a system, determining the settling time is a very important parameter that must be evaluated. This is done using Lyapunov function.

Lyapunov function is a key concept in the analysis of stability in dynamical systems. A Lyapunov function is a scalar function that is used to study the stability of an equilibrium point of a system. It serves as a tool to determine whether the system's behaviour converges to or diverges from the equilibrium point over time. If a Lyapunov function can be found for a system, and it satisfies certain criteria, it can provide valuable insights into the system's stability properties. Specifically, a Lyapunov function is positive definite, such that it is greater than zero for all non-zero arguments, and its derivative along system trajectories is negative definite, indicating a stable equilibrium. Therefore, Lyapunov functions play a critical role in assessing the stability of dynamical systems in various fields such as control theory, robotics, and nonlinear dynamics.

Define a positive definite Lyapunov function $V(x)$ whose total time derivative is negative definite $-\dot{V}(x)$, and let the parameter T be the maximum time for the ratio

$T = \max \frac{V(x)}{-\dot{V}(x)}$. The by simple manipulation, we have;

$$V[x(t_s)] \leq V[x(0)]e^{-\frac{1}{T}t_s} \quad (3.21)$$

Thus the settling time t_s can be evaluated as the upper bound of $V(x)$ for any initial condition given at $t = 0$.

Important parameters in the analysis of settling time in change of demand of power consumption in a household include and not limited to overshoot load, settling time,

allowable tolerance, settling ratio, oscillation period among others. These parameters are illustrated in the figure below and discussed in detail in (Ali, 2011).

The parameters in question are to be investigated and analysed, so that the optimal thresholds are obtained through simulation. These threshold values form the guiding values for the control system, which ensures that the desired power loading of every consumer is achieved. This is achieved through the use of smart grid, and feedback control mechanism. Neural network therefore is fundamental in the achievement of the desired results.

Define the following parameters with respect to step response as shown in Figure 9 below;

- (a) Rise time t_r as the time needed for the load to change from a given low value to the desired high value. This can also be used to define fall time as t_{-r}
- (b) Maximum load overshoot Max_l representing the peak load value of the specific response curve, with Min_l as the minimum undershoot of loading
- (c) Settling time t_s denoting the time required for the oscillation to settle within tolerable allowance. This represents the time that the loading takes to settle and persist within specified error band, usually $\pm 2\%$ to $\pm 5\%$.
- (d) Delay time t_d denoting the time it takes the power to achieve the desired value from the initial value. This is measured from the time the loading is switched, either up or down.
- (e) Peak time t_p representing the time needed for the response to achieve the first peak or overshoot or undershoot value of the loading.

- (f) Steady state error e_{ss} representing the difference between the preferred final loading value and the real response. This value is achieved from the settling time to the next level of interruption or change in desired value.

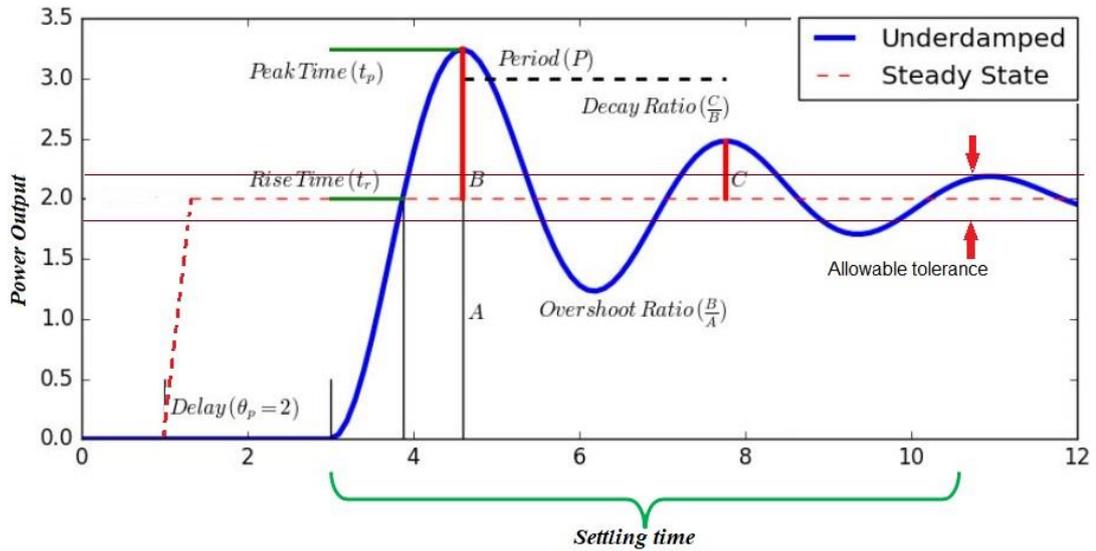


Figure 9: Power input and output response function. Source:(Ali, 2011)

These parameters are used to design the controller that is able to give a feedback and control the output so as to reduce the overshoot, reduce the delay time, reduce the settling time and maintain the power within the desired acceptable error bracket.

CHAPTER FOUR

MODEL SIMULATION, TESTING AND DISCUSSION OF RESULTS

In this chapter, data from random consumers is used on a designed control model, and the behaviour of the smart grid is simulated to show energy mix and optimization of power to satisfy the consumer's desires. The analysis of data is done using the methods discussed in chapter three, and the results presented, both in tables and graphs.

4.1 Description of the SMART grid model in SIMULINK

Creating a Simulink model for a smart grid involves representing the various components and operations of the smart grid using Simulink blocks and connecting them to form a comprehensive simulation. The model may include power generation sources, energy storage systems, transmission lines, distribution networks, control algorithms, and communication protocols. By simulating the smart grid in Simulink, it becomes possible to analyse and optimize the performance, efficiency, and reliability of the grid under different operating conditions and scenarios. Smart grid Simulink models are valuable for research, development, and testing of advanced grid control strategies, renewable energy integration, demand response, and other grid modernization techniques.

For a single input single output (SISO) , the general feedback system is represented by the following transfer function and illustrated in Figure 10 below, where $G(s)$ denotes the system or the plant or in this case the consumption unit, which could be an household, small scale industry or large scale commercial setup, $H(s)$ represents the feedback component, a signal from the consumption unit which describes the

difference between the supplied power and the demand, $E(s)$ is the design variable, and $U(s)$ is the input component.

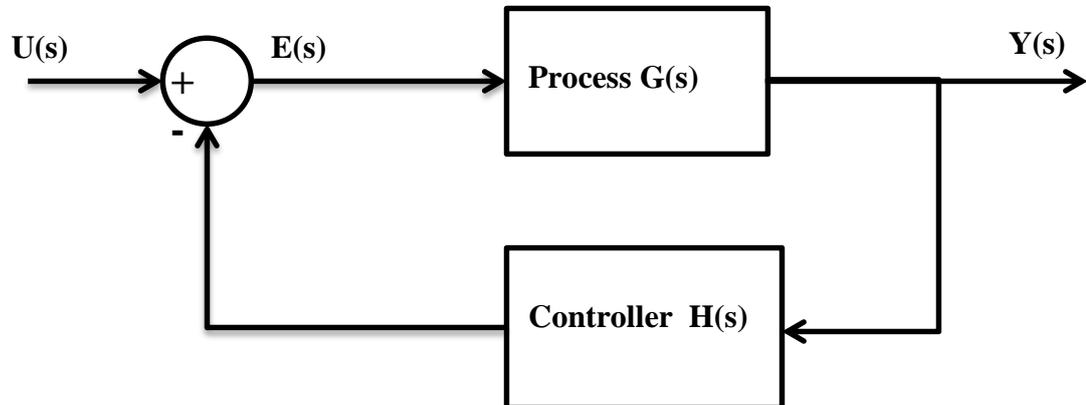


Figure 10: Single loop Feedback Control System. Source: Author

Note that for a closed loop feedback system, the following equation which represents the system characteristic equation is always satisfied, that is;

$$1 + G(s)H(s) = 0$$

The complex function obtained from the characteristic equation above is given by

$$D(s) = 1 + G(s)H(s)$$

Whose roots define the characteristics of the closed loop transfer function. The system is stable if all the roots of $D(s)$ lies on the left side of the complex s -plane. A simple closed contour Γ_s is chosen such that it encloses all the roots of the transfer function. This contour is called the Nyquist curve, a polar plot of the function $F(s)$, the analytic equation inside or on the closed curve Γ_s , which encloses the number of poles $Z = P - N$ where P denotes the number of poles and Z the number of zeros.

4.2 Stability of the SIMULINK Model

One of the key methods for assessing the stability of a Simulink model is through linearization. This involves approximating the system's linear behaviour around an operating point by a linear model. Additionally, stability analysis tools such as the Control System Toolbox in MATLAB provide various methods to analyse the stability of Simulink models. These methods include frequency response analysis, root locus analysis, and pole placement. Furthermore, simulation techniques such as adjusting the step size, solver type, and tolerances can also help in evaluating the stability of Simulink models. Conducting sensitivity analysis to assess the impact of parameter variations on stability is another important method. Overall, these methods provide valuable insights into the stability characteristics of Simulink models, ensuring reliable and effective system performance.

The Nyquist stability criterion is a powerful tool for analysing the stability of linear time-invariant (LTI) systems. It provides a graphical method for assessing the stability of a system based on its transfer function. By plotting the frequency response of the system on a polar plot, the Nyquist criterion enables engineers to determine the stability of the system by examining the encirclements of the critical point $-1 + i0$ in the complex plane. If the Nyquist plot encircles the critical point in a counterclockwise direction, the system is deemed unstable; if it encircles it in a clockwise direction, the system is stable. Additionally, the number of encirclements provides insight into the system's relative stability. The Nyquist stability criterion is widely used in control system design to ensure the stability of feedback control systems and is an essential concept in classical control theory.

Nyquist stability criterion states that, a feedback control system of equations is stable if and only if for the contour Γ_p , the number of counterclockwise encirclements of the point $(-1, 0)$ is equal to the number of poles $p(s)$ with positive real part. Thus, the Nyquist circle gives information on phase and gain stability, that is; how far the system is from stability. This is an indication of the degree of stability, and how much adjustment required on the system to gain stability or not to lose stability.

4.3 Formulation of Neural Network of the Model Equation

Formulating a neural network as a system of differential equations involves defining the dynamical behaviour of the network's individual neurons and the interactions between them. Each neuron's activity is typically described by a differential equation that captures its response to input signals, internal state, and network connections (Li & Peng, 2006). The network's connectivity can be represented through a system of coupled differential equations, describing how the activities of one neuron affect others. The specific form of these equations depends on the neural network architecture, such as feedforward, recurrent, or convolutional networks. By formulating neural networks as systems of differential equations, it becomes possible to analyse their dynamic behaviour, study learning processes, and investigate emergent phenomena, providing insights into their computational capabilities and information processing abilities (Li & Peng, 2006).

In this research study, model equations representing power mix consumption and feedback control is given in Equation (3.1 a,b,c,d), and they are expressed in Neural Network feedback control system in equation (3.8). The details of matrices A, B, C, D are given in equation (3.9) and the corresponding transfer function is given by the

state equation (3.14), while the response equation, with matrix $D = 0$ is given in equation (3.9), thus the transfer function system is of the form;

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (4.1)$$

Where the coefficient matrices $A = \begin{pmatrix} -\eta_H - \mu_H & 0 & 0 \\ 0 & -\eta_S - \mu_S & 0 \\ 0 & 0 & -\eta_C - \mu_C \end{pmatrix}$, $B = \begin{pmatrix} \lambda_H \\ \lambda_S \\ \lambda_C \end{pmatrix}$, and $C = (\phi_H \ \phi_S \ \phi_C)$.

The transfer function of a system can be obtained using the resolvent algorithm by employing techniques from the field of control theory and linear algebra. The resolvent algorithm involves calculating the resolvent matrix of the system, which is derived from the system's state-space representation (Martini et al., 2020). By applying the resolvent algorithm, it becomes possible to directly obtain the transfer function of the system from its state-space model without explicitly converting it to the transfer function form. This can be particularly useful for systems with complex dynamics represented in state-space form, providing an efficient method for determining the transfer function directly from the system's structural and dynamic properties. The resolvent algorithm offers a powerful approach for obtaining transfer functions in control system analysis and design, especially for systems with large state-space dimensions or intricate interconnections (Martini et al., 2020).

The transfer function is obtained from the linear state model (3.9) using resolvent algorithm (Hsu et al., 1997; Vu, 2008) as follows;

Taking the Laplace transform of equation (4.1) and assuming $x(0) = \mathbf{x}_0$, we obtain the equation

$$\hat{y}(s) = C(sI - A)^{-1}\mathbf{x}_0 + [C(sI - A)^{-1}B + D]\hat{u}(s) \quad (4.2)$$

The first term on the right hand side is the natural response which depends on the initial state \mathbf{x}_0 and the second term is the forced response which depends on the input $\hat{u}(s)$.

In the case of zero initial state, that is $\mathbf{x}_0 = 0$,

$$\hat{H}(s) = C(sI - A)^{-1}B + D$$

And thus;

$$\hat{y}(s) = \hat{H}(s)\hat{u}(s) \quad (4.3)$$

Taking the inverse Laplace transform of $\hat{H}(s)$ in equation (4.3) we get

$$H(t) = Ce^{At}B + D\delta(t) \quad (4.4)$$

Which is the impulse response matrix of equation (4.1). For SISO system, the matrix $H(t)$ reduces to a scalar transfer function of the system given by,

$$\hat{h}(s) = c(sI - A)^{-1}b + d$$

The transfer function matrix in equation (4.4) can be written as

$$\hat{H}(s) = \frac{DC(sI - A)^{-1}B + |sI - A|}{|sI - A|} \quad (4.5)$$

Where $|sI - A|$ is the characteristic polynomial of matrix A , and $(sI - A)^+$ is the adjoint of the matrix $(sI - A)$, or a matrix of cofactors of $(sI - A)$, which is the polynomial in s of order $n - 1$ or less.

The cofactor matrix is expressed in terms of its components as;

$$(sI - A)^+ = Q(s) = Q_1s^{n-1} + Q_2s^{n-2} + Q_3s^{n-3} \dots + Q_{n-1}s + Q_n \quad (4.6)$$

And the characteristic polynomial is expressed as;

$$|sI - A| = \alpha_0s^n + \alpha_1s^{n-1} + \alpha_2s^{n-2} + \dots + \alpha_{n-1}s + \alpha_n$$

Where the coefficient constant scalars α_i are defined from resolvent algorithm as;

$$Q_1 = I$$

$$\alpha_1 = -tr(AQ_1)$$

$$Q_2 = AQ_1 + \alpha_1I$$

$$\alpha_2 = -\frac{1}{2}tr(AQ_2)$$

$$Q_{n-1} = AQ_{n-2} + \alpha_{n-1}I$$

$$Q_n = AQ_{n-1} + \alpha_{n-1}I$$

$$\alpha_n = -\frac{1}{n}tr(AQ_n)$$

In this case, the trace is the sum of elements along the main diagonal, and consequently,

$$\det(A) = (-1)^n \alpha_n$$

And

$$A^{-1} = -\frac{Q_n}{\alpha_n}$$

In our workings in equation (3.9), we obtain

$$\alpha_1 = -(a_{11} + a_{22} + a_{33}) \quad (4.8a)$$

$$\text{Denote matrix } A = \begin{pmatrix} -\eta_H - \mu_H & 0 & 0 \\ 0 & -\eta_S - \mu_S & 0 \\ 0 & 0 & -\eta_C - \mu_C \end{pmatrix} := \begin{pmatrix} -a_{11} & 0 & 0 \\ 0 & -a_{22} & 0 \\ 0 & 0 & -a_{33} \end{pmatrix}$$

where we define the elements $a_{11} = \eta_H + \mu_H$, $a_{22} = \eta_S + \mu_S$, and $a_{33} = \eta_C + \mu_C$.

$$\text{Define } Q_2 = A + \alpha_1 I = \begin{pmatrix} -(\eta_H + \mu_H) + \alpha_1 & 0 & 0 \\ 0 & -(\eta_S + \mu_S) + \alpha_1 & 0 \\ 0 & 0 & -(\eta_C + \mu_C) + \alpha_1 \end{pmatrix}$$

and therefore

$$\alpha_2 = -\frac{1}{2} \text{tr}(AQ_2) = -\frac{1}{2} (a_{11}^2 - a_1 a_{11} + a_{22}^2 - a_1 a_{22} + a_{33}^2 - a_1 a_{33})$$

$$\alpha_2 = -\frac{1}{2} (a_{11}^2 + a_{22}^2 + a_{33}^2 - \alpha_1^2) \quad (4.8b)$$

$$Q_3 = AQ_2 + \alpha_2 I = \begin{pmatrix} a_{11}^2 - a_{11}\alpha_1 + \alpha_2 & 0 & 0 \\ 0 & a_{22}^2 - a_1 a_{22} + \alpha_2 & 0 \\ 0 & 0 & a_{33}^2 - a_1 a_{33} + \alpha_2 \end{pmatrix}$$

Where $a_{11} = (\eta_H + \mu_H)$, $a_{22} = (\eta_S + \mu_S)$, $a_{33} = (\eta_C + \mu_C)$ and therefore,

$$\begin{aligned} \alpha_3 &= -\frac{1}{2} \text{tr}(AQ_3) \\ &= -\frac{1}{2} \begin{pmatrix} -a_{11}^3 + a_{11}^2 \alpha_1 - \alpha_2 a_{11} & 0 & 0 \\ 0 & -a_{22}^3 + a_1 a_{22}^2 - \alpha_2 a_{22} & 0 \\ 0 & 0 & -a_{33}^3 + a_1 a_{33}^2 - \alpha_2 a_{33} \end{pmatrix} \end{aligned}$$

$$= -\frac{1}{2}(-a_{11}^3 + a_{11}^2\alpha_1 - \alpha_2 a_{11} - a_{22}^3 + a_1 a_{22}^2 - \alpha_2 a_{22} - a_{33}^3 + \alpha_1 a_{33}^2 - \alpha_2 a_{33})$$

$$= -\frac{1}{2}(-a_{11}^3 - a_{22}^3 - a_{33}^3 + \alpha_1(a_{11}^2 + a_{22}^2 + a_{33}^2) - \alpha_2(a_{11} + a_{22} + a_{33}))$$

$$\alpha_3 = -\frac{1}{2}(-a_{11}^3 - a_{22}^3 - a_{33}^3 + \alpha_1(a_{11}^2 + a_{22}^2 + a_{33}^2) - \alpha_1\alpha_2)$$

$$\alpha_3 = -\frac{1}{2}(-a_{11}^3 - a_{22}^3 - a_{33}^3 + \alpha_1(a_{11}^2 + a_{22}^2 + a_{33}^2) - 3\alpha_1\alpha_2) \quad (4.8c)$$

Therefore, the characteristic equation becomes

$$\Delta s = s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3 \quad (4.9)$$

While

$$Q(s) = (sI - A)^+ =$$

$$\begin{pmatrix} s^2 - (a_{22} + a_{33})s + a_{22}a_{33} & 0 & 0 \\ 0 & s^2 - (a_{11} + a_{33})s + a_{11}a_{33} & 0 \\ 0 & 0 & s^2 - (a_{11} + a_{22})s + a_{11}a_{22} \end{pmatrix}$$

expressed briefly as $Q(s) = \begin{pmatrix} q_{11} & 0 & 0 \\ 0 & q_{22} & 0 \\ 0 & 0 & q_{33} \end{pmatrix}$, where $q_{11} = s^2 - (a_{22} + a_{33})s +$

$a_{22}a_{33}$, $q_{22} = s^2 - (a_{11} + a_{33})s + a_{11}a_{33}$, and $q_{33} = s^2 - (a_{11} + a_{22})s + a_{11}a_{22}$,

and therefore, the transfer function matrix $\hat{H}(s) = \frac{CQ(s)B}{\Delta s}$ simplifies to

$$\hat{H}(s) = \frac{(\phi_H \quad \phi_S \quad \phi_C) \begin{pmatrix} q_{11} & 0 & 0 \\ 0 & q_{22} & 0 \\ 0 & 0 & q_{33} \end{pmatrix} \begin{pmatrix} \lambda_H \\ \lambda_S \\ \lambda_C \end{pmatrix}}{\Delta s}$$

$$\hat{H}(s) = \frac{\phi_H q_{11} \lambda_H + \phi_S q_{22} \lambda_S + \phi_C q_{33} \lambda_C}{s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3} \quad (4.10)$$

Where, $\phi_H q_{11} \lambda_H = \phi_H \lambda_H (s^2 - (a_{22} + a_{33})s + a_{22} a_{33})$, $\phi_S q_{22} \lambda_S = \phi_S \lambda_S (s^2 - (a_{11} + a_{33})s + a_{11} a_{33})$ and $\phi_C q_{33} \lambda_C = \phi_C \lambda_C (s^2 - (a_{11} + a_{22})s + a_{11} a_{22})$ or in expanded form is;

$$\begin{aligned} \phi_H q_{11} \lambda_H &= \phi_H \lambda_H (s^2 - (a_{22} + a_{33})s + a_{22} a_{33}) \\ &= \phi_H \lambda_H [s^2 - (\eta_S + \mu_S + \eta_C + \mu_C)s + (\eta_S + \mu_S)(\eta_C + \mu_C)] \end{aligned}$$

$$\begin{aligned} \phi_S q_{22} \lambda_S &= \phi_S \lambda_S (s^2 - (a_{11} + a_{33})s + a_{11} a_{33}) \\ &= \phi_S \lambda_S (s^2 - (\eta_H + \mu_H + \eta_C + \mu_C)s + (\eta_H + \mu_H)(\eta_C + \mu_C)) \end{aligned}$$

And

$$\phi_C q_{33} \lambda_C = \phi_C \lambda_C (s^2 - (\eta_H + \mu_H + \eta_S + \mu_S)s + (\eta_H + \mu_H)(\eta_S + \mu_S))$$

So the numerator of $\hat{H}(s)$ in equation (4.10) equals to

$$\begin{aligned} CQ(s)B &= \phi_H \lambda_H [s^2 - (\eta_S + \mu_S + \eta_C + \mu_C)s + (\eta_S + \mu_S)(\eta_C + \mu_C)] \\ &\quad + \phi_S \lambda_S [s^2 - (\eta_H + \mu_H + \eta_C + \mu_C)s + (\eta_H + \mu_H)(\eta_C + \mu_C)] \\ &\quad + \phi_C \lambda_C [s^2 - (\eta_H + \mu_H + \eta_S + \mu_S)s + (\eta_H + \mu_H)(\eta_S + \mu_S)] \\ &= \beta_2 s^2 - \beta_1 s + \beta_0 \end{aligned}$$

Where $\beta_2 = (\phi_H \lambda_H + \phi_S \lambda_S + \phi_C \lambda_C)$, $\beta_1 = \phi_H \lambda_H (\eta_S + \mu_S + \eta_C + \mu_C) + \phi_S \lambda_S (\eta_H + \mu_H + \eta_C + \mu_C) + \phi_C \lambda_C (\eta_H + \mu_H + \eta_S + \mu_S)$, while $\beta_0 = \phi_H \lambda_H (\eta_S + \mu_S)(\eta_C + \mu_C) + \phi_S \lambda_S (\eta_H + \mu_H)(\eta_C + \mu_C) + \phi_C \lambda_C (\eta_H + \mu_H)(\eta_S + \mu_S)$.

Alternatively, we can express the coefficients as; $\beta_2 = (\phi_H \lambda_H + \phi_S \lambda_S + \phi_C \lambda_C)$, $\beta_1 = \phi_H \lambda_H (a_{22} + a_{33}) + \phi_S \lambda_S (a_{11} + a_{33}) + \phi_C \lambda_C (a_{11} + a_{22})$, and $\beta_0 = \phi_H \lambda_H a_{22} a_{33} + \phi_S \lambda_S a_{11} a_{33} + \phi_C \lambda_C a_{11} a_{22}$.

4.4 Canonical Signal flow Simulation diagram

A canonical signal flow diagram is a visual representation of a system's interconnections and signal paths using a standardized layout and notation. It provides a clear and structured depiction of the flow of signals through the system, enabling easy comprehension and analysis of the system's dynamics. Canonical signal flow diagrams adhere to specific conventions, such as consistent signal flow direction, standard symbols for components and operations, and a systematic layout that follows the direction of signal flow. By presenting the system's architecture in a standardized and easily interpretable format, canonical signal flow diagrams facilitate communication, understanding, and analysis of complex systems in various domains such as control theory, signal processing, and communication systems.

Canonical signal flow diagram is therefore the most convenient way of representing a continuous differential model in a graph. From equation (4.10), the numerator and the denominator can be expressed as in equation (3.11) reproduced in equation (4.11) below.

$$\hat{H}(s) := \frac{Y(s)}{U(s)} = \frac{(\beta_2 s^2 - \beta_1 s + \beta_0)E(s)}{(\alpha_0 s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3)E(s)} \quad (4.11)$$

Multiply equation (4.11) through by s^{-3} and divide through by α_3 yields

$$E(s) = \frac{U(s)}{b_n} + [b_{n-1}s^{-1} + b_{n-2}s^{-2} + \dots + b_1s^{1-n} + b_0s^{-n}] \quad (4.12)$$

Using equation (4.11) and (4.12) the following simulation diagram with s^{-1} denoting integration and b_n denoting gain is presented. The standard form also referred to as the observer canonical form flow diagram is given in Figure 11 below.

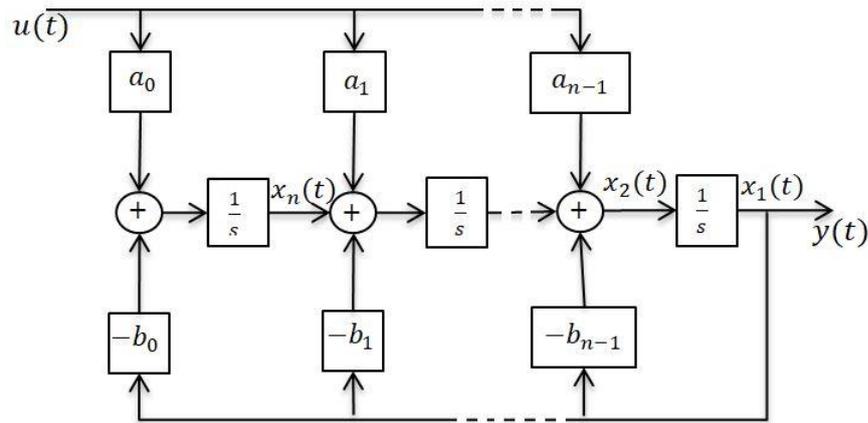


Figure 11: Canonical form Simulation diagram in state variable representation.
Author.

4.5 Data from Random Consumers and Response Function

The following data in Table 2 relates to 10 consumers, of which six are domestic users, 2 SME and 2 heavy commercial users of electricity with options of using alternative source of power, namely; solar and gas for heating and cooking. Table 2 shows the summary of the type and daily cost of energy used, that is; electricity (KWh), solar (KWh) or gas (BTU). The details of energy consumption is presented in Appendix II.

Table 2: Units and Cost of energy Consumption by a sample of domestic, SME and Commercial users per day

No	Consumer	Type	Average Daily Consumption			Daily Cost (in Ksh)			TOTAL COST
			Electricity KWh	Solar KWh	LPG Gas In Kg	Electricity	Solar	LPG Gas	
1	Consumer 1	Domestic	10.3 KWh	12KWh	0.93 Kg	133.51	8.88	236.08	378.47
2	Consumer 2	Domestic	21.4 KWh	-	0.43 Kg	278.03	0	109.15	387.18
3	Consumer 3	Domestic	18.7 KWh	10 KWh	0.15Kg	243.53	7.40	38.08	289.01
4	Consumer 4	Domestic	32.9 KWh	-	0.43 Kg	427.61	0	109.15	536.76
5	Consumer 5	Domestic	12.8KWh	20 KWh	-	166.01	14.80	0	180.81
6	Consumer 6	Domestic	23.5 KWh	-	0.43 Kg	306.06	0	109.15	415.21
7	Consumer 7	SME	45.8 KWh	-	-	595.75	0	0	595.75
8	Consumer 8	SME	38.1KWh	50 KWh	5.31 Kg	494.87	37.00	1347.92	1879.79
9	Consumer 9	SME	86.3 KWh	-	-	1122.70	0	0	1122.70
10	Consumer 10	Commercial	96.6 KWh	275 KWh	1.67 Kg	1256.20	203.50	423.92	1883.62
11	Consumer 11	Commercial	1720 KWh	-	-	22,360.00	0	0	22,360.00
12	Consumer 12	Commercial	587.1KWh	-	13.0 Kg	7632.30	0	3300.00	10932.30
TOTAL			2693.5	367 KWh	22.35 Kg	35016.57	271.58	5673	40961.6
AVERAGE			224.5	30.6 KWh	1.86 Kg	2918.05	22.63	472.80	3413.47

In Figure 12, the parameters in Table 3 are denoted as; $a_3 = \alpha_3, a_2 = \alpha_2, a_1 = \alpha_1, a_0 = \alpha_0, b_3 = \beta_3, b_2 = \beta_2, b_1 = \beta_1$ and $b_0 = \beta_0$. Using the values on Table 2 and parameters in Table 3, simulation yields the following graph.

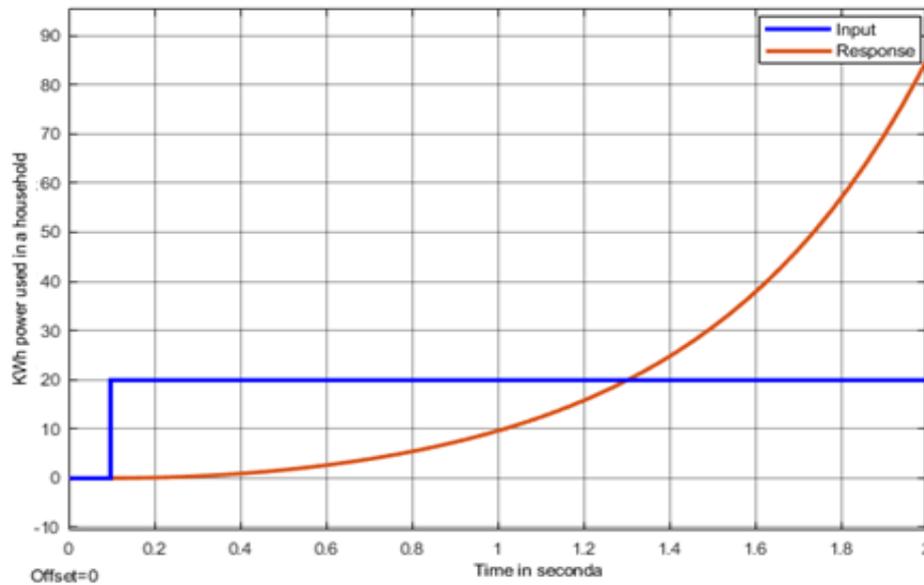


Figure 13: Graph showing newly connected consumer and amount of energy supply

Note that a newly connected energy user with consumption of 20KWh is supplied initially with zero power but within a short time (1.2 seconds), the supply is beyond the demand and continually exceeds the demand. This excess supply is the concern of this research, in the sense that, the amount of supply should approximately coincide with the demand, so as to minimize wastage, and provide additional access to new clients.

In order to achieve this, a Proportional Integral Derivative (PID) controller, is connected to the SIMULINK block diagram to regulate the response function to changes in input. A PID controller is a device used to regulate the response to a step function representing new connection or increase in power consumption appliances. The PID control Simulink block diagram is illustrated below.

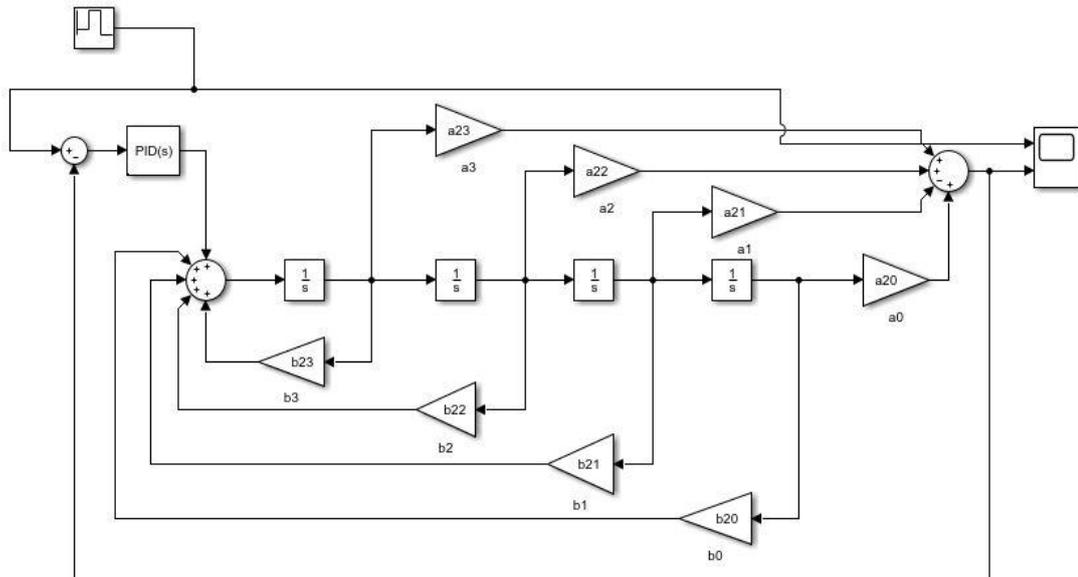


Figure 14: SIMULINK block diagram with PID controller to regulate feedback response supply function

Note that the PID controller is connected using a feedback line from the consumer, to evaluate the error and adjust the input (supply) accordingly, to avoid excess or shortage in the supply of energy.

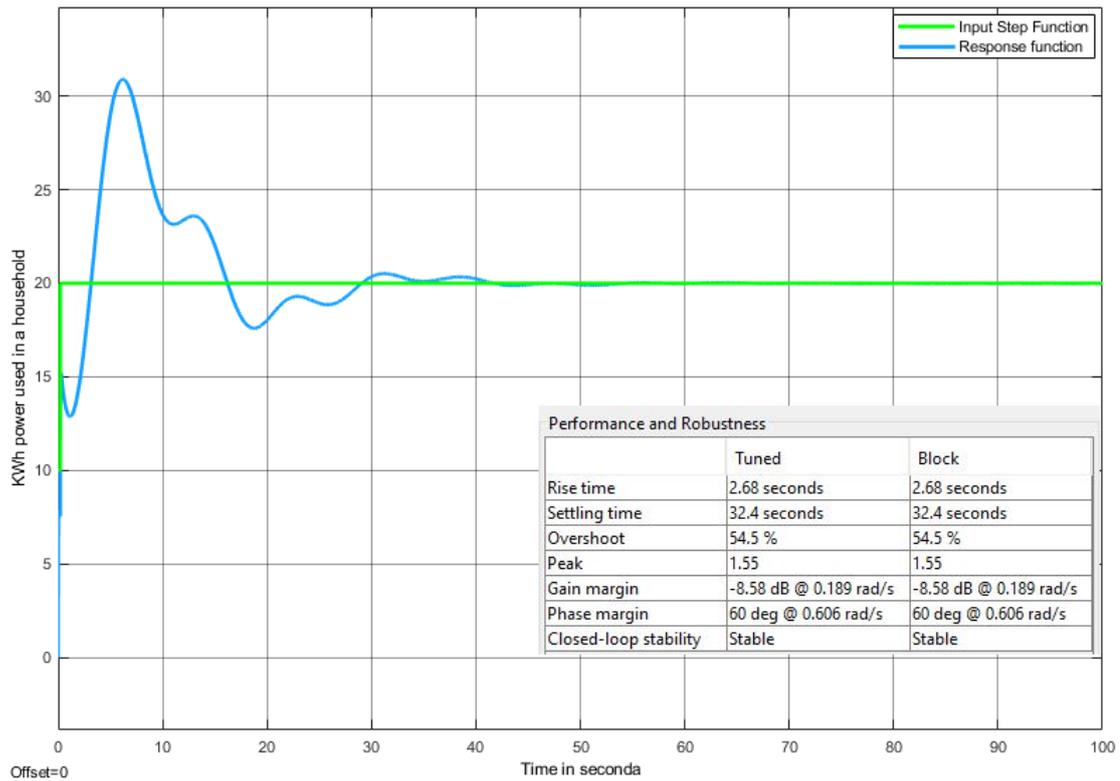


Figure 15: The Simulation response function with PID for newly connected client. Source: Author

It is noted that the response function (energy supply) is not growing exponentially as in Figure 15, but the response is periodic and exponentially converging towards the desired demanded energy.

Important parameters for stability and robustness are mainly settling time and overshoot percentage. It is noted in this figure that the system is stable but initial overshoot runs to 54.5% with a settling time of 32.4 seconds. This implies that appliances with 20KWh rating will be supplied with excess energy of over 54.5%, that is 30.9KW which is an excess. This will result either to overheating and thus damage of appliance or and wastage of energy excessively supplied for 32 seconds.

PID (Proportional-Integral-Derivative) tuning is the process of adjusting the proportional, integral, and derivative gains of a PID controller to achieve the desired

system response. The goal of PID tuning is to optimize the controller's performance, ensuring stability, responsiveness, and minimal steady-state error in regulating the system output (Wang et al., 1999). The tuning process typically involves iteratively adjusting the proportional, integral, and derivative terms based on the system's characteristics, such as its dynamic response, stability margins, and disturbance rejection requirements. Different tuning methods, such as trial-and-error, Ziegler-Nichols, and auto-tuning algorithms, can be employed to systematically adjust the PID gains and achieve the desired control behaviour (Wang et al., 1999). PID tuning is a critical aspect of control system design and is utilized in various applications, including industrial control processes, robotics, and motion control.

The excess overshoot and settling time is achieved through training of the neural network system so as to obtain the optimal values. With several simulation training, the optimal values of responses are achieved when the desired overshoot is below 20% and settling time below 5 seconds, which is the range that most appliances can withstand. The response function results obtained after training is illustrated below.

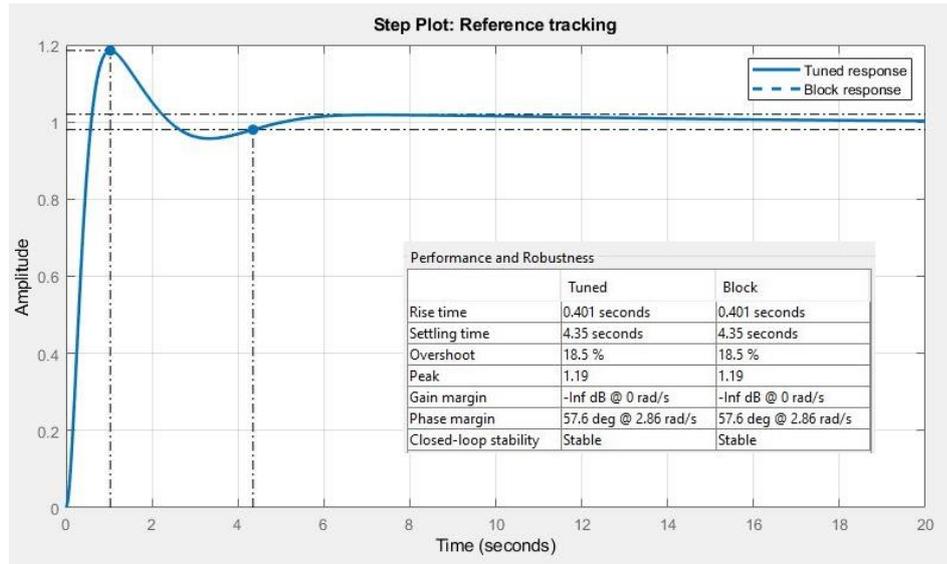


Figure 16: Optimal response function after training. Source: Author

This is the desired response function with stability parameters as rise time of 0.41 seconds, settling time of 4.35 seconds, and an overshoot of 18.5%. This meets the desired threshold limits, and thus the demanded energy and the corresponding supply is illustrated in the figure below. Figure 16 shows the response to changes in the quantity of energy demand, for a consumer with variable needs as time changes.

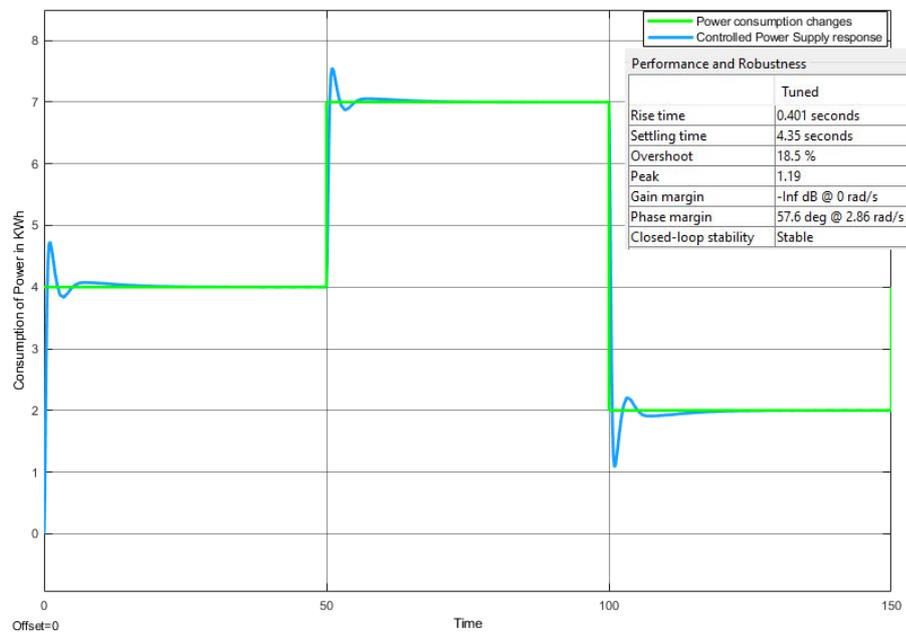


Figure 17: Simulation response of changes in energy demands with optimal settling time and overshoot

Note that from the simulation illustrated in Figure 17, the response of supply due to changes is consistent to the optimal response of settling time of 4.35 seconds and an overshoot of 18.5%, with a rise time of 0.401 seconds. This is within desirable range and cannot harm the appliances, and at the same time limits wastage due to the excess supply of energy.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

This chapter gives a summary of the findings obtained in this research, and recommendations, both for implementation and that for further research. These are presented in three sections, namely; conclusions; recommendations for implementation; and recommendations for further research.

5.1 Conclusion

This research was intended to evaluate the optimal level of energy mix that reduces waste and increases utility of energy, while keeping into account the conservation of the environment. The idea focuses on the combination of energy sources in this case, electricity, solar, and LPG gas to determine the best way to minimize cost. Due to the lack of access to all forms of energy by every individual, the evaluation was approached from the perspective of minimizing waste. The main question was how to align the quantity of energy supply to the level of individual demand.

The process was simulated and analyzed using a smart grid, a device created using neural networks that evaluates the error (the difference between supply and demand) and appropriately gives a feedback signal to the producer to adjust the quantity of supply accordingly. In this research, it was found that an appropriate feedback model can critically dampen oscillations in supply caused by changes in demand, achieving an overshoot of 18.5%, which diminishes to an acceptable tolerance level of less than 2% within 5.35 seconds. These parameters guarantee the stability of the system for either self-regulated or automated control.

The findings also showed that the cheapest source of energy is solar energy, followed by LPG gas, and lastly electricity, at a ratio of 6:129:1 for electricity, solar, and LPG

gas respectively. This implies that a unit of energy from LPG gas can serve six units of electricity or the equivalent of 129 units of solar. This makes solar cheaper by over 136% compared to electricity and over 29% compared to LPG gas. In other words, the cost of producing 129 units of electricity is enough to produce 2,709 units of solar energy or 21 units of LPG gas.

5.2 Recommendation for Implementation

From the findings obtained, it is recommended that finances be channeled to solar power, which was found to be 136% less expensive than electricity. Given the advantage of Kenya's equatorial location, it is more appropriate to invest in solar energy and integrate it into the national grid.

In addition, there is a need to introduce technological advancements into the national grid to transform it into a smart grid with an automated feedback mechanism that appropriately adjusts energy supply to match demand. This will provide timely information for planning production capacity upgrades, while in the meantime maximizing the utilization of available energy to serve as many people as possible.

This transformation can be achieved through the introduction of smart meters, smart sockets, and smart electronic gadgets, combined with a responsive feedback signal mechanism.

5.3 Recommendation for Further Research

This research was limited by the scope of data collection, which did not cover the entire country. Instead, one local village was selected, served by a 1000 kW transformer supplying 12 consumers: 6 domestic, 3 small and medium-scale

entrepreneurs (SMEs), and 3 commercial consumers. Of these, 20% were connected to solar, 30% to LPG gas, and the remaining 50% relied solely on electricity.

It is recommended that more consumers be connected to solar energy. Further research should investigate the optimal energy mix, taking into account the times of day and types of energy to be used. Such an investigation would provide valuable insights into switching frequency and quarterly allocation per area. These findings would, in turn, support strategic planning for the location and connectivity of residential, industrial, and light industrial sectors.

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APPENDICES

Appendix I: Commercial Consumer energy utility

Consumer 1

Learning Institution			
PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Hot water and Cooling	1000	300	13000
IT and Office Equipments	1200	300	15600
Lighting	4200	600	54600
Heating	12213	600	158769
	17613	1500	228969

Consumer 2

Heavy Industries			
PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Hot water and Cooling	3000	300	39000
IT and Office Equipments	3600	300	46800
Lighting	12000	600	156000
Heating	36000	600	468000
	51600	1500	670800

Appendix II: SME Consumer energy demands

A(User_1_January)			
PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	36	60	468
Cookers	0	0	0
Kettle	0	0	0
Refrigerators	0	0	0
Entertainmnet Equipments	72	120	936
Air Conditioners	0	0	0
Laundry/ Dishwashing	0	0	0
Iron Boxes	11	10	143
Water Heaters	3.3	3	42.9
Room Heaters	0	0	0
	86.3	133	1121.9

A (User_1_February)			
PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	34.8	58	452.4
Cookers	195	65	2535
Kettle	0	0	0
Refrigerators	0	0	0
Entertainmnet Equipments	60	100	780
Air Conditioners	0	0	0
Laundry/ Dishwashing	75.6	54	982.8
Iron Boxes	8.8	8	114.4
Water Heaters	2.2	2	28.6
Room Heaters	300	200	3900
	641.6	429	8340.8

A(User_1_March)

PARTICULARS	CONSUMPTION Per Month		
	KWh	Time (Hours)	COST Per Month
Lighting	30	50	390
Cookers	135	45	1755
Kettle	3	2	39
Refrigerators	59	118	767
Entertainmnet Equipments	0	0	0
Air Conditioners	0	0	0
Laundry/ Dishwashing	84	60	1092
Iron Boxes	11	10	143
Water Heaters	0	0	0
Room Heaters	270	180	3510

562	415	7306
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A(User_1_April)

PARTICULARS	CONSUMPTION Per Month		
	KWh	Time (Hours)	COST Per Month
Lighting	36	60	468
Cookers	180	60	2340
Kettle	4.5	3	58.5
Refrigerators	60	120	780
Entertainmnet Equipments	24	40	312
Air Conditioners	350	100	4550
Laundry/ Dishwashing	84	60	1092
Iron Boxes	11	10	143
Water Heaters	3.3	3	42.9
Room Heaters	270	180	3510

986.8	576	12828.4
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Appendix III: Domestic User Energy demands

A (Household_1)			
PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	36	60	468
Cookers	180	60	2340
Kettle	4.5	3	58.5
Refrigerators	60	120	780
Entertainmnet			
Equipments	72	120	936
Air Conditioners	420	120	5460
Laundry/ Dishwashing	84	60	1092
Iron Boxes	11	10	143
Water Heaters	3.3	3	42.9
Room Heaters	540	360	7020
	1374.8	856	17872.4

B (Household_2)			
PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	108	180	1404
Cookers	0	0	0
Kettle	13.5	9	175.5
Refrigerators	300	600	3900
Entertainmnet			
Equipments	60	100	780
Air Conditioners	2520	720	32760
Laundry/ Dishwashing	0	0	0
Iron Boxes	5.5	5	71.5
Water Heaters	0	0	0
Room Heaters	0	0	0
	2899	1434	37687

C (Household_3)			
PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	180	300	2340
Cookers	0	0	0
Kettle	45	30	585
Refrigerators	25	50	325
Entertainmnet			
Equipments	18	30	234
Air Conditioners	0	0	0
Laundry/ Dishwashing	0	0	0
Iron Boxes	3.3	3	42.9
Water Heaters	165	150	2145
Room Heaters	270	180	3510
	706.3	743	9181.9

D (Household_4)

PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	180	300	2340
Cookers	60	20	780
Kettle	0	0	0
Refrigerators	0	0	0
Entertainmnet Equipments	0	0	0
Air Conditioners	0	0	0
Laundry/ Dishwashing	0	0	0
Iron Boxes	6.6	6	85.8
Water Heaters	16.5	15	214.5
Room Heaters	225	150	2925

308.1	191	4005.3
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E (Household_5)

PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	90	150	1170
Cookers	0	0	0
Kettle	0	0	0
Refrigerators	0	0	0
Entertainmnet Equipments	360	600	4680
Air Conditioners	0	0	0
Laundry/ Dishwashing	0	0	0
Iron Boxes	6.6	6	85.8
Water Heaters	16.5	15	214.5
Room Heaters	0	0	0

383.1	621	4980.3
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F (Household_6)

PARTICULARS	CONSUMPTION Per Month KWh	Time (Hours)	COST Per Month
Lighting	102	400	1326
Cookers	5	1	65
Kettle	8	5	104
Refrigerators	12	20	156
Entertainment Equipment	120	400	1560
Air Conditioners	350	100	4550
Laundry/ Dishwashing	180	120	2340
Iron Boxes	12	6	156
Water Heaters	5	15	65
Room Heaters	450	300	5850

1142	967	14846
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Appendix IV: Plagiarism Awareness Certificate

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